

HANDBOOK *of* THE HISTORY OF LOGIC

VOLUME 1

GREEK, INDIAN AND ARABIC LOGIC

Edited by

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CONTENTS

PREFACE

With the present volume, the *Handbook of the History of Logic* makes its first appearance. Members of the research communities in logic, history of logic and philosophy of logic, as well as those in kindred areas such as computer science, artificial intelligence, cognitive psychology, argumentation theory and history of ideas, have long felt the lack of a large and comprehensive history of logic. They have been well-served since the early sixties by William and Martha Kneale's single volume *The Development of Logic*, published by Oxford University Press. But what such a work cannot hope to do, and does not try to do, is provide the depth and detail, as well as the interpretive coverage, that a multi-volume approach makes possible. This is the driving impetus of the *Handbook*, currently projected to run to several large volumes, which the publisher will issue when ready, rather than in strict chronological order. Already in production is the volume *The Rise of Modern Logic: From Leibniz to Frege*. In process are volumes on *Mediaeval and Renaissance Logic*, *The Many-Valued Turn in Logic*, and *British Logic in the Nineteenth Century*. Others will be announced in due course.

As with the present volume, the *Handbook*'s authors have been chosen for their capacity to write authoritative and very substantial chapters on their assigned topics; and they have been given the freedom to develop their own interpretations of things. In a number of cases, chapters are the equivalents of small monographs, and thus offer researchers and other interested readers advantages that only a multi-volume treatment can sustain.

In offering these volumes to the scholarly public, the Editors do so with the conviction that the dominant figures in the already long history of logic are the producers of theories and proponents of views that are possessed of more than antiquarian interest, and are deserving of the philosophical and technical attention of the present-day theorist. The *Handbook* is an earnest of a position developed by the Editors in their Editorial, "Co-operate with you logic ancestors", *Journal of Logic, Language and Information*, 8:iii–v, 1999.

The *Handbook of the History of Logic* aims at being a definitive research work for any member of the relevant research communities. The Editors wish to extend their warmest thanks to the *Handbook's* authors. Thanks are also due and happily given to Jane Spurr in London and Dawn Collins in Lethbridge for their indispensable production assistance, and for invaluable follow-up in Amsterdam to our colleagues at Elsevier, Arjen Sevenster and Andy Deelen. The Editors also acknowledge with gratitude the support of Professor Bhagwan Dua and Professor Christopher Nicol, Deans of Arts and Science, University of Lethbridge, and of Professor Mohan Matthan, Head of Philosophy and Professor Nancy Gallini, Dean of Arts, University of British Columbia. Carol Woods gave the project her able production support in Vancouver and is the further object of our gratitude. The

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LOGIC BEFORE ARISTOTLE: DEVELOPMENT OR BIRTH?

Julius Moravcsik

INTRODUCTION

"What is the origin of logic as a distinct discipline?" is a complex and partly confusing question. It is confusing because some might misinterpret it as asking for a date at which people discovered the difference between sound and unsound reasoning. But presumably people have been thinking logically, at least in relatively simple contexts, since the origin of humanity. Material elements have behaved "physically" much before the rise of physics as a discipline, and people, at times, have argued logically much before the first system of logic was presented. There is a difference between the two cases. Physics did not start with everything "physical" beginning to think about what normal physical functioning is. Humans, with a certain sense of detachment, started raising that question. Reason had to be applied to natural processes in space and time. In the case of logic, however, reason had to be applied to reason. This application required that people reflect on their own thought processes and that of others. This reflection then had to be coupled with separating the art of logical reasoning from other subjects. However, this separation was not like the separating of two natural sciences, e.g., chemistry and biology. "Separation" in our case has two aspects.[1] We need to separate logic from other disciplines dealing with argumentation and communication, such as rhetoric, advertisement generation, and others. But there is also another sense. For in the case of logic we need to bring our reflection on language to a new, higher level of abstraction. We need to consider language, like mathematics, as an abstract system, and then isolate a higher level abstract quality, namely valid and invalid inference discriminability. Language in practice is a series of sounds. We next abstract from that the phonological, syntactic, and semantic elements. We then consider the grammar and semantics, and attempt to impose on some of this logical structure. So we delineate the valid inference patterns. To justify this effort we need to bring in rules of valid inference. But these do not consist of independent elements. Logic emerges when we can relate our rules of inference and present them as a coherent system. Finally, we reflect on the abstract features of such systems in order to understand what logic itself is. Nobody has come up

[1] Netz, R., 1999, *The Shaping of Deduction in Greek Mathematics*, (Cambridge UP), chapter 1.

Handbook of the History of Logic. Volume 1
Dov M. Gabbay and John Woods (Editors)

with an informative and explanatory definition of what logic is. The same holds for mathematics. Logic and mathematics evaluate sequences of elements from a completely detached view. By this we mean that the point of view is not influenced by considerations of utility, pleasure, and other such human interest-relative factors.

It would be misleading to characterize the emergence of the first system of logic as either an invention or a discovery. It is not an invention like artifacts, such as wheel, car, or less concrete, such as an alphabet. On the other hand, it is not like the discovery of a mountain nobody knew about, or a chemical element. There are elements of both invention and discovery in the formulation of a logic. We discover necessary relationships between abstract elements that can be characterized linguistically or conceptually. We have, however, options as to how to characterize these relationships. At that point, inventiveness enters the scene. We cannot credit the discoverer of a river we did not know, with originality. Nor do we credit a logician with originality when he first presents the law of non-contradiction. But we do credit him with originality in view of the particular ways in which he shapes rules for deduction and proofs.

If we think — as we should — of logic as a system of justifiable rules of inference that demarcates the valid from the invalid ones, then we should say that Aristotle was the first creator of logic in Western culture, and that this achievement came spontaneously. As Carl Hempel used to say, it was "a free creation of the human mind". It took a special leap in cognition to arrive at this high level of abstractness, and to formulate the laws of syllogisms the way Aristotle did. On the basis of these considerations one can say that there was no logic in the West before Aristotle, and that emergence was spontaneous, not a matter of gradual development.

Nevertheless, we should not think of the rise of logic as taking place in an intellectual vacuum. Logic presupposes a set of concepts that provide its back ground and elements. These concepts are interrelated.[2] They do not surface isolated from each other. Their respective developments need not be interrelated, but the final products must be linked that way.

In the following we shall trace the developments of some of these concepts. We must not think that once these concepts are parts of a culture, that logic *must* emerge. The conceptual background we will trace constitutes a necessary but not also sufficient background for the kind of ingenious work Aristotle did.

Thus we divide the question: "was there logic before Aristotle?" into two sub-questions. First: "did anyone produce a system of logic comparable to that of Aristotle prior to the work of the Stagirite?" and "what, in any, jumps in level of abstraction, and development of new concepts were required for the rise of logic?" The answer to the first question is negative. To the second question we respond by showing what levels of abstraction were needed, and how concepts developed that formed the needed background for logic. In the next section we turn to a brief sketch of this conceptual background.

[2] *Ibid.* chapter 3.

1 DEVELOPING THE CONCEPTUAL FOUNDATIONS

The conceptual foundation has two aspects. One of these is the development of the needed detachment and objectivity. One can assess arguments in a number of ways. For example, how persuasive these are, how colorfully they are presented, how easy it is to understand them and so on. We need to eliminate such considerations when we assess something as valid or not. Thus we should trace the development of the notions of generality and objectivity. Detailed work in this area is yet to be done. We need to trace the move from "this must be right because the gods say so" to "this must be a sound deduction because each step can be justified by the rules of inference." In general terms, the tracing of this development was started by Bruno Snell, with the felicitous title to one of his chapters, "from myth to logic".[3]

Before we start on the details, it is worth clarifying where we should look for relevant evidence. For example, the Kneales suggests that we should look primarily at texts of geometry and the natural sciences that started around that time.[4] They suggest that literary sources are less likely to contain logically sound texts. But it seems that if we regard the use of logic as a universal human practice that is to be codified, then we should look at literary texts as well There are many arguments in Homer's *Iliad.* The problem lies not so much with the deductive links but with the premisses; clearly a matter outside the province of logic. Many would not accept the premiss that everything that is loved by the gods is necessarily good, but whether the other beliefs that surround this one are consistent with it or not is a matter of logic. Some discussions involve sketching alternatives among which the characters must choose. For example, who should have more prestige and power the tribe, the one of royal blood, or the most successful warrior in the tribe?[5] Some notion of what argument supports what — apart from what one would like to be the case — seems implicit in the exchanges. Such material can be seen as leading to the development of the notion of consistency. Some might be tempted to compare agreements, and see something more general and abstract that these have in common, even if this cannot be yet articulated as logical consistency. For the latter concept to be used, one needs the notion of logical form, more abstract than anything we need in other disciplines except mathematics. We have no idea what enables humans to discern the same "form" in so many arguments with different vocabularies and dealing with different topics, different domains. Some suddenly found the similarities, on this abstract level, puzzling. This puzzlement need be resolved; hence the notion of logical form. To take these conceptual steps may seem to many today easy. This would blind us to the difficulty with which different cultures sooner or later managed to form the right notion. A good comparison would be that with grammar. Logic needs some sort of a grammar for the internal analysis of sentences. Yet the first formal complete logic was formulated only in

[3]Snell, B., 1953 *The Discovery of the Mind*, tr. T. Roseuirneyer, (Harvard UP), p. 9.
[4]Kneale, W. and M., 1962 *The Development of Logic*, (Oxford UP), p. 1.
[5]Homer, *Iliad*, Bk.I., lines 180–280.

120 B.C., quite a bit later than Aristotle's activity.[6] Yet Aristotle does rely on grammatical distinctions and composition rules.

The logical form of a sentence explains some of the semantic features of that unit. And yet the relation between logical deduction and explanatory power is complex. Part of the problem is that there is no complete agreement on what explanatory power in general is. Some think that this can be explained within the formalisms of today's logic,[7] while others deny that.[8] So one could point to the understanding of a mathematical proof as a holistic cognitive phenomenon, going beyond mere understanding of logical relations.

It is safe to say that not all deductive patterns have explanatory power. Our drive towards logic is not based solely on finding sound patterns for explanation. Logic is also involved in human efforts to prove something, to sustain an argument in debate, and to find an unassailable stand which nobody can legitimately attack. There is no one drive for logic. The motivation is pluralistic.

In Aristotle's way of thinking there is clearly a strong link between explanation and demonstration and hence also deduction. All men are mortal, all Greeks are men, therefore all Greeks are mortal. The term "men" carries the key explanatory power in this. configuration. It is through being human, of having human nature? that the Greeks have their mortality.

Yet Aristotle himself points out that the explanatory power does not come from the form alone. As he says, one must select the "middle term" — "human" in our example, very carefully.[9] There will be a number of co-extensive middle term candidates; so how do we choose the "right one"? Aristotle thinks that this is a matter of insight; we need intuitive understanding of what is puzzling us in a context to see what "really" explains.

So explanation and deduction overlap. Some explanations are not deductions, and some deductions are not explanations. But the overlap is important, or at least it seemed to Aristotle the most important part of our rational activities. His predecessors did not link the two, but Aristotle saw a way in which — he hoped — one could.

One of the expressions the development of which led to the notions of evidence and thus also to premiss, is "signs". Signs show in Homer what the gods want, signs suggest happiness or tragedy, signs signal to Priam that he can trust, at the end, Achilles.

Once we trust regularities in nature, signs need not be arbitrary. We use these in order to predict storms and other weather conditions. Interestingly, both of these uses indicate something that will give us — or so they thought — certainty, but for quite different reasons. In one case the certainty is derived from religious thinking. The source of the certain, the assurance, is beyond typical rational

[6] Dyonisius Thrax

[7] Salmon, W., 1989, "Four Decades of Scientific Explanation" in *Minnesota Studies in the Philosophy of Science*, (Minneapolis University of Minnisota Press) pp. 3-196.

[8] Manders, K., "Diagram Contents and Representational Granularity", mimeographed paper, University of Pittsburgh Philosophy Department.

[9] Aristotle, *Post. An.* Bk II.

understanding. In the other case the source is the regular observation of general truths in nature. In modern times we link the predictability of rain, storm and other such phenomena to probabilistic reasoning, but it was not construed that way in ancient times. This turned out well in a way, for in this development kinds of certainty paved the path for forging the necessity - and correlated One might think that the sciences that promise only probability developed first, and the ones bringing certainty only later. But exactly the opposite is the case. Mathematics and geometry had their early flowering and the "natural' sciences only later.

One might think that the development was "from probability to certainty", but in fact the reverse took place. Humans reach for what promises certainty and settle for the probable only when the methodology and selection of the right domain of objects is not at hand.

We shall now look into the developments, using Bruno Snell's work, the title of which we cited already. In the light of the last paragraph we can see now myth and logic not only as *terminus ab quo* and *terminus ad quem*, but also as sharing an important characteristic, namely the promise of certainty.

There is not enough evidence to trace out the separation between the two sources of certainty. In many countries the two live side by side, and in Plato's *Timaeus* the rough equivalent to some of what are today natural sciences is introduced within a religious framework. One can speculate about how questioning of a well known sort, namely, going from the particular to the general helped to sort out different kinds of certainty. In the one case, people presumably started to ask questions about the reliability of specific divine commands and alleged forecasts. But after that they could ask questions about the nature of these pronouncements in general. For example, an important critic, Plato, treats them as a group, but here generality is not available. Divine orders, as also divinities, remain particular. On the other hand, after we agree on specific rules of deductive inference for some patterns, we can ask for more general justifications, and a system of logic will provide this for us.

In our tracing the development of what was necessary background for logic we will side-step the question of whether at earlier stages logic was seen as necessary rules of thought, or necessary rules about how elements of reality function, or an autonomous discipline dealing with its own unique domain. For these are metaphysical questions, and do not touch on the nature of early systems of logic as general theories about validity. Aristotle's work suggests that he saw logic as having both metaphysical grounding and reflecting necessary features of thought.[10] It is reasonable to suppose that the first clear examples of demonstration and proof in the Greek world came from geometry, or rather what was then the combined subject of mathematics and geometry. Here abstraction and rigor combine and reach the same high level that logic does.[11] Nevertheless, we must not make the mistake of thinking that logic grew out of the practice of geometry and mathematics. For one can conceptualize geometrical demonstrations as limited to the

[10] Aristotle *Metaphysics*, Bk. Gamma.
[11] Netz, R. *Op.cit.*, in general.

particular domain of mathematical and geometrical entities. "He need not look at reason having a universal domain in order to do rigorous geometry. This does not mean that geometry might not have influenced Aristotle in his construction of logic. In fact, there are signs suggesting that Aristotle had an independent conception of logic, but wanted logical demonstrations to be in some ways analogous to geometrical demonstrations.[12]

Thus we can see that neither grammar nor geometry should be interpreted as the forerunners of logic. As we say, the first grammar was constructed by Dyonisius Thrax around 120 B.C. — quite a bit after Aristotle's logic. But apart from the temporal issues, we can see why neither of these subjects provide all that logic presupposes. Geometry does not because its methodology is not sufficiently general, and grammar not, because though it moves on the required level of abstraction, and is sufficiently general, it lacks the rigor at least in earlier times that logic requires.

In the case of geometry the interest in explanatory power and in demonstration comes happily together. Furthermore, geometry can be seen also as a paradigm for at least some types of deductive reasoning.

In summary, then, we can interpret the three interests, in explanatory power, in proof and deductive reasoning and argument assessment, as stimuli for the development of that set of concepts. Thus the vocabulary that is necessary for a system of logic came into being. Analagously we can speculate that the combination of these interests would motivate people to work toward the development of the logical concepts and vocabulary. Thus the three interests can be seen as underlying logic, and psychologically, as underlying human efforts, conscious or otherwise for formulating logic.

2 CONCEPTS AND VOCABULARY PRESUPPOSED BY LOGIC

We have sketched the salient interests that would lead people towards constructing a system of rules generating and assessing logical validity. We now turn to the vocabulary that logic requires.

First we examine the notion of truth. Truth is clearly needed, for without it we could not articulate notions like premiss, conclusion, and consequence. We must assume that grammar already provided the notion of a sentence, and thus we can understand the way in which truth is attached primarily to sentences. Truth is also what we need as a contrast to falsehood, a notion to be discussed later.

Presumably some notion of truth existed since the dawn of human history. It may not have been separated from some general notion of what it is to describe something correctly. Furthermore, in its early forms truth was not separated from what is true in evaluative ways. Something can be a true or genuine diamond, friend, a true alumnus, a genuine Egyptian artifact, and so on. The evaluative aspect emerges in contexts in which we wonder whether to apply this term to a

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[12] Aristotle, *Prior Analytics*.

friend or mere well-wisher. One can only speculate on whether there was a notion of a true sentence as a truly genuine real sentence, i.e. one that did its job and gave a good representation of a part of reality in which we are interested.

In any case, we need to abstract various aspect of this notion of true F, or real F, in order to work towards forging the truth that logic wants. First, we need to take away the positive evaluative aspect. A true proposition or sentence may be bad news, or describing evil doings. Secondly, we need to change the gradational aspect of truth into a non-gradational one. What is genuine can be a matter of degrees, and the same holds for a true friend or true spring weather. But a sentence cannot be really true or not really true, or just half-true. If it were of that sort, it could not do the job that logic demands of it.

We need now a further level of abstraction. We use descriptions as good *for* a certain community. Descriptions function in contexts and with qualifications. Some of these are relational, others introduce pragmatic contexts of description. All of this applies especially to nouns designating artifacts like 'table'. How much damage can an object endure and still qualify to be a table? What is a table for a certain community need not serve as such for others. But when the word 'table' occurs in a logical construction we abstract from all of this. Either there is a table or not, and either it functions appropriately in an intended premiss like "all tables are..." or should be replaced with another equally context and gradation independent term (noun, verb, adjective).

We are still not quite finished with our account of "assent to truth as used in logic". Plato has a characterization of truth in the *Sophist*,[13] and there are indications that the formula comes from earlier times. "the true sentence expresses things that are, *as they are*". The emphasis of the sentence being about and describing real things need be taken away if we are to see "true" as a purely logical notion. But apart from that, there is the promising but troublesome expression" as these are". We need to give this an interpretation that transcends the differences between philosophical theories of truth, such as correspondence, coherence, pragmatic, redundancy etc, "theories". As Tarski noted the logician's notion of truth is independent of all of this. Furthermore, it is a notion that can characterize sentences in systems the domain of which may turn out later to be seen as illusory.[14]

Plato knows that he is not offering a reductionist definition of truth. Nor did Aristotle attempt such an account. Once we reached beyond all of the abstractions listed above, we van only say: "what is left" is the truth required for logical constructions and inferences.

It is interesting to ponder the two very different views that emerged concerning this "ascent". According to one view, Platonic in origin, we "purify" language and our concept of truth as we reach the level of abstraction needed for logic. Purification is no longer a much used concept, but the modern term idealization

[13] Plato, *Sophist*, 263b c 1936.

[14] Tarski, A. "The Concept of Truth in Formalized Languages", in *Logic, Semantics and Metamathematics*, 1956. pp. 152 278 (Oxford: Clarendon Press).

will do just as well. According to the alternative view, we oversimplify meanings and deprive the term of all of its richness when we restrict it to the use needed for logic. On the one hand, one can argue that without the restriction no logic, no great expressive power. On the other hand, one can argue also that with the abstractions we lose a lot of the flexibility, metaphoric power, simple, and other such literary devices that enrich languages so much. It is quite wrong to think of these devices as just decorative elements. They carry meaning, help to think about the more indeterminate aspects of what we talk about, and are very important as vehicles for gradual changes of meaning for words either in scientific or everyday or literary contexts. It is an interesting peculiarity of natural languages that we cannot have it " both ways". Thus in our uses of language we choose to stress in some contexts this and in others that aspect of meaning.

It is natural for us to turn now to another important concept of language without which logic cannot be conceived, namely that of negation and falsehood. These are distinct concepts, but at times their extensions overlap. The notion of truth makes no sense without a notion of falsehood. Falsehood could have well originated in connection with normative notions like honesty. In Sophocles' *Philoctetes*, Odysseus is trying to persuade the young Neoptolemus to lie to Philoctetes. Lying must carry, at least implicitly, the notion of falsehood, for presumably to lie is not to tell the truth; "the way real things actually". Successful prediction is also an ancient notion, whether in connection with the diumation of priests or weather forecasts (these two might overlap). So we can look at various practices such as being honest, being good at forecasting weather, or not, and from such notions abstract the notion of falsehood. The failures of practices like the ones mentioned would — on detached analysis — yield the notion of falsehood. It is impossible for us to construct what would be by even lax standards reasonable hypotheses as to when these abstractions became explicitly the objects of cogitation. Falsehood must have been an essential ingredient in the conceptual framework within which mathematics and geometry were practiced as sciences and so conceived consciously by the practitioner. But even so, it takes an additional step of abstraction and generalization to extend the notion of falsehood to assessment of descriptive speech in general.

As we turn to negation, we must draw an important line between that notion and some others following in our discussion, and truth and falsehood. For truth and falsehood are not conceived at any stage in history as forces of nature. These are not metaphysical concepts. But negativity and one thing following another are. (If so the one points out that much later for Frege the True and the False are objects, one should separate this purely abstract notion of objectification, installed as a part of a highly abstract system of semantics from the kind of metaphysical or natural posits about which we talk here.)

In tracing the notion of negation, we should start with the notion of opposites, in particular opposing natural forces like fire and water (Heraclitus), or moistness and dryness. Was their incompatibility construed as necessary and *à priori*, or as just an extreme case of clashing natural forces, is an unanswerable question.

From the notion of opposing natural forces to the logical notion of negation, one has to climb a long and steep road. First, opposites need not exclude each other completely. The weather maybe "stormy" with what the British call so characteristically — "sunny intervals". Or it could be between stormy and calm. With hot and cold, a thing can be hot in some respect, and not hot in others.

By the time we see opposition illustrated in Plato, we come to examples like tall and short. These are what are called later qualities. Furthermore, semantically they behave like adverbials. A tall monkey may be a short jungle-living animal. The distinction between natural forces and the more quality-oriented classification is not an all-or-nothing affair. What about light and darkness? Whether one regards these as forces or not, depends on one's physics, rudimentary as this may be.

Plato wrestles a lot with the "negative". His thinking about negation is strongly influenced by the Parmenidean attack of this as not-being. This explains also why his first attempt of characterization not in terms of sentence-negation, but predicate negation. He is anxious to point out.[15] that the not-fine is as much a part of reality as the fine. Fine things and not-fine things are different, but both existing. Furthermore, the difference is a special "contrast" that Plato does not define any further. Nor has anyone since. Hence the problem is swept under the rug. In what consists the negativity of the not-fine? Otherwise? This is ???.

One cannot insist that we arrive at a completely adequate account of negation and falsity only when e.g., negation is completely divorced from metaphysics. For example, a constructive step forward is the realization that the completely negative (predicationally) entity is conceptually impossible. So the negative concerns always some aspect of what we talk about. With negation we indicate in a unique way that what we talk about is different from that which the corresponding positive description would represent. The early treatments of negation in metaphysical and ontological terms is responsible for the early concentration on predicate negation, before subsequent consideration of what is logically prior, namely sentence negation.

Some might regard it as a step forward when negation is considered a purely logical operation of contrasting what is taken in a language or system as positive with the negative. But Aristotle himself can be certainly described of having a system of logic. Yet he could assign in the case of predicate negation an important difference between the positive and the negative. According to his view a negative predicate, e.g., not-human, was indefinite.[16] Given what we know about Aristotle's views about unities of predicates and their significance, we can represent this Aristotelian view in the following terms. A positive predicate like "is a human" (or in term logic "human") has a unity that is seen by considering the conceptually related principle of individuation; if we understand what 'human' is then w also understand what it is to be 1, 2, 3, etc. human(s), even if in some concrete

[15] Plato, *Sophist*, 258 259.

[16] For a discussion of this notion see Thompson, M. "On Aristotle's Square of Opposition", *Philosophical Review*, 1953 pp. 251 265.

context the counting is difficult to carry out; e.g. battle fields. But the predicate "not-human" has no individuation principle attached to it. In principle we should be able to answer the question: "how many humans in this room?" but there is no correct answer in principle to the question: "how many not-humans in this room?" We can count the not-human in an infinite number of ways, each equally good or bad. In this respect positive and negative predicates differ. (This does not interfere with the logical operation of double negation yielding a positive.)

We shall now turn to the last of the pillars that is needed in order to have a conceptual framework within which logic can be conceived. This is the notion of "p following from q" where 'p' and 'q' represent descriptive sentences. In short, we need the notion of logical consequence. How does language build up this notion? We shall show in terms of the key words used gradual emergence in ancient Greek of the needed notion.

First we shall consider the word "akolouthein".

In its earlier less abstract uses it means following someone in a general physical sense; for example, soldiers following others in rows. The stress from our point of view is not merely the concrete domain of application, but also the element of *order* associated with the term. When armies are set up, soldiers and their rows are occupying designated places, and traverse designated routes. Thus it is not surprising that we find also usages in which the word denotes natural phenomena following each other such as cloudy sky followed by rain.[17] Here the notion of order has more force. The "following" is a matter of the laws/regularities of nature.

We find also uses in which the word stands for guidance and obedience. In each of these cases, the key force is not just sequencing, but things following each other because of natural order or rational human order (the wiser, or in position, demanding obedience from the lesser.)

Finally meaning is raised to an abstract level, and our word designates sequence in argument. This is abstract but too wide and not sufficiently structured. Once the final stone in the diadem: x following logically from y. In other words, the notion of logical consequence. Hence a key cog in justifying inferences.

The other expression Aristotle uses in this connection is "sumbainein". This means originally "standing with feet together". But other senses emerge, such as joining something, and come to agreement. Thus in this case too we see both movement towards abstract levels and differentiation of ingredients. Things are joined according to a certain order, and their "agreement" signifies harmony of elements. We see also the use of this word for consequence, and necessarily joining things. Eventually the word denotes inevitable sequences, and thus becomes a fine vehicle for Aristotle to designate logical consequence.

We have, then, key ingredients in the conceptual framework within which Aristotle's syllogistic logic was formulated. We turn now to a basic notion absolutely necessary for explicating logical relations, namely predicates or terms (for our purposes we will not need to make here fine distinctions.) The basic structure of logical formulae in modern symbolic logic is the same as in the logic of Aristotle.

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[17] Snell, *Op.cit.*, p.212.

In the premisses and conclusions relations between predicates or terms are represented. First let us see on what level of abstraction we need to construe these terms. In a sentence like "All A's are B's" the A and the B need be taken as having potentially: Universal application; must be independent of subject matter; and should be precisely delineated, without polysemy or ambiguity.

We need to deal with two further factors, if only to lay these aside: First, it is not relevant to their employment in schemes of the sort just indicated that our specification or of the content of the terms is in most cases dictated by human interest, bias of our perceptual system, etc. One can reason logically with pure concepts, detached from human interest and with ones reflecting bias. This will not affect what is called logical form. Secondly, this characterization of terms/predicates is neutral with regard to ontology. The Platonist and the nominalist will have to present both an interpretation of "A" and "B" that allows these expressions to figure in the purely logical characterizations of "some A" or "no B".

All of this may sound trivial to a philosophic or mathematical audience, but we must make a real effort to try to imagine a world of ideas in which these levels of abstractions are not yet present and are not picked up by adequate vocabulary. And yet, it is important to stress again that we are not accusing the first humans to have had a materialist bias. Rather, they used languages in which many distinctions fundamental to the delineation of logic have not yet been made. The fact that the abstract has not been separated from the non-abstract does not mean that each relevant word had only concrete entities in its domain of designation.

Undoubtedly, there are many ways of sketching speculatively the development of the terms of logic. In the following, we rely heavily on the scholarly work of Bruno Snell.

Any natural language with sufficient communicative power to serve as describing reasonably vast areas of reality must contains words of general power, and hence words describing things with oversimplifications. Thus the meanings of 'lion' or 'horse' ignore all of the specific differences between specimens within the respective species. This is governed by two conditions. First, separate words for every qualitative difference between specimens would create languages with absurdly large vocabularies. Secondly, the ignoring of specific differences is dictated by the needs of the projected linguistic community. As is well known, some of the tribes in northern regions have many words for different kinds of snow. This is because these differences play roles in the securing of practical necessities in their daily life. Thus modes of life-style at times push towards oversimiplifications and at other times towards generating many senses for the same word.

Snell discusses in the early parts of his "from myth to logic" natural kind terms like 'lion' and 'horse'.[18] That does not mean that he thinks of these as the earliest words, but that these are the kind of word (noun) that plays crucial roles in the development of the notion of predicate. Brief reflection should show us why this is the case. With kind-nouns like "lion" and "horse", application is an either-or proposition. We will not say things like "a kind of lionish thing", or "more or less of

[18] *Ibid.* pp. 201 207.

a horse" (mythical entities excluded). But when we turn to verbs we find a different situation. Is someone walking? There are clear cases showing the affirmative, and on the other hand, clear cases of being in a stationary position thus deserving the negative. But in between there are many cases that can be interpreted either way. Furthermore, there are different criteria for walking depending whether it is supposed to be an event of walking for a baby, a normal healthy adult, a recovering patient, or a moon-walker. Similar cases can be shown for most other verbs raining, singing, dividing, melting, boiling, etc.

In order to use verbs as predicates or subjects in syllogistic logic we must idealize, and abstract away from in-between cases and relativity to subject. At the same time, we must admit that verbs with their cases and argument places provide most of the structure of the sentence. Thus what is fundamental from the point of view of constructing a logic is not fundamental from the point of view of explicating what gives the sentence its basic syntactic and thus also partial semantic structure. These reflections should not lead us to the conclusion that verbs are intrinsically vague and plagued by polysemy, and cannot be sharpened in their meanings to serve as terms in syllogistic reasoning. The point is, however, that we must do a lot of abstracting before they can serve this way. In the case of the types of nouns we considered we need to abstract away from differences between individual specimens of a species, but these are all on the most elementary level of abstraction, and easy to ignore.

We do find verbal meanings sharpened in certain contexts in which the verb with many others is used within a certain practice, or applied jointly with others to illuminate a certain domain of entities. For example, in mathematics, there is no room for considering many senses of 'divide'. The relevant meaning is that which we use in connection with the mathematical notion of 'division. (The same holds for 'addition', subtraction', etc.) Thus we can say that one should look for certain disciplines or domains with respect to which certain words, or even word families, need to have sharpened meanings. Such might be: mathematics, geometry, chemistry, and other sciences, or applied fields of the sort we today call engineering. But we should not think of precision limited to the sciences. Certain financial transactions and other economic exchanges, once precise units of what serves in the exchange are determined, require also precise meanings (i.e. sharply delineated (ones). So one might conjecture that we build logic once we have become accustomed to think within certain fields very precisely. But this might not be right. For what logic requires is precisely to think universally, to understand logical form regardless of subject matter. This contrasts with the restricted rigorous thinking of geometry or certain parts of economics. Logic is a set of rules of inference which we judge to be within valid patterns, regardless of what the "A's" and "B's" stand for.

Let us, then return to consider the development of other parts of the vocabulary, and the devices with which we can raise the semantic content, and thus the thinking with these concepts, to higher levels of abstraction.

It might be thought that the kind concepts we form are based on resemblences

we notice. Since there is an infinite number of these in any context, we should add "salient" to this proposal. But even with the emmandation it seems weak. For what unites our concepts of both human creations and many natural kinds that are in some way related to human interest is their *functional* aspects. We call many things that are quite different in shape, material constituents, and aesthetic looks houses, because these buildings offer shelter and a place where what in a specific context can be regarded as suitable for carrying out usual human functions can indeed be carried out.

The problem is that the more functional a concept is, the more likely it is to be subject to vagueness and polysemy. It is no surprise that Aristotle thought geometry to be providing good examples of deduction. Abstract generality, no vagueness, are inherent in geometrical notions. The problem we are discussing is how to carry out high level abstractions in order to have all or most parts of a natural language be capable of producing elements that can functions as terms in syllogisms.

There are some kinds of cases where Snell is right and resemblance rules, but that is because we supplement similarity with a "built-in similarity space", as Quine pointed out.[19]

In the case of terms like 'horse' and 'lion', we are in a fortunate semantic situation. On the one hand, the terms by themselves can function well in logical inferences, once we abstract the layers of meaning relating to human interest and differences between specimens. On the other hand, from an early stage of development on, we used these nouns with the equivalents of '...like' (as in 'lion-like') to set up what seemed to the linguistic community salient similarities, with the exact nature of the similarity left open. In this way on could use solid "noun-blocks" and with the additions give flexibility, room for further development, and beauty to language. Consider "white as snow", or "sweet as honey", used in Homeric literature. Maybe if they had discovered something sweeter than honey, they would have used another comparison to describe the objects under consideration. So the development of the descriptive vocabulary moves on from comparisons to simile to metaphor, and that, in turn can become "calcified" and turn into a noun or adjective with precise meaning. Much technical vocabulary develops this way, also in philosophy. E.g., Plato uses the Greek equivalent to "partaking" to mark the unique indefinable relation between Forms and what modern philosophy calls instances. What starts as a metaphor becomes a part of Plato's technical vocabulary.

Another important way in which abstract thinking develops even though we do not tend to think of it this way is by analogies which, when sharpened, can be restated in terms of proportions. We live in cultures in which basic units of measurement for length, area, time, and three-dimensional content are taken for granted. This separates us sharply from the Greek culture in which such exact units were not available for a long time. Thus the basic notion for measurement was proportion. For example, - though it is hard for us to fathom this - the

[19]Quine W.V., *Word and Object*, (John Wiley, New York) 1960. p. 83.

Parthenon was built using constantly proportional instructions for building.

In order to raise this mode of thinking to how we would understand mathematics proper, and with this the production of the mathematics of exact measurement, we need to focus on how proportions can be raised from the purely empirical to the mathematized.

Not all proportional thinking can be handled in this way. For example, as Snell notes, in the *Gorgias* Plato set up the following proportion.[20] Rhetoric is to philosophy as cooking is to medicine. The proportional statement is articulated in order to shed light on philosophy. Of course, in this form whatever conception of philosophy emerges, it cannot be used in syllogistic thinking. But the *problem* of making the characterization formulated by the proportional statement explicit and sharp, fuels the mind sooner or later to try to characterize all of the terms in this statement in explicit and unambiguous ways, so that Plato's conception of philosophy could play roles in further deductive arguments. One might think that in order to shape an acceptable vocabulary for logic, one must take as fundamental those attributes that are quantitative in character. But this does not follow. The concepts must have principles of individuation. We understand what 'horse' means when we know how to count horses. But that does not make the meaning of 'horse' quantitative in the sense in which what linguists call "mass terms", such as terms for colour, smell, weight, etc., quantitative.

These reflections show how much work must go into reaching the level of abstraction at which the relevant concepts and terms needed as the background for logic, can emerge. As said before, merely having the vocabulary for logic does not necessitate the emergence of a system of logic. But in this case a drive for more generality in explanations, uniting criteria for good proofs, arguments, and some kinds of explanations, forces the mind also towards higher levels of abstractness. Hence the idea of a system of logic in which the rules form nets of justification, and subsequently, the reflections of what different systems of logic might have in common. In this way we arrive at the notion of theory of logic. I do not think that Aristotle reached this level, but dealing with that thorny issue is someone else's job.

Before we leave this developmental sketch we need to say something about quantifiers. These are essential parts of the logical vocabulary. On the other hand, the needed words do come from ordinary language, and the changes needed for these to turn into technical expression are not formidable. Let us take the existential quantifier "some". Most of our everyday uses of this term are contextual; some students, some luck, some countries, etc. But even common sense flirts with the use of 'some' that is designed to cover all of reality, such as the claim: "there are some things money cannot buy." But note that this use requires generality only. It does not require a leap to a higher level of abstractness. One might insist that the logical use of 'some' entails thinking of the range of application all possible entities, abstract or concrete. But such a range is neither officially acknowledged nor excluded from the everyday use.

[20] Snell, *Op.cit.*, p.221.

We find the same situation with regard to the universal quantifier. Zeus is described in one of the Greek dramas as "the all-conquering" one. This hardly specifies in a sharp way what 'all' covers, nor does it rule out anything. To obtain the logical sense all we need to do is consider the religious sense of 'all' as in mythology, and sharpen the delineation of the domain. In this interesting way, one can think of aspects of religion as preparing the way to logical vocabulary.

After this positive sketch of how the background of logic requires development, even if the first system of logic does not, we will turn to devices that some historians have wrongly taken to be forerunners of logic.

3 LOGIC AND DEFINITIONS

In their influential book on the history of logic the Kneales say about Plato that "he is undoubtedly the first great thinker in the field of the philosophy of logic. He treats... three important questions that arise as soon as we begin to reflect on the nature of logic...", and he thinks one of these is "what is the nature of definition, and what is it that we define?"[21] In this brief section I would like to show that this view is mistaken, and that in fact, whatever questions one can raise about definitions, the notion of definition itself is independent from logic. We can think about the concepts we need for the construction of a system of logic and construct a system of logic, without bringing in the notion of definitions at all.

As our deliberations above have shown, what we require for logic is: clear, precise, well understood terms, without spatio-temporal or pragmatic relativity. Providing definitions for the terms figuring in a syllogistic inference may be at times a good means to achieve the above presupposition, but it is not a necessary means, and at times even if we have definition their availability does not entail that the presupposition is met.

The myth that there are serious links between logic and definition is likely to have a historical origin. The Kneales report that Aristotle thought Plato to be much interested in definitions[22] and that this in turn led Aristotle to think much about definitions in relation to logic. This historical reconstruction rests on shaky grounds. Historians tend to confuse two questions: "was Plato interested in answering 'what is *F*' questions" where '*F*' stands for what we would call today an important property such as justice, friendship, insight, etc.? and "was Plato primarily interested in finding definitions for important qualities, or properties, be these in the ethical or mathematical realm?" (These were not as separate for him than for the modern world, for reasons into which we cannot enter here.) In my case, the answer is affirmative to the first question and negative to the second.

There are many ways to answer a question of the form: "what is it to be a positive integer? or "what is justice?" For one, a philosopher might give a variety

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[21] Kneale, W. and M., *Op.cit.*, p.17. 22).

[22] *Ibid.*, p. 21.

of unique and necessary descriptions of one of these concepts, without being able, or feel the need for, defining them. There are also many ways of leading an audience to understand a basic notion without defining it. An obvious example is 'language'. There is no general definition for this word, and yet linguists and philologists manage quite well (making progress in their disciplines, relying on a common understanding within the profession of what language, and a language is). Plato never defines 'number', and yet reading the relevant texts one comes to grasp what Plato's notion of number was. At other times a notion is basic and primitive, or undefinable in a system, but we come to understand it by considering its use, and thus how it plays roles in a variety of types of sentences.

The Kneales stress that for Plato it was possible to come up with definitions that are non-arbitrary and informative. But this shows once more how the questions about definitions are independent from those concerning logic. As long as the presupposition stated above is met, whether the terms used in syllogisms are arbitrary or not, and whether their connections are trivial or informative does not affect logic at all. One can construct logics while believing that all definitions are always arbitrary. Then again, one can do logic with different assumptions. These issues do not affect the task of proposing rules of valid inference over a domain and a set of sentences.

What the Kneales might have had in mind, and what should be said, is that the issue of arbitrariness will affect, not the potential to generate a logic, but the potential of the definitions used to introduce terms to have explanatory power. But as we saw earlier, the question of what is a logic and what has explanatory power are distinct, though in results there can be an overlap. Some explanations can be phrased in terms of inferences, and some, but not all, can have explanatory power. E.g., "the larger the state, the more likely a high level of corruption; so-and-so is a large state; therefore there is a likelihood of high level of corruption."

Perhaps those insisting on conceptual connections between definitions and logic have in mind that both involve sketching links between concepts. A definition typically has the form: "$A = B + C$". Thus we "carve up" a concept, and hope that B and C are sufficiently clear, and thus some illumination is fostered by this scheme. Likewise, a syllogistic argument reveals certain conceptual relations. But this is a very thin common denominator. Many other investigations and enterprises also reveal conceptual relations. For example mere classificatory systems, or drawing contrasts. Furthermore, the ability to provide adequate definitions does not entail having also the ability to *reflect* on the nature of definitions and definition giving. My claim is here, in any case, that neither of these intellectual exercises has anything significant to do with logic.

This defence of the sudden emergence of logic and its preconditions does not say that once we have a logic, it cannot be expanded. Surely, that is the way to construe the relation between syllogistic logic and modern symbolic logic. The latter does not replace the former, but provides a larger framework on the same level of abstractness within which the data syllogistic logic explains can be explained along with lot more.

A few more reflections on definitions. Some of these may be covering a "closed" domain, but others do not. Perhaps an account of the series of positive integers in terms of a starting point and the successor function will generate just the right class. But consider the definition of "vehicles for transportation". Given changes in technology and modes of transportation, there is no way of delineating a sharply specifiable class as the extension that this term and its meaning gather up. Many definitions are of this sort. E.g., 'cooking'. Whatever definition we generate, will it cover cooking with a microwave oven? And if it does. then why not just any mixing of edibles, e.g., cereal with yogurt? Currently we think that cooking should involve heating and some transformation of substance; but modes of heating and degrees as well as kind of transformation of substance need be left open, given developments of skill and technology.

In summary, definition is neither a necessary nor sufficient condition for the kind of vocabulary within which a system of syllogistic logic can be formulated. Perhaps, the historians' misjudgments arise from a confusion between what it takes to have a logic and what it take to have an axiomatic system capturing a theory, e.g. Euclidean geometry. The second enterprise demands a lot more than the first. We can reflect on validity and have a logic for a set of sentences without having an axiomatic system. And as was stressed earlier, having a rigorous way of dealing with one specific domain, e.g. geometry, does not yet indicate that the researchers have a conception of logical analysis completely regardless of any particular domain.

To conclude this section, let us consider some of the things that Aristotle says about definition, and see if these texts suggest any historical dependence of thinking about logic.

In *Post. An.* Bk II # 7, Aristotle considers relationships between definition, demonstration and essence. His conclusion is that one cannot demonstrate essences by definition. His basic point is that a definition is merely a picture of a configuration of conceptual relations. Thus it is an articulation, but not a demonstration. In order for it to be a demonstration it would have to contain *inferences*. But a definition is not a series of inferences, even if its ingredients could — in some other contexts — serve as terms in syllogistic arguments. Furthermore, definitions cannot prove existence, but deductions with explanatory power must end up with conclusions about conceptual relations among ingredients that exist. The tone of this section does not in any way suggest that Aristotle thought of logical inferences as a result of thinking about definitions. On the contrary, the comments about definitions seem to be directed to those who did think that there was a close relationship, perhaps even development. Aristotle's comparison stresses the differences, and not the surface similarities.

The discussion continues and in # 10 new light is shed of the relationship between definitions and syllogistic deductions. In 94a ff. he concentrates on "real definitions". Unlike the modern interpreters of the real-nominal distinction, for Aristotle the key issue is that the real definitions carry existential import. In *this* sense we define kinds that exist in nature and not just verbal expressions. He then

explains what definitions are by contrasting these with deductions. The English "demonstration" is sufficiently polysemous so that one can see what disturbed Aristotle's audience. On can think of a demonstration is merely drawing a picture of conceptual relations, or as a dynamic process that draws out of material initially understood other material that we did not think was contained in the premisses.

Deductions and inferences are not static pictures. These represent dynamic processes in which new information comes to light, even though we also learn that the information was already potentially in the premisses. In this way inferences are not subject to what was called much later "the paradox of definitions" ("if the definition is informative, it cannot be right; if it just repeats what we knew already, it is trivial".) So in definitions, no inferences; and if no inferences, no logic. Aristotle seems to have worked out the theory of what a logic is quite independently of definitions; and in the passages briefly surveyed he contrasts deductions with definitions, to show how the former are instructive and informative in ways in which the latter cannot be, though they may be useful for other purposes.

4 LOGIC AND THE METHOD OF DIVISION

The Kneales say that Plato's Method of Division must have influenced Aristotle's thinking about logic.[23] Others too have seen in the past links between the Platonic method and Aristotle's logic. We can divide the question into two parts: first, is there any conceptual link between the Divisions and syllogistic logic, and second, is there any evidence that Aristotle thought there to be such links? The following will support negative answers to both of these questions.

Plato's Method of Division is introduced in *Phaedrus* 265d-e. In this passage the Method is introduced both as collecting the right elements under the appropriate genera, and as the correct way of dividing generic Forms into the "right" species. In places Plato construes the project as that of coming up with "right namings". Clearly any generic Form can be divided into species in an infinite number of ways. But Plato thinks that some of these are better at reflecting real differences and similarities between Forms than others. As he says in the *Politicus*, we must cut "along the right joints".[24] There seems to be, then, three tasks that the Method must fulfill. First, to carve out what in later literature are called "natural" kinds. Second to present a correct conceptual anatomies of generic Forms so that the more correct subdivisions will mirror the more important conceptual relations under a genus, thus vindicating indirectly also the positing of a given genus. Thirdly, by dividing genera, and then subdividing the results of the first cut, as well as those that follow, to end up with a series of characteristics, with more and more narrow extension, all of which can be collected and thus give a "definition" or unique necessary characterization of a given item under investigation. Examples include statesmanship and sophistry. (See, e.g., *Sophist* 258c-d). Since, e.g., sophistry is

[23] *Ibid.*, p. 10.
[24] Plato, *Politicus*, 252 d-263b.

illuminated by seven different divisions and none of these is declared as fraudulent, we can assume that the divisions are not meant to provide one correct account of a given element, to the exclusion of all others. On the other hand, the fact that much more space is devoted to one of the characterization of sophistry suggests that Plato did not regard all divisions to be of equal explanatory value.

If we are to support the Kneales' conjecture, we would have to argue in the following way. In at least one of their employment divisions can lead to definitions. But definitions are linked to logic. Therefore, divisions are linked to logic. In reply it should be pointed out that, as we saw, no conceptual dependency can be established between definitions and logic. We can have a fine syllogistic logic as long as the terms used are clear. For this we need not use definitions, though in some contexts these can be used to achieve clarity of terms.

Looking closer at what steps we find in divisions and in logical deductions, we see important differences. Suppose someone wants to "divide" the generic form, "discipline", ("art"?) into two sub-species, e.g., productive and exchange-oriented. How is this cut achieved? There is no way of *deducing* these two from the genus, and more importantly, there is no evidence that Plato thought that the cut is the result of deduction. Rather, the cut is a matter of making the "right" conceptual divisions. Looking at it one way, repeated cuts along many lines should lead to appropriate classifications. Once we have these, we can use these as the basis for constructing — with additional premisses— deductions. But we cannot test such classifications with deductions. Suppose that we divide humanity into Greeks and non-Greeks. As long as we did not leave out any element, the division is, in terms of logical form, correct. Whether it gives us insight is then a quite different question that mere logical devices will not answer.

Thus Plato's purpose with his Method is quite different from the purpose of a syllogistic logic. It is conceptual clarification, with the criteria for success being given by the notoriously intractable notion of insight or wisdom.

Aristotle's purpose with syllogistic logic is to present ways of argument that regardless of subject matter will always enable us to see consequences of conceptual relations that we assumed at the outset. Logic will show us appropriate inference patterns, regardless of the nature of the premisses with which we start. The premisses may come from any domain, and they may be true or false. Logic is concerned with inference (topic-neutral), while Plato's Divisions aim at appropriate in this or that conceptual domain. (Maybe he saw, by the time he wrote the *Philebus*, that in different domains we need different types of divisions. Some function along the genus-species line, others in terms of quantitative and measurable difference.)

So much for one of our two questions. We now ask whether there is any evidence that Aristotle regarded Plato's Division as either suggesting logical form, or functioning as a help in deductive arguments or our checking these.

The key evidence is supposed to be *Post An.* 96b25 ff. But in this passage Aristotle makes it quite clear that divisions do not prove or demonstrate anything. He does want to point out a way in which divisions are nevertheless helpful. But

in the examples he gives all we see is that divisions are useful as a starting point for forming definitions. They help us in "collecting" all of the attributes that will explain the essence of the given concept under investigation. As we saw, this does not show either that there is any conceptual relation between definitions and logical inferences, or that Aristotle thought so.

Divisions help to put attributes in the right order, i.e., as attributes with less and less extension. says also that divisions are a great way of checking whether we got all of the crucial attributes that jointly define the sought essence. But he should have added that this is so only if we did the division "appropriately"; i.e. not added anything superfluous and not left out key attributes. But what guarantees appropriate dividing? Certainly not anything having to do with deductions. No logical proof can be given that element E must be a part of the division. This judgment is based on conceptual intuitions, that lead us to view various elements as part of essence. Judgments of essentiality may be crucial for certain ways of viewing classifications, but these intuitions are quite different from logical intuitions of validity, consistency, or inconsistency. It is true that segments of divisions can be explicated in structures on' the basis of which deductions can be formulated. For example, maybe all As are Bs, and all Bs are Cs. This, so far, has nothing to do with inferences. Given this conceptual map, one can infer that all As are Cs. In this context the Method of Division can provide data for deduction, but lots of other investigations provide data as well. We can draw logical inferences on the basis of conceptual relations presented in legal presentations. This hardly shows that Aristotle was influenced in his thinking about logic by law.

In sum, Aristotle's comments on Divisions can be best interpreted as negative and defensive. He wants to show how different the purpose and carrying out of a division is from drawing logical inferences. His remark that divisions can help in checking proposed definitions are only meant to show that there may be some intellectual use of these structures. We have evidence that divisions were practised a great deal in Plato's Academy. Aristotle says the more or less appreciative things about divisions, to suggest that all the people who spent so much time and energy in constructing divisions did not wasted a lifetime.

5 SUMMARY

In comparison with other disciplines, logic moves on a higher level of abstraction. In physics, for example, there are specific events to account for, then generalizations that can stretch into lawlike ones, and after layers of these, a cohesive system, or theory. In logic we need to analyze and evaluate individual statements, and such analyses are generalized in terms of consistency and validity. Thus we have rules of construction and inference. Placed together in a coherent way this yields a system of logic. We can then reach a higher level of abstraction and look at systems of logic. Hence theorems about completeness, incompleteness, etc. We can view the natural sciences as forming theories by talking about entities that

give causal or other types of explanation of sensible particulars. Logic obviously cannot be viewed this way. Furthermore, logic is not some sort of an abstraction from sensory experience, nor can it be defined in terms of other disciplines or interest. Logic has — like mathematics – autonomy and the highest level of abstractness.

We saw that logic needed a set of terms, vocabulary items, of its own. But we must not think that once we have some of these terms like predicate, negation, then we automatically have a logic. To rise to the level of logic we need a special free creative move of the human mind. Thus logic is not a slow development of ideas, though once we have logic, it can be expanded, as we saw in the case of modern symbolic logic.

We need certain linguistic arid thus also. conceptual development in order to reach the level at which the vocabulary of logic can be forged. Much of this chapter is devoted to a sketch of how the required vocabulary items can be seen as a last jump in an otherwise long conceptual and semantic development. The Greeks reached the required level in one way. Other cultures with other languages might reach it in different ways.

Our main point is that the rise of logic is both a matter of development and the matter of instantaneous creation. The required vocabulary must have a historical process preceding it. Once that is in place, the possibility of constructing a logic is there. Aristotle was the first to understand the autonomy of logic, and the way it opens up a magic world of endless explorations of a unique mode of reflection, construction, justification, and argument evaluation.

APPENDIX 1: DID ARISTOTLE BASE LOGIC ON SOLID FOUNDATIONS?

As we saw, Aristotle's logic is not only syllogistic logic, but in terms of its internal anatomy it is a term logic. The premisses and conclusions contain different relationships between what one might call today the extensions delineated by the terms in the arguments. This entailed regarding predication as a key connecting element between the terms within arguments. Thus Aristotelian logic depends for its sound foundation on the intelligibility of predication. As long as we construe the relations between the terms as presented in an argument as overlaps of different sorts between extensions, predication may not seem problematic. The class of all animals contains the class of humans and that class the class of all Greeks, for example. Still, Aristotle is concerned with not so much relations between classes (if one can call the denotations of the terms that) as with the unity of sentences of subject-predicate form. The copula does not merely relate terms, it is also responsible for the unity of the relevant sentences. The sentence that we can assess as true or false is not merely a collection of parts. It is in some sense more than the mere sum of parts. And this "more" is indicated by the copula of predication. Then pushed to the limit, Aristotle retreats to metaphor. In *Met.* Z, l04lblD-20 he compares a sentence to a syllable. The syllable is not a mere sum of letters. The

letters in certain juxtapositions yield a syllable, without adding another element as the connector. It is clear that Aristotle chooses this metaphor so as to avoid infinite regress. If the copula denotes another element, then there must be a connection between this element and the predicate, and so on. Is this a satisfactory answer? Our evaluation depends on what we are willing to take as primitive.

Predication was also a problem earlier, for Plato. But what made predication problematic for Plato was different from what puzzled Aristotle. It is worth mentioning in this context that it is senseless to ask simply: "is predication a sufficiently clear notion?" This question can be raised meaningfully only within a conceptual framework. In different frameworks different factors might make predication problematic. Is there a framework in which predication can account for the facts that it must explain in semantic analysis, and still there are no features of the framework that will render predication puzzling? The jury is still out on that one.

Plato's puzzle was different from Aristotle's because his analyses of sentences of subject-predicate form were different. There are passages that show without doubt that in in modern parlance we must represent Platonic predication as tying subjects to intensional elements. For example, according to Plato everything partakes of Being, Sameness, and Difference, but there are three distinct Forms involved, not just one.[25] Thus we cannot take predication as just relating classes. Predication, in important contexts., relates spatial particulars to timeless, non-spatial, in principle recurrent, entities. Hence the special problem of what partaking is, the relation Plato introduces to make the combining of subject and predicate possible. *Parmenides* 131 shows that Plato is concerned with explaining what partaking is. Not surprisingly, his descriptions are typically negative. It is not part-whole relations, it is not physical engulfing, and so on. The only positive account turns out, not surprisingly, to be metaphoric. In the *Sophist* he introduces the notion of a Vowel-Form, and regards Being (in the predicative sense) as a prime example. Thus Being has as its sole function to relate things; it functions like vowels connecting consonants.

So much — briefly – for ancient wisdom on the topic.. Do we do better when looking at modern proposals? The most profound statement of the worry can be found in the writings of Frege. He compares the subject-predicate form to that of function and argument. Functions are incomplete, unsaturated, as Frege says, and thus their joining with arguments leads to intelligible completeness. Is this a better solution than the earlier ones? Is Frege's metaphor better than the earlier ones by Plato and Aristotle? Arguments still rage on that issue.[26] In his masterly review of twentieth-century proposals Frank Ramsey adds Wittgenstein's proposal according to which objects in atomic propositions hang in one another like the links of a chain.[27]

Ramsey himself attempts to draw the distinction by viewing the necessity of the

[25] Plato, *Sophist* 255b-d.d

[26] *Essays on Frege, 1968*, ed. I. Klemke, (Chicago: University of Illinois Press.)

[27] Ramsey, F. 1931, "Universals". in *The Foundations of Mathematics*, pp. 112 135; p. 129.

subject-predicate for as arising only in atomic propositions, and he construes these as having an ontological structure of a property being predicated (or universal) of a spatio-temporal individual. Thus the issue in Ramsey's analysis boils down to this: Is there a non-question-begging way of distinguishing universals from particulars? Borrowing an idea from Whitehead, Ramsey sketches in a few pages how one can consider a particular "adjectivally" e.g., regard the particular object as adjectival to events Needless to say, we can continue this by moving from one type of events to others, and so on.

One could continue and enumerate more recent proposals such as those by Strawson and others, but these are basically variations on the same theme and Ramsey's application of the Whiteheadian idea remains intact.

Why, then, should we look at predication in this puzzled way? The origin of these puzzles come from the monism of Parmenides who declared predication to be incoherent. He claimed that statement of subject-predicate form entail also negative statements. To say that x has F, makes sense only if we add to it that in virtue of this it is also something negative, a not-G for example. Parmenides questions the intelligibility of not being G. Plato responded by saying that in the contrast of what is fine and what is not fine two classes (collections?) of existent entities are juxtaposed. This hardly answers Parmenides who would question the ontological status of the negativity of the not-fine, "not-G", etc.

One might wave one's hand at other modern treatments of predication. It, and with it class membership, or instantiation, are basic primitives, we need then in our pluralistic analysis of reality, and as long as employment of predication does not lead to formal contradictions it is legitimate to use it. Whether such a complete separation of what is to be regarded as legitimate in logic and what are viable ontologies underlying the use of logic and language is intellectually conscionable or not cannot be discussed here. The fact is that after 2400 years all philosophers have come up with as response to the Parmenidean challenge is a bouquet of four metaphors. If there is immortality, then Parmenides is right now chortling in his coffin.

APPENDIX 2. LOGIC AND GRAMMAR: EPISTEMĒ, TECHNĒ, EMPEIRIA?

We can learn about the status of logic by a comparison with grammar, and especially the discussions about grammar in the early stages of its establishment in the Greek scholarly world.[28]

Was grammar to be regarded as an *epistemē* (branch of knowledge) or *technē* (rational discipline) or ere *empeiria* (set of empirical conjectures)? The difficulty surrounding these debates is that these three terms changed their meaning over

[28] Steinthal, H. 1891, *Geschichte der Sparchwissenschaften bei den Griechen und Roemern*, Vol. II. (repr. Hildescheim, Germany), pp. 173 178.

time. For Plato in the *Gorgias* the issue was, given a certain putative discipline, is it genuine knowledge or mere empirical beliefs? This question cannot be applied to grammar in a straightforward way, because the nature and scope of grammar was construed by different authors in different ways. On the one extreme one can view grammar as a descriptive study that is to record all of the language uses by a linguistic community. In this way of looking at it we come up with descriptions many of which are not even lawlike. Furthermore, many of the rules of grammar and the formulae allowed strike one as arbitrary.

At the other extreme one can think of grammar as a normative discipline. It does not describe how we do speak, but how we ought to speak. The question emerging at this stage is: what is the authority underlying the "ought"? Some will say the speech of the poets, philosophers, and scientists. Others might accept this but add in any case that at the base of grammar we find certain necessary features that enable grammar to reflect some of the basic patterns of reality. Needless to say, there are many possible positions in between.

Reflecting the ambivalence is the use of the phrase "*epi to polu*". For a staunch empiricist this means "the usual' or "for the most part" where these notions are spelled out probabilistically. For Plato and Aristotle this was not the appropriate meaning. For Aristotle the phrase represented what in modern English we call the generic use. An animal does something "*epi to polu*" if in doing so it expresses an aspect of its essence; e.g., "beavers build dams". This means, roughly, "the normal, healthy beaver." It need not be the majority of beavers (maybe many are sick), or the statistically predictable ones. Correspondingly if you want a grammar to reflect how people speak "*epi to polu*", you want to write a purely descriptive grammar, or one that tells people what linguistic use in the case of this language is at its best.

Interestingly, the more sparse the grammar, the more plausible the normative interpretation seems. Thus Plato in *Sophist* 258–259 introduces a complex that is roughly equivalent to actor/agent vs. action/property as fundamental to sentences expressing truth or falsity, and thinks that this reflects the metaphysical relations between Forms. Thus the structure posited is both necessary and justified. Needless to say, one cannot call this a complete grammar. Some such structure remains necessary and normative in Aristotle's treatment of basic combinations. But as Aristotle adds to the "grammar" e.g., endings, the ground for regarding his basic structures as based on features of reality seem weaker, and with that, of course, also the normativity.

How does all of this apply to logic? In principle one could start an enterprise of discovering and describing how people in fact reason. This is clearly not Aristotle's enterprise. The system of logic Aristotle articulates is not a piece of psychology. It shows us how we ought to reason, should reason, and not what we in fact do, though one might say that in Aristotle the "ought" in this context translates into "how humans at their best reason". His teleological conception of reality endowes logic at its base with both necessity and normativity. It is interesting thus to note that the necessity and normativity of certain combinations of linguistic units have

as their origin Plato's reflection on minimal grammar-like combinations which, in turn rest on their alleged isomorphism with some fundamental ontological relations among the most basic constituents of reality, namely the Forms. Denuded of the Platonistic metaphysics, but keeping some form of the necessary in reality and its reflection on language yields these two essential aspects of logic necessity and normativity.

ARISTOTLE'S EARLY LOGIC

John Woods and Andrew Irvine

1 BIOGRAPHICAL BACKGROUND

Aristotle is generally recognized as the founder of systematic logic, or of what he called "analytics." For over two thousand years he was logic's most influential writer.

Although there were precursors, especially with respect to the study of dialectic or the art of public argument, Aristotle was the first to systematize universally valid logical laws. As Julius Moravcsik points out, "[we do not] credit a logician with originality when he first presents the law of contradiction.[1] But we do credit him with originality in view of the particular ways he shapes rules for deduction and proofs."[2]

Aristotle was also responsible for the remarkable accomplishment of developing logic in at least two distinct ways, including his almost complete theory of the syllogism and his complex and sophisticated theory of modal logic. In addition, he is noted for his work in axiomatics, and there is some evidence that he also began investigating what is now called propositional logic, although he did not develop these investigations systematically. His claim near the end of *On Sophistical Refutations* that he is the primary creator of the discipline of logic is therefore quite justified (*On Sophistical Refutations*, 15, 174^a, 20).

Aristotle was born in 384 BCE, in Stagira in northern Greece, and it is from here that he received his nickname, "the Stagirite." His father, Nicomachus, had been court physician to the Macedonian King, Aourntas II, but both his parents died when Aristotle was still a boy. In 367 BCE, Aristotle was sent to Athens to study under Plato. He remained at Plato's school, the Academy, first as a student, and later as an instructor, for almost twenty years. Initially, the students at the Academy made fun of this new foreigner who spoke with a lisp, but Plato himself was impressed and nicknamed Aristotle "the intelligence" of the school. Upon Plato's death, Aristotle left Athens, perhaps for political reasons, or perhaps because he had not been appointed Plato's successor and was dissatisfied with how the Academy was now being directed. After travelling for awhile he married the (adopted) daughter of a former classmate who had become an Aegean king and in whose court Aristotle for a time served.

[1] *Cf.* Plato at *Republic* 436B: "It is obvious that the same thing will never do or suffer opposites in the same respect in relation to the same thing at the same time."

[2] Moravcsik, this volume, ch. 1.

Handbook of the History of Logic. Volume 1
Dov M. Gabbay and John Woods (Editors)

In 342 BCE Aristotle was appointed by Philip II of Macedon to tutor his son, the future Alexander the Great. From Aristotle's point of view, the appointment could not have been a pleasant one. He was expected to provide guidance to the young Alexander in a palace noted for its savagery and debaucheries, and at a time when both the father and son were intent upon assassination and conquest. When Alexander finally claimed his father's throne in 336 BCE, Aristotle left the court. Some reports say that Aristotle left well endowed by Alexander; others say that he was lucky to escape with his life. In any event, his influence could not have been too great since Alexander soon ordered Aristotle's nephew, Callisthenes, hanged for refusing to bow before the new king.

When Aristotle finally returned to Athens in 335 BCE, he opened his own school, the Lyceum, in a grove just northeast of the city. At the Lyceum he planted a botanical garden, began both a library and a natural history museum, and taught philosophy. The school was also known as the *Peripatos* or "strolling school" since, at least occasionally, Aristotle lectured to his students while strolling about the school's grounds. It is for this reason that even today Aristotle's followers are known as "peripatetics." In 323 BCE, following the death of Alexander, anti-Macedonian feeling in Athens increased and, because of Aristotle's ties to Alexander's court, the accusation of impiety was raised. Recalling the fate of Socrates—who had been put to death after being found guilty of similar charges - Aristotle returned to Chalcis, his mother's hometown, saying that he did not want Athens to "sin twice against philosophy." He died the following year, in 322 BCE.

Aristotle's interests were universal. His writings represent an encyclopedic account of the scientific and philosophical knowledge of his time, much of which originated with Aristotle himself and with his school. His extant writings cover logic, rhetoric, linguistics, the physical sciences (including biology, zoology, astronomy and physics), psychology, natural history, metaphysics, aesthetics, ethics and politics.

His logical writings, referred to as the *Organon*, consist of six books; the *Categories*, *De Interpretatione* (*On Interpretation*), the *Prior Analytics*, the *Posterior Analytics*, the *Topics*, and *De Sophisticis Elenchis* (*On Sophistical Refutations*).

Transmitted from the ancient world in large part through Arab scholars, Aristotle's writings shaped the intellectual development of medieval Europe. His logical treatises occupied a central place within the medieval curriculum and it was during this time that Aristotle came to be known as the source of all knowledge. Even so, the first modern critical edition of Aristotle's writings did not appear until 1831. The scholarly practice of citing Aristotle's work by a series of numbers and letters still refers to the page, column and line numbers of this nineteenth-century edition.

While it has often been claimed that Aristotle's dominating authority over such a long period hampered the development of logic (just as it did the development of science), such an observation cannot properly be taken as criticism of Aristotle himself. Instead, it is a telling incrimination of those of less talent and imagination who were to follow his remarkable accomplishments.

2 MOTIVATION

The Greek intellectual revolution can be characterized in large part by its discovery of a new method of enquiry and demonstration. This new method, called *logos*, shared its name with a perceived rational purpose thought to underlie the entire universe. Thus in one sense, *logos* represented the laws and regularities governing all of nature. In another, it represented the process of reasoning by which these laws and regularities were to be discovered. This new method of reasoning originated in physics and, with cosmology as a bridge, soon began to influence all branches of knowledge. Eventually, however, it was to collapse into a kind of intellectual pathology, typified by the efforts of pre-Socratic philosophers such as Heraclitus and Parmenides. Pathological philosophy was *logos* run amok and, for all its quirky theoretical charm, *logos* was soon being regarded as an intellectual disgrace. Left to its own devises, it threatened to destroy science and common sense alike.

It is widely agreed that the destructive arguments of Heraclitus and Parmenides, as well as those of many of the most able of the Sophists such as Protagoras and Gorgias, turn on the mismanagement of ambiguity. In the case of Heraclitus, his repeated equivocations typically take the following form:

> If ν is Φ in one sense and not-Φ in another sense, then ν is both Φ and not-Φ.

On the other hand, Parmenides' equivocations are typically driven by a misconception which is the dual of Heraclitus' error:

> If ν is Φ in one sense and not-Φ in another sense, then ν is neither Φ nor not Φ.[3]

Since Heraclitus and Parmenides both appear to accept a common major premiss, namely that

> For all ν and for all Φ, ν is Φ in one sense if and only if ν is not-Φ in another sense.

there arose the two great pathological metaphysics of the ancient world. For Heraclitus, the world turns out to be thoroughly inconsistent (or, as modern logicians would say, absolutely inconsistent) while for Parmenides the world turns out to be thoroughly indeterminate (or, as modern logicians would say, non-truth-valued).

No doubt it will strike the modern reader as puzzling that these blunders were accorded such high respect by thinkers as able as Plato and Aristotle. How could anyone be fooled by such blatant equivocations? In answer, there are two possibilities. One is that the predecessors of Plato and Aristotle could not *see* the

[3] Lawrence Powers calls these, respectively, the *Heraclitean Rule* and the *Parmenidean Rule* [Powers, 1995, ch. 2].

equivocations that they were guilty of. The other is that they indeed had the intuition that they somehow had mismanaged ambiguity, but that for some reason these intuitions appeared to be untrustworthy.

Like Plato, Aristotle wrote copiously, and he is at the very height of his intellectual powers in his discussions of these types of pathological philosophy. According to Aristotle, *logos* had been used in ways that denied both the reality of the empirical world and the meaningfulness of language. Aristotle saw Plato's forms as an important attempt to de-pathologize philosophy but, largely for reasons set out in the *Parmenides*, he also sees Plato's project as a serious failure. Aristotle is thus left with the following fundamental question: *How can* logos *be made to behave*?

Aristotle's answer was to invent logic, and to use logic as a constraint upon *logos*. Logic would be a technique, or a set of techniques, for facilitating the use of correct reason, for constraining and taming *logos*.

It is interesting that the first two monographs of the *Organon*, the *Categories* and *On Interpretation*, seem not to be about logic at all, never mind their occasional references to logical concepts and principles. The *Categories* contains an elaborate taxonomy of types of change, and *On Interpretation* is a theory of grammar for the Greek language. In both these works, Aristotle also devotes considerable attention to the phenomenon of ambiguity and to the deductive corruptions to which it gives rise; but when it is borne in mind that Aristotle is taking up the challenge of de-pathologizing philosophy, and that he seeks to do this by holding all theoretical reasoning to the standards of a correct logic, it is not surprising that he should start with an examination of change and ambiguity. Heraclitus and Parmenides both emphasize arguments that exploit change and ambiguity, and Aristotle is of the view that such arguments turn pivotally on errors in the ways in which change and ambiguity are to be analyzed. As a result, he begins his great reform of philosophy with an attempt to conceptualize these matters correctly. Plato had been shrewd enough to see that some of these pathological arguments go wrong because of equivocation the conceptual mismanagement of ambiguity; but Plato's own attempts at repair reveal that he lacks a competent understanding of equivocation (*Republic* 479B ff.). So, again, it is wholly tempting to see in the works of both Plato and Aristotle a response to the Heraclitean–Parmenidean challenge. This is especially true of Aristotle. In these first two books he is struggling to produce a theory of ambiguity and a set of protocols for its avoidance.

Following the *Categories* and *On Interpretation* come two works of signal importance -the *Topics* and *On Sophistical Refutations*. These two monographs are closely connected; in fact, some scholars are of the view that *On Sophistical Refutations* is either a ninth chapter of, or an appendix to, the *Topics*. By "topic," Aristotle means a "strategy" or "scheme of argument"; so we must not confuse this word with our word, which means "subject-matter." The importance of these books consists primarily in Aristotle's insight that there exists a model of correct argument which has a wholly general application. This model of argument is the

syllogism, and with it comes a precise answer to Aristotle's question of how *logos* can be made to behave.

Showing that all correct reasoning—all legitimate use of *logos*—conforms to Aristotle's theory of the syllogism would be a stunning accomplishment. It would establish that errors in reasoning arise solely for reasons other than the use of topic-neutral models of argument, to recur to *our* use of the word "topic." In this, Aristotle stands apart from more familiar Socratic denunciations of the teacher-generalist. He proposes to make good on the Sophist's central insight, that there are correct modes of reasoning and correct ways of arguing about anything and everything.

As we proceed, there will be several ways in which we will be able to judge the theoretical fruitfulness of Aristotle's invention of the syllogism. That logic was invented by a philosopher is a significant fact. Many a profession could claim the indispensability of clear thinking for sound practice. So why was logic not invented by an admiral or a general, or by a physician or a physicist? Why indeed was logic not invented by a mathematician: why is Aristotle *not* the Frege of the ancient world?

Logos is nothing if not a corrective to common sense. *Logos* has an inherent obligation to surprise. It began with the brilliant speculations of the Pythagoreans—the original neopythagoreans, as one wag has put it—with regard to a number-theoretic ontology. Apart from the physicists, the great majority of influential practitioners of *logos* before Plato allowed *logos* to operate at two removes from common sense. The first was the remove at which speculative science itself would achieve a degree of theoretical maturity. But the second remove was from science itself. The first philosophers were unique among the practitioners of *logos* in that they created a *crisis* for *logos*. In the hands of the sophists, philosophy had become its *own* unique problem. It was unable to contain the unbridled argumentative and discursive fire-power of *logos*. In fact, philosophy has had this same sort of problem—the problem of trying to salvage itself from its excesses—off and on ever since. Thus, logic was invented by a philosopher because it was a philosopher who knew best the pathological problematic that philosophy had itself created.

3 ORIGINS

It is widely accepted that Aristotle's main contributions to logic begin—and some would say end—with the *Prior Analytics*.[4] We are of a different view. It is often remarked that Aristotle may have composed the *Organon* in the following order: the *Categories*; *On Interpretation*; *Topics I VII*; *Posterior Analytics I*; *Topics VIII* and *On Sophistical Refutations*; and *Prior Analytics* and *Posterior*

[4]Thus it is with scarcely an exception that many of the leading contemporary commentaries concentrate on the *Prior Analytics*. In these writings one finds little to suggest that Aristotle's earlier treatises might warrant detailed critical scrutiny, even as a fledgling venture into logical theory. See, for example, [Lukasiewicz, 1957]; [Kneale and Kneale, 1962]; [Patzig, 1968]; [Smiley, 1973]; [Corcoran, 1974a]; [Kapp, 1975]; [Lear, 1980]; [Thom, 1981]; and [Frede, 1987].

Analytics II.[5] Yet if this is so, a certain caution is called for.[6] If, for example, *Posterior Analytics I* does indeed precede not only *Topics VIII* and *On Sophistical Refutations* but the *Prior Analytics* as well, it cannot strictly be true that logic originates with the *Prior Analytics*, since book one of the *Posterior Analytics* represents a considerable anticipation of many of the formal structures contained within the *Prior Analytics*.[7] Alternatively, on the chronological ranking of Barnes, in which the *Topics* and *On Sophistical Refutations* precede the two *Analytics*, themselves written in fits and starts over a more or less unified later period,[8] much the same point can be made. Thi si so, even though it remains the dominant contemporary view that the *Topics* and *On Sophistical Refutations* are treatises on dialectic and that, being so, they are not a serious contribution to logical theory.[9] Concerning *On Sophistical Refutations*, Hintikka proposes that

> [i]nstead of being mistaken inference-types, the traditional fallacies were mistakes or breaches in the knowledge-seeking questioning games which were practised in Plato's Academy and later in Aristotle's Lyceum. Accordingly, they must not be studied by reference to codifications of deductive logic, inductive logic, or informal logic, for these are all usually thought of as codifications of inferences [Hintikka, 1987, pp. 211–238].[10]

[5][Forster and Furley, 1955, p. 4]. See also [Kneale and Kneale, 1962, pp. 23–24]; [Rist, 1989, pp. 76 82] and [Dorion, 1995, pp. 25–27].

[6]We note, in passing, an interesting contention between Solmsen and Ross. Solmsen holds the chronological claim in the form of a priority of dialectic (*Topics*) and apodeictic (*Posterior Analytics*) over syllogistic (*Prior Analytics*). This is stoutly resisted by Ross. But contra Ross, see [Barnes, 1981]. (*Cf.* [Solmsen, 1929]; [Ross, 1949]; and [Forster and Furley, 1955].)

[7]For example, [Boger, 1998a] holds that Aristotle's work on the fallacies in *On Sophistical Refutations* presupposes the mature theory of the *Prior Analytics* (*cf.* [Boger, 1998b]).

[8][Barnes, 1993, p. xv]. It is, however, a mistake to attribute a full-blown chronology to Barnes: "Here and there ... we can indeed make chronological claims which have a certain plausibility to them; and some of these claims are not without philosophical significance. (For example, we believe that the core of the theory of demonstration which is expounded in *An. Post* was developed before the polished theory of syllogistic which is expounded in *An. Pr*; and we believe that this has some bearing on the way we should interpret some of Aristotle's views about the nature of science). But claims of this sort will rarely be made with any confidence; they cannot yield a chronology of Aristotle's writings; and they will not amount of anything which we could call an intellectual biography" [Barnes, 1995, pp. 21–22].

[9]For example, see Corcoran: "Aristotle presents [his logical] theory almost completed, in chapters 1, 2, 4, 5 and 6 of the first book of *Prior Analytics*, though it presupposes certain developments in previous works—especially the following two: first, a theory of form and meaning of propositions having an essential component in *Categories* [Corcoran, 1974b, ch. 5, esp. pp. 234 267]; second, a doctrine of opposition (contradiction) more fully explained in *Interpretations* (chapter 7). Bochenski has called this theory [of book one of the *Prior Analytics*] 'Aristotle's second logic,' because it was evidently developed after the *relatively immature* logic of *Topics* and *On Sophistical Refutations* ..." [Corcoran, 1974a, p. 88], (emphasis added). *Modus ponens* and *modus tollens* are recognized at *Topics* 111^b, 17 13 and 112^a, 16 13, and opposition and negation are discussed at *Topics* 143^b, 15 ff.

[10]*Cf.* [Woods and Hansen, 1997].

Hence,

> in a sense all Aristotelian fallacies are *essentially* mistakes in questioning games, while some of them are *accidentally* mistakes in deductive ... reasoning [Hintikka, 1987, p. 213] (emphasis added).[11]

Yet why should this be so? Hintikka believes that it is because *On Sophistical Refutations* is a dialectical work, not a logical one. As against this, Hamblin attributes to Aristotle the view that

> [d]ialectic as a mere technique [is] unessential to the pursuit of truth.[12] At times ... [Aristotle] even thinks of it as a hindrance: he is in the process of discovering Logic which, he thinks, enables a man to achieve as much by solitary thought as in social intercourse [Hamblin, 1970, p. 60].

Hence,

> Aristotle's *On Sophistical Refutations* can then be regarded as a first step in constructing the relevant logical theory [Hamblin, 1970, p. 59].

Hamblin himself thinks this is a retrograde step, at least when it comes to handling the fallacies:

> [I]n our attempt to understand Aristotle's account of fallacies we need to give up our [and Aristotle's] tendency to see them as purely logical and see them instead as moves in the presentation of a contentious argument by one person to another [Hamblin, 1970, pp. 65–66].

In contrast, we find this tension between dialectic and logic somewhat misconceived. Even apart from Aristotle's work on the fallacies, Hamblin is certainly right to say that Aristotle's dialectical works mark the beginning of logical theory. *On Sophistical Refutations* has as its primary target an analysis of sophistical refutations with which an unrepentant tradition (mis)identifies Aristotle's list of thirteen fallacies.[13] Whatever we may think of this identification, a theory of sophistical refutation requires a theory of refutation. Aristotle obliges with an obscure definition:

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[11] *Cf.* [Woods and Hansen, 1997, pp. 217–239].

[12] This, *en passant*, is nothing that Aristotle would have accepted. Dialectic is an indispensable instrument of negative knowledge, of the discovery of what is not the truth, itself essential to the pursuit of truth. What *is* true is that Aristotelian dialectic cannot demonstrate positive truth with certainty. On the other hand, dialectic is also a kind of induction (*epagôgê*), a method of examining all sides of an issue in ways that sometimes gets inquirers to see the self-evidence of first principles.

[13] For helpful discouragement, see [Hansen, 1992].

> For to refute is to contradict one and the same attribute—not the name, but the object and one that is not synonymous but the same—and to confute it from the proposition granted, necessarily without including in the reckoning the original point to be proved, in the same respect and relation and manner and time in which it was asserted ... Some people, however, omit one of the said conditions and give a merely apparent refutation ... (*On Sophistical Refutations*, 167^a, 23–29).

There follows an account of refutations which presupposes an account of syllogisms, and which eventuates in a formal codification, together with something tantamount to a completeness proof in the *Prior Analytics*.[14] What is distinctive about the *Prior Analytics* is thus not the doctrine of the syllogism; rather, it is, as we would say today, the discovery of a proof procedure for completeness [Lear, 1980, ch. 2, pp. 15–33]. In other words, the project of the *Prior Analytics* is formal and reductive. Aristotle gives two treatments of the syllogism. In the one, syllogisms are considered informally, and generally. In the other, they are subjected to formal constraints by which they are fitted for the particular theoretical purposes of the *Prior Analytics*.[15]

The question of the chronology of the *Organon* is thus part of the larger issue of the extent to which Aristotle's philosophy underwent significant development. On the developmental view, Aristotle recognized that certain of his later doctrines contradict and displace earlier doctrines.[16] The developmental perspective is resisted by some scholars, who hold that, by the time they were complete, Aristotle saw his writings as forming a consistent and unified whole.[17] Still others hold that the developmental and anti-developmental perspectives are reconcilable.[18]

For our purposes it is unnecessary to decide these larger questions. We shall say that the logical theory of the *Prior Analytics* presupposes Aristotle's theory of syllogisms (and—tacitly—a theory of validity, too). These theories appear implicitly in the *Topics* and *On Sophistical Refutations*. (In passing, we note that to say that a theory T presupposes a theory T^*, is to say that T could not be true without T*

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[14]Aristotle's own attempt, which does not quite succeed, is to be found at *Prior Analytics* 23. However, [Corcoran, 1972] has shown how to repair Aristotle's proof.

[15]"Indeed there is an ambiguity in Aristotle's use of the word 'syllogism' similar to that in the modern use of the word 'deduction.' There is first the use of 'syllogism' in the broad sense ... This corresponds to our use of 'deduction' in the general sense of an informal argument in which the conclusion is a logical consequence of the premisses ... Second, there is the use of 'syllogism' in the narrow sense, used to describe the formal inferences and chains of inferences that Aristotle isolated [in the *Prior Analytics*]" [Lear, 1980, p. 10]. *Cf.* [Mignucci, 1991, p. 25]: "[T]he definition of 'syllogism' at the beginning of the *Prior Analytics* (24^b, 18–22) refers to the generic meaning of the word, and it does not apply to the special inferences of which Aristotle offers the theory in the following chapter." It appears that the distinction between syllogisms in the broad and narrow sense was not recognized by mediaeval commentators. Thus, "[T]he mediaevals never doubt that [Aristotle] means [by 'syllogism'] the same in the *Topics* as in the *Prior Analytics* ..." [Green-Pederson, 1984, p. 20].

[16]See here [Jaeger, 1923]; English translation in [Robinson, 1948].

[17]See, for example, [Cherniss, 1935].

[18]See [Graham, 1987]. See also [Scott, 1971] cited in [van Benthem, 1994, p. 133].

being true, whereas T^* could be true without T being true. The relationship is preserved even when T^* is repeated in T, as is, to a large extent, the logical theory of the *Topics* and *On Sophistical Refutations* in the *Prior Analytics*.) It is well to note that the presupposition claim carries no implication of temporal priority.

The *Topics* is a handbook of dialectical argument that presupposes a distinction between dialectical *propositions* and dialectical *problems*. They differ in three ways: by way of content, logical form, and function. As to content, "a dialectical proposition [or premiss] consists in asking something that is reputable to all men or to most men or to the wise" (*Topics* 104^a, 9-10). Dialectical propositions are thus those that are believed to have a *prima facie* degree of credibility because they are universally or widely held, or because they are held by someone whose opinion deserves respect. The Greek term for such a proposition is *endoxon*. In contrast, a *dialectical problem*

> is a subject of inquiry that contributes either to choice and avoidance, or to truth and knowledge, and does that either by itself, or as a help to the solution of some other such problem. It must, moreover, be something on which either people hold no opinion either way, or most people hold a contrary opinion to the wise, or the wise to most people, or each of them among themselves. (*Topics* 104^b, 1–6).

Dialectical problems differ in content from dialectical propositions in that what marks them as problems is that their status is unsettled. (We note in passing that, being questions, so-called dialectical propositions are not a type of proposition in what we are calling Aristotle's technical sense.)General or expert opinion is not clear on what the answer to the problem is. Dialectical problems lack the very thing that makes a proposition an *endoxon*.

The logical form of a dialectical proposition is, "Is A B?'; for example, "Is two-footed terrestrial animal the definition of man?" (*Topics* 101^b, 29–30). All dialectical propositions have to have this form, and they must be answerable by Yes or No (*Topics* 158^a, 16–17). In contrast, the logical form of a dialectical problem is that of a disjunctive proposition, "Is A B or is A not-B?"; for example, "Is two-footed terrestrial animal the definition of man or not?" (*Topics* 101^b, 32-33). A question of this form cannot be answered (non-vacuously) by a simple Yes or No. The answerer must choose one of the two disjuncts, thereby committing to one of two propositions, either "A is B" or "A is not-B." We can see that the logical forms of both dialectical propositions and problems determine the logical forms of the answers to be given.

We can also distinguish dialectical problems and propositions by their different functions in refutations. The function of a dialectical problem is to give rise to a dialectical discussion; in opting for one of the two possible answers, a thesis is established that will be the target of a refutation. The function of dialectical propositions, to be answered by a Yes or a No, is to provide the grounds for the possible refutation of the thesis by being the premises from which the refutation is fashioned.

Aristotle contrasts dialectical arguments with other kinds of arguments in at least two places. In *On Sophistical Refutations* (2, 165^a, 37–165^b, 12) he lists four kinds of arguments used in discussion: (1) scientific arguments which reason from first principles appropriate to a subject and not from opinions of the answerer; (2) dialectical arguments which reason from generally accepted opinions to a contradiction; (3) examination arguments which reason from opinions held by the answerer; and (4) contentious arguments which reason from, or seem to reason from, opinions which are, or appear to be, generally accepted.

The object is then

> to discover some faculty of reasoning about any theme put before us from the most reputable premisses that are [*endoxa*] ... we therefore proposed for our tretise not only the aforesaid aim of being able to exact an account of any view, but also the aim of ensuring that in defending an argument we shall defend our thesis in the same manner by means of views as reputable as possible. (*On Sophistical Refutations* 183^a, 37-183^b, 6).

In places Aristotle seems to advance something stronger. His strategies enable a reasoner to reason about anything whatever, independent of its subject matter (*On Sophistical Refutations* 170^a, 38; 171^b, 6–7).[19] In other places still, he appears to confine himself to arguments from definitions and, thus, to arguments that are not entirely topic-neutral (*Topics* 102^b, 27; 120^b, 10 ff.). This does not cancel the claim that the arguments under review are always those that reason about reputable opinions. For it is possible that anything about which it is possible to argue independently of its content is a possible object of opinion by experts or by the many or by the wise, but there is nothing in the *Topics* or *On Sophistical Refutations* to suggest that the strategies advanced there have application to such arguments only on the assumption of some such possibility as this. For example, Aristotle explicitly recognizes demonstrative arguments, *i.e.*, arguments from premisses that are true, primary, appropriate to their subject matter and better known, or more intelligible, than the conclusions that they sanction (*Topics*, 141^a, 29; 158^a, 36–37; *On Sophistical Refutations* 165^b, 1 and 172^a, 19). Although it is true that Aristotle contrasts demonstrative arguments with dialectical arguments, it does not follow that strategies for the engagement of dialectical arguments have no application to demonstrations.

On Sophistical Refutations concerns itself with various ways in which the ends of argument can be subverted, sometimes deliberately, and with strategies for avoiding and evading these pitfalls. The example of refutation dominates this work. Aristotle specifies thirteen respects in which a refutation can go wrong, ways in which the purported refutation is not really a refutation but only appears to be one. These are his sophistical refutations, and here the word "sophistical"

[19] *Cf.* [Allen, 1995]: "But dialectic, the faculty of arguing about all matters, remains possible, for it falls to the dialectician to know the refutation arising through topoi, which are common by bearing all subject matters (*On Sophistical Refutations* 170^a, 34–170^b, 1)."

carries the meaning of "sham" or "counterfeit." Aristotle's list of thirteen is famous to this day. Traditionally, his sophistical refutations have been divided into two categories, which Latin translators have labelled *in dictione* and *extra dictionem*. In the first are *equivocation, amphiboly, combination of words, division of words, accent* and *forms of expression*. The other category comprises *accident, secundum quid, ignoratio elenchi, consequent, non-cause as cause, begging the question* and *many questions*. It is customary for commentators to think of the *in dictione* cases as sophistical refutations that "depend on language," and of the *extra dictionem* as "depending on considerations other than linguistic ones." However, this may not be the distinction that Aristotle intends.

Every putative refutation is an argument of a certain kind, in a sense of argument which, for Aristotle, is always a linguistic entity. So it may be said that any argument, good or bad, owes its goodness or badness to linguistic factors. Thus, what Aristotle has in mind is not a distinction between linguistic and non-linguistic considerations, but rather a distinction that turns on whether or not an argument is *spoken*.

For example, an argument is brought down by the sophism of *accent* when it contains a word which, when pronounced one way has one meaning, and when pronounced another way has a second meaning. If the argument in question were written down, the ambiguous word might not reveal its intended meaning, since it would have only one *spelling*. But if the argument were spoken, the word would be disambiguated by its different pronunciations. This is a matter *in dictione* precisely because a problem that might cripple the argument in written form would be cleared up in speaking it.[20] So, whether or not it will bear close scrutiny in every case, Aristotle's intention is to capture a class of argument mistakes that could be avoided by speaking the arguments in question. In contrast, the category *extra dictionem* would be made up of argument mistakes that cannot be be spotted or avoided in this same way, as for example with begging the question.

It is apparent that not everything falling under any of the thirteen subcategories in Aristotle's two lists is a sophistical refutation. A sophistical refutation is an argument which appears to be a refutation but, in fact, is not. Many arguments that do not even pretend to be refutations are arguments that are made bad by their instantiation of one or another of the structures in Aristotle's list. Any such argument is a bad argument and Aristotle thinks that it is made bad, and often also is made to appear good, by its instantiating one of the thirteen conditions. When this happens the argument in question is a *paralogismos* or a *fallacy*, and whereas it is Aristotle's view that a sophistical refutation is always a fallacy, it is not his view, nor is it true, that a fallacy is always a sophistical refutation. As a result, we see here, too, that the theoretical apparatus of *On Sophistical Refutations* has an application that extends beyond the kinds of argument denoted by the title of that work.

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[20] A charming example of accent is given by Powers: "The workers were unionized and therefore contained no extra electrons," which exploits the fact that "unionized" also means "non-ionized" [Powers, 1995, ch. 7].

In its ordinary use in Greek, *syllogismos* can be translated as "computation" or "reckoning." In Aristotle's logical writings, it is given a more technical meaning. In its broad or generic conception a syllogism is an argument in which a conclusion is derived of necessity from premisses, subject to further conditions to which we shall recur. Syllogisms in the narrow sense are triples of categorical propositions which are reducible to the first syllogistic figure as, for example, is the following:

All Greeks are human
All humans are mortal
―――――――――――――
Therefore, all Greeks are mortal.

Aristotle's programme in the *Prior Analytics* has an ambitious objective. It is to prove that all imperfect syllogistic forms reduce to the first syllogistic figure.[21] The very coherency of Aristotle's mature programme requires that he have had a sufficiently well-articulated conception of syllogisms in the broad sense to enable the reductionist strategy to be judged.

There is, we say, a prior theory of syllogism in the broad sense. The theory is presupposed by the *Prior Analytics* and is found in the *Topics* and *On Sophistical Refutations*.[22] What is more, we find in the *Topics* a clear presentation of the operation of argumental conversion, a subject to which we shall return in due course. Also evident, as we have noted, is a well-managed distinction between the relations of contradictoriness and contrariety, and early treatment of *modus tollens* and *modus ponens*. The output of the *Topics* and *On Sophistical Refutations* serves as input for the formalizing and metalogical devices of the *Prior Analytics*. It may rightly be said that the inputs to the reductive devices of the *Prior Analytics* must have structural features which enable the devices to engage them. In the *Prior Analytics* these structural features can be thought of as logical forms. There is no reason to suppose that the syllogisms of the early parts of the *Organon* lack logical forms. What is true is that the theory of syllogism in the broad sense is not a theory that manipulates those logical forms, at least overtly. We shall say, therefore, that the theory of generic syllogisms is a *pre-formal* theory.

4 SYLLOGISMS IN THE GENERIC SENSE

Whatever else they are, syllogisms are valid arguments, or sequences of *propositions*, meeting certain further conditions:[23]

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[21] *Cf. Prior Analytics* A1, 24^b, 27; A23, 40^b, 20.

[22] In the *Topics*, syllogisms are discussed at 100^a, 25–27. *Cf. On Sophistical Refutations* 164^b, 28 ff., and *Rhetoric* 1356^b, 16 17; 1357^a, 8 ff.; 1358^a, 3 ff., among other places.

[23] Even this is not quite without controversy. Aristotle reserves the term *protasis* for the premisses of syllogisms. In fact, *protasis* is often translated as "premiss." This leaves the question of how to characterize conclusions. In as much as the conclusion of one syllogism might well be the premiss of another, there is a reason to hold that a *protasis* is a proposition irrespective of its role in any given syllogism.

> For a deduction [a *syllogismos*] rests on certain statements such that they involve necessarily the assertion of something other than what has been stated, through what has been stated (*On Sophistical Refutations* 165^a, 1–3).

In several of the treaties of the *Organon*—for example in the *Categories* (see 2^a, 35-2^b, 7)—Aristotle attempts to bring forth an account of propositions. Inchoate as it certainly is, and hardly consistent in all details, Aristotle's treatment imposes significant constraints on what is to count as a proposition. The core notion is that in a proposition *a single thing is predicated of a single thing.* For example, in book one of the *Posterior Analytics* a proposition is "one thing said of one thing" (72^a, 9). Barnes suggests that this "one-one" principle, as we may call it, might have been designed to rule out equivocal predications (see *Metaphysics* 4, 1006^a, 32) or multiple predications (see *Topics* I, 6; *On Sophistical Refutations* 169^a, 8–9; 14–20; 181^a, 36-39 and *On Interpretation*, 18^a, 18 23). At *On Interpretation* 18^a, 13–14, Aristotle also writes that "a single affirmation or negation is one which signifies one thing about one thing." Barnes then directs us to a later passage (20^b, 12–21) where it is suggested that the "one-one" rule is designed to hold subjects and predicates to the expression of metaphysical unities. *On Sophistical Refutations* also has it that a proposition "predicates a single thing of a single thing" (169^a, 7) and requires that "one must not affirm or deny several things of one thing nor one thing of many, but [only] one thing of one thing" (181^a, 38). Further, "since a deduction starts from propositions and a refutation is a deduction, a refutation, too, will start from propositions. If, then, a propoistion predicataes a single thing of a singel thing, it is obvious that this fallacy [of Many Questions] too consists in ignorance of what a refutation is; for in it what is not a *proposition* appears to be one." (*On Sophistical Refutations* 169^a, 12-156; emphasis added). In a note to *Prior Analytics* 24^a, 16–24^b, 15, Smith also points out that Aristotle

> developed a theory according to which every such sentence [*i.e.* proposition] either affirms or denies one thing of one thing, so that a single assertion always contains a single subject and a single predicate. (In *On Interpretation*, he always explains more complex sentences either as having complex subjects or predicates or as really equivalent to groups of sentences) [Smith, 1989, pp. 106-107].

It was not Aristotle's intention to preclude plural propositions. What seems to be meant is that declarative sentences cannot be propositions unless they are *connective free*, with the exception of something like predicate-negation. So, whereas

(α) All men are mortal

is a proposition,

(β) All men are mortal or Socrates lives in Athens

and

(γ) All men are mortal and Madeleine lives in Vancouver

are not. On the other hand

(δ) No men are non-animals

is a proposition.

There is a significant sense in which arguments such as that from (α) to (β) fail. Their failure does not consist in there being countermodels for them. They fail in the theory of syllogisms on a non-deductive technicality. They either deploy or authorize the derivation of non-propositions, of statements that are not propositions in Aristotle's technical sense. Even so, the reason for the failure is deductively salient. It permits, even if it does not invite, the conjecture that Aristotle's conception of validity is indistinguishable from our own. On this view, a valid argument is any finite sequence of statements whose last member is necessitated or entailed by those that precede it. Further, since some statements are also propositions in Aristotle's sense, an argument is valid when its premisses entail its conclusion, even when some of its statements are propositions and others not. Where the validity rules fail, when they do, is in the theory of syllogisms. As we may now say, a syllogism is a valid argument, *all* of whose statements are propositions. Bearing in mind the translation of *protasis* as "proposition," we propose to call such arguments *protaseic arguments*. Thus the $\wedge$-introduction rule fails in the sense that no valid argument satisfying it can be a *protaseic* argument. Unlike validity, the property of being a protaseic argument is not closed under the standard deduction rules. (We are so using "valid" that an argument is valid just in case its premisses necessitate its conclusion; and we are using "necessitate" to mean what modern logicians mean by "implies" or "entails," and these conventions will remain in force unless otherwise indicated.)

Aristotle's propositions in the technical sense will strike the modern reader as something of a curiosity. What motivates so restricted a conception? *On Interpretation* bears directly on this question. There Aristotle advances the semantico-grammatical thesis that all statements reduce to simple statements in ways that preserve content. Simple statements are those that obey the one-one rule. Thus, they are propositions in Aristotle's technical sense. The thesis of *On Interpretation* (17^a, 13; 18^a, 19 ff., 24) is that all statements reduce to propositions (assuming reduction to be reflexive), and this we might call the thesis of *propositional simplification*. Thus "proposition" is a technical term for Aristotle, made so by the daring thesis of propositional simplification. If the thesis is true, it is hugely important. It isolates a sentential minimum adequate for the expression of all statements of Greek.

Aristotle's requirement that syllogisms be made up of propositions now seems to be explicable. It greatly simplifies the task of specifying the class of syllogisms and isolating their key properties. On this view, the propositional simplification thesis achieves the same economies in the theory of syllogisms as it achieves in the theory of grammar.[24]

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[24] *Cf.* [Smith, 1989, p. 35]: "Therefore in studying categorical sentences [Aristotle] took himself to be studying *what can be said*, without qualification. This last point is essential in

A further indication of Aristotle's motivation can be found in the *Topics*, as we have said. Aristotle's object, also stated at the beginning and repeated at the conclusion of *On Sophistical Refutations*, is to discover a method from which we will be able to syllogize about every issue proposed from *endoxa*, *i.e.*, reputable premisses, and, when compelled to defend a position, say nothing to contradict ourselves (100^a, 20 22; 183^a, 37–183^b, 6). A position to be defended is called by Aristotle a *problem*, which he divides into four kinds, each corresponding to a different predicable. The four predictables are genus, accident, (unique) property and definition. Every investigation of a problem involves determining whether a predicable belongs to a subject as genus, as accident, as unique property or by definition. If it is characteristic of such predications that they involve the attribution of one thing to one thing, it may be that Aristotle is embedding this characteristic in his technical notion of proposition. Whatever the motivation, the restriction to propositions is a fact about how Aristotle's syllogisms are to be constructed.

It is possible that Aristotle was influenced in his conception of an elementary proposition by Plato's contention (*Sophist* 252C4 ff.) that a statement has minimally a name (*onoma*) and a verb (*rhema*). The function of a name is to refer to something; but if we want to "get somewhere" (262D5) we must add to the name a verb. Only then do we say (*legein*) something. The result is a sentence (*logos*) (262D5–6). Modern readers may see this as anticipation of Frege's notion of the unsaturatedness of predicates, since here too the utterance of a predicate fails to "get somewhere" unless completed by a name or subject expression. In *On Interpretation* Aristotle repeats the view that a *logos* is composed of an *onoma* and a *rhema*, and no formula of whatever kind or degree of complexity is a sentence unless it contains a verb (*On Interpretation*, 17^a, 11 15). As stated, the doctrine puts no obvious *a priori* limits on the complexity of names and verbs. At *On Interpretation* 17^a, 39, it is implied that a subject term can be either general ("man") or singular ("Callias")—but *cf.* 17^b, 3.

On the other hand, sentences whose predicates are singular terms or proper names are not predications strictly speaking. They are ungrammatical. This excludes would-be premisses such as "All wives of Socrates are Xanthippe," even though Xanthippe, in fact, is Socrates' one and only wife. Also, names of accidents may appear in predicate position but not in subject position. When an accident name appears to occur in subject position it serves as the name not of the accident but, rather, of the thing in which the accident inheres (*Categories* 5^b). It is not red that is coloured, but *red things*. So adjectives are admitted into the basic *onoma/rhema* structure.

It is hard to see that these developments leave the "one thing" predicated of "one thing" doctrine with any meaning except this: that propositions in what we have been calling Aristotle's technical sense are statements that conform to

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understanding Aristotle's theory of validity. In fact, this is a theory of validity for arguments composed of categorical sentences, but since Aristotle thought that all propositions could be analyzed as categoricals, he regarded the syllogistic as the theory of validity in general."

the *onoma/rhema* structure of *elementary* sentences (hence, they have *one* name of whatever degree of complexity and *one* verb of whatever degree of complexity). "Whatever degree of complexity" of course is complexity consistent with the one name/one verb structure. Thus "If Socrates is wise and Plato is wise, then Socrates and Plato are wise" is disqualified, but not because it contains an adjective. Rather, it is because it contains a connective in virtue of which the one name/one verb structure is violated.

It would be helpful at this point to revisit the claim that in the *Topics* and *On Sophistical Refutations* syllogisms are inherently dialectical. Aristotle asserts that the *Topics* is a genuinely original piece of work, forwarding conceptions and insights that were entirely new. He repeats the point in *On Sophistical Refutations*:

> Of the present inquiry, on the other hand, it was not the case that part of the work had been thoroughly done before while part had not. *Nothing existed at all.* ... If, then, it seems to you after inspection that, such being the situation as it existed at the start, our investigation is in a satisfactory condition compared with the other inquires that have been developed by tradition, there must remain for all of you, our students, the task of extending us your pardon for the shortcomings of the inquiry, and for the discoveries thereof your warm thanks. (*On Sophistical Refutations*, 183^b, 34–184^b, 8, emphases added).

Yet the *Topics* announces itself as concerned with a method "from which we will be able to syllogize about every issue proposed from *endoxa*," a method, therefore, for the construction and presentation of dialectical arguments. Neither dialectical nor refutation arguments are anything that originated with Aristotle. Where, then, does the vaunted innovation of the *Topics* lie? Our view is that the original contribution is the syllogism, developed in such a way as to elucidate the deductive substructure of real-life arguments in their everyday uses as disputes about received opinions, as arguments that refute an opponent's claim, and so on. If this is right, Aristotle lays claim to being the first systematic developer of *applied logic*. That this is indeed right is suggested by the following considerations.

It is interesting to ask whether someone might be taught how to perform argumentative tasks properly and efficiently, or offered guidance under which his performance of them is improved. Aristotle's answer is Yes. The *Topics* contains a catalogue of propositions of possible use, in the sort of argument community that Aristotle is addressing, in the derivation of target conclusions. Here the basic idea is to find a set of acceptable propositions relevant to the issue under contention, and the *Topics* attempts to give guidance on how to find such sets. There follows a catalogue of rules and what might be called *set-piece* arguments (or schemas of arguments) which take acceptable premisses to target conclusions. Bearing in mind that Aristotle sometimes claims to be giving this guidance in such a way that it can be followed by arguers who have no knowledge of the content or subject matter of the disputed question, it is an audacious feature of the *Topics* that it offers advice of a highly abstract nature, of a kind that might be described as

"transcommunal," that is, effective in an arbitrary community of arguers. For example, let C be any target irrespective of its content; then the task is to find premisses, $P_1, ..., P_n$, which, whatever their contents, are acceptable according to acceptability criteria $K_1, ..., K_n$, are relevant to C, and are such that C follows from them. The employability of such rules for this task presupposes the possibility of recognizing the properties of premiss-acceptability, premiss-relevance and premiss-consequence independently of premiss and conclusion content. If we were intent on using such a strategy for the construction of a refutation, it must be possible, first, to identify the thesis to be refuted. This is done operationally: it is some proposition proclaimed by the one party, and which the other party challenges. The refuter's premisses in turn are acceptable if and only if they are conceded by his opponent. They are relevant to the target conclusion, which is the contradictory of his opponent's thesis, if (loosely) they are about the same sort of thing as the thesis in dispute. Further, they must satisfy the premiss-consequent condition if a subset of those premisses consists of the premisses of a syllogism for the negation of the disputed thesis. So there is a content factor here at work. Arguers must know enough about the subject matter of their contention to know whether a given would-be premiss is a proposition about that same subject matter; but if their argument is being conducted in a topic-neutral way, they need not understand that content.

In all cases, whether abstract or concrete, the overall approach of the *Topics* is *abductive*. It seeks to answer the question, "What is the optimal set of premisses from which to conclude a target conclusion?" The *minimal* answer is that a set of optimal premisses is any set, S, from which the target conclusion, C, is derivable. The fuller answer not only cuts S down to a relevant and acceptable subset, but it also cuts down the consequence relation in ways that we have yet to examine.

The *Topics* contains an abundance of (often confusing) instructions about how to optimize the derivation of target conclusions. To this end suggestions abound for premiss searches, and rules of derivation, as well as sample derivations, are provided. Aristotle was scornful of the methods of the Sophists. He says, in effect, that all that they offer the would-be arguer is set-piece arguments. Aristotle sees nothing wrong with set-piece arguments, but on his view, they cannot constitute an adequate methodology of successful argumentation. Catalogues of set-piece arguments are deficient in two respects. They lack a systematic account of why they are successful, if they are. Further, they lack systematic principles of extrapolation to contexts and subjects of disputation for which the catalogue contains no set-piece arguments as guides. It is in respect of these two deficiencies that Aristotle's claim to originality should be understood. In saying that the *Topics* and *On Sophistical Refutations* constitute a wholly original innovation, that there is something in these monographs that did not exist before, Aristotle invites us to consider precisely those features that are absent from the sophist's methodology and present in his own. Of course, there is a great deal in these works that had existed before. There is the notion of dialectical argument, and of combative or eristic variations of it, concerning which there is a huge preceding literature,

not least of which are the deep and detailed discussions of dialectical reasoning in several of Plato's dialogues (*Meno* 86E–89C, *Phaedo* 95E7–107B, *Republic* 510D - 511D, 527A6 B1, 533B–534D), and Zeno's celebrated paradoxes, which Aristotle took very seriously.

Then, too, it is a commonplace that when a conclusion is correctly derivable from some premisses, there is a relation from premiss to conclusion in the absence of which the derivation would not be correct. But it is certainly not true that Aristotle was the first to recognize this commonplace. So, again, where does the innovation lie? When we recall that Aristotle's strategic rules include rules for premiss searches and rules for the construction of derivations, it is clear that Aristotle sees himself as specifying a type of argument whose conditions blend and incorporate these two sets of rules. The type of argument in question Aristotle calls syllogisms. The necessitation requirement is a condition on derivation. A target conclusion is correctly derived from premisses only if it is necessitated (or implied) by them. The requirement that a conclusion must not repeat a premiss is a premiss-search (and premiss-eligibility) rule. The requirement that conclusions be derived from and through (or because of) its premisses is another premiss-eligibility rule, and so on.

On the face of it, this is not all that exciting. It can scarcely be imagined that the definition of the syllogism would have struck any of Aristotle's contemporaries as a *discovery*: a useful tidying up of something commonly employed by disputants perhaps, but surely not an original theoretical insight. The received wisdom in our own time is that it was certainly not a discovery, or much of one anyhow, and that the real innovations in Aristotle's work in logic do not present themselves until the *Prior Analytics*, what with its perfectibility result. But Aristotle was not stupid, nor was he given to misplaced self-congratulation. When he says that there is a wholly new theoretical twist to the *Topics* and *On Sophistical Refutations* it would be a little short of insulting to ascribe this innovation to the *definition* of *Topics* 100^a, 25–27 and *On Sophistical Refutations* 165^a, 1–3. Far more likely is that Aristotle's originality lies in the *uses* to which he is able to show that syllogisms can fruitfully be put. That is, we may suppose that when Aristotle wrote these treaties there were, in what might broadly be called the study of argument, various open questions which no known account of argument was able to handle satisfactorily. These included the following:

(1) When we refute someone, how can we be sure that our refutation is correct, and how can we get the refutation to *stick*, *i.e.*, at a minimum, how can we guarantee our opponent's acquiescence?

(2) When we argue against a position, how can we be sure not to have begged the question against that position in our selection of premisses?

(3) Some people are of the view that argument is just word play and clever self display, and that at bottom arguments do not get us anywhere; they do not facilitate the realization of deep ends. Is this right and, in particular, can argument ever lead to knowledge?

(4) As any well-educated Greek knew, arguments abound in which the conclusion is an utter violation of commonsense and deep scientific conviction, but which seems with equal conviction to be correctly derived from acceptable premisses. How are these paradoxical arguments to be answered? *How is the problem of pathological philosophy to be solved?*

Aristotle's innovation then consists in this: He is able to marshall, or so he claims, the argumentative structures he has dubbed "syllogisms" in such a fashion as to enable the satisfactory answering of each of these questions. Furthermore, these answers are given in such a way as to reveal that their satisfactoriness depends indispensibly on features of embedded syllogistic structures. In this, as we have said, Aristotle is the first applied logician. He is the first to show how answers to these and other practical questions are rooted in what can only be called the logical structure of deductive reasoning. For this to be true, it must also be true that the definition of syllogisms is in some sense a surprisingly deep one.

We have said that few of Aristotle's colleagues would have supposed the definition to be all that deep, novel or surprising. What sharp contemporary Sophist would have been bowled over by it? In fact, this is both right and wrong. It is right in so far as the definition would strike the Sophist as intuitive and familiar. It is wrong in so far as it turns out to be the case that structures defined by the definition of syllogism have certain properties whose significance is not transparent in the definition, and other properties whose existence is not transparent in the definition. Here is a modern example, and a contentious one.

Someone might define the entailment relation in the "classically" semantic way: Φ entails Ψ just in case it is in no sense possible that Φ and $\ulcorner \neg\Psi \urcorner$ are both the case. On hearing it, people might say, "Of course," or "Yes, that's what it is all right." If the producer of the definition turned expectantly to his colleagues for praise as an innovator, he would be disappointed. But suppose he went on to observe, "Well, this being so, it follows impeccably that an impossible statement entails every statement." "Ah," says a colleague, "your definition has hidden depths!"

Likewise, our task will not have been completed until it is shown how the syllogism facilitates Aristotle's programme in applied logic. Without this connection, it is open to a critic to complain that exposing the details of syllogistic structures is conceptual complexity for its own sake, and that Aristotle has contrived his account of syllogisms to no good end. In this context it is not our purpose to emphasize Aristotle's doctrine of *paralogismoi*. (But see [Woods and Hansen, 1997] and [Woods and Hansen, in progress].) Suffice it here to say that Aristotle takes the *paralogismoi* of *On Sophistical Refutations* to be arguments that appear to be refutations but, in fact, are not. A refutation is a syllogism meeting certain specific conditions. Aristotle's view is that in making, or accepting, a sophistical refutation one commits the fallacy of mistaking it for such a syllogism. Since syllogisms are not inherently dialectical structures, and since they do inhere in the very concept of a sophistical refutation, and of the fallacy that attaches to the making or accepting of a sophistical refutation, the concept of fallacy is not exhausted by merely dialectical factors.

If we have succeeded in showing that syllogisms are not inherently dialectical structures, it is nevertheless left open for someone to claim that *fallacies* are inherently dialectical. We do not think that this is so, but if we are right, we face the heavy weather of fallacies such as Begging the Question and Many Questions, each of which, for the modern reader, is as dialectical as it gets.[25] Even so, except for brief remarks in the section to follow we shall not, as we say, be much concerned with this question.

However, before leaving this section, we shall say our piece about the similarities and dissimilarities between and among *syllogisms, fallacies* and *sophistical refutations.* A *syllogism* is a valid argument meeting the additional constraints we have already listed, together with others yet to be discussed. Thus the class of syllogisms is a nonconservative restriction of the class of valid arguments. A *fallacy* is an argument that appears to be a syllogism but which in fact is not. A *sophistical refutation* is an argument that appears to be a refutation but which in fact is not. When an argument merely appears to be a refutation it owes this appearance to the fact that it embeds something that appears to be a syllogism but is not, or to the fact that it embeds a syllogism whose conclusion appears to be, but is not, the contradictory of the original thesis whose refutation is sought. Thus an argument is a sophistical refutation when it appears to be a refutation but embeds either *a fallacy* or a non-fallacy with the *wrong conclusion.*

Real-life arguments involve more than the production of syllogisms. With refutations as an example, there are also constraints on how premisses are selected. In this case, the refuter must draw his premisses from concessions given by an opponent in answer to the refuter's Yes-No questions. Or, as another example, demonstrations consist of syllogisms whose premisses must be drawn from the descendent class of a science's first principles under syllogistic consequence. In both cases there is more to the real-life argument than a mere sequence of premisses and conclusion. In each case there are additional conditions on premiss-eligibility. These are not themselves *syllogistic* conditions. This enables us to see that an argument might be a perfectly good syllogism and yet be a perfectly bad refutation or demonstration (or instruction argument or examination argument). If we think of the syllogism embedded in a real-life argument as its strictly logical component, then it is clear that most real-life arguments also satisfy non-logical constraints. It is also clear that in some cases, but not all, these non-logical constraints include conditions that can be called dialectical in ways that we have been considering.

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[25]Thus we have [Hamblin, 1970, pp. 73-74]: "The Fallacies of Begging the Question and Many Questions depend in conception, more than any other kinds, on the context of contentious argument ... The Fallacy of Many Questions can occur only when there is actually a questioner who asks two or more questions disguised as one." See also [Hintikka, 1987, p. 225]: "[O]ne thing is clear of the so-called fallacy of many questions. It cannot by any wildest stretch of the imagination be construed as a mistake in inference. It will thus bring home to the most hardened skeptic the impossibility of seriously construing Aristotelian fallacies in the twentieth century sense, *i.e.*, as tempting but invalid inferences." But *cf.*: "It is not clear in Aristotle's writings that the so-called fallacy of many questions is thought of *by him* just as a violation of presuppositions of questions" [Hintikka, 1987, p. 224] (emphasis added).

5 WHY THE FALLACIES ARE IMPORTANT

It may strike some readers as odd that Aristotle develops his generic account of the syllogism to stabilize the distinction between good arguments and good-looking arguments. In the absence of such a distinction, a general theory of argument would certainly be significantly disabled. Aristotle's optimism may incline us to think that the theory of syllogisms now makes this a usable and principled distinction, and that the theory of argument can now proceed apace. Yet clearly this would be to misjudge Aristotle's own view of the matter, as is evidenced by his very concept of fallacy.

As we have said, in its broadest sense a fallacy is something that appears to be an argument of a certain type but which, in fact, is not an argument of that type. In its use in *On Sophistical Refutations*, a fallacy is an argument that appears to be a *syllogism* but is not, in fact, a *syllogism*. We see, then, that the concept of syllogism is bedevilled by the same uncertainty that affected the more general concept of argument. Aristotle thinks that a good argument is one that is, or subsumes, or is in some other way intimately related to, a syllogism. But just as it is not always possible to distinguish a good argument from a good-looking argument, we also have it that it is not always possible to distinguish between a syllogism and something that only looks like a syllogism. Syllogisms were to be the means of removing the indeterminacy between good and merely good-looking arguments. Yet syllogisms are afflicted by this self-same indeterminacy. How, then, can syllogisms perform their restorative function in the general theory of argument?

Aristotle will overcome this problem in what rightly can be said to be the greatest technical achievement of the *Prior Analytics*, namely, the (almost sound) proof of his *perfectibility thesis*. Aristotle distinguishes between perfect and imperfect syllogisms. It is an oddly expressed distinction in as much as it is not the case that an imperfect syllogism is any less a syllogism than a perfect one. What Aristotle intends to capture with this distinction is the contrast between arguments that are *obviously* syllogisms and arguments that, while they are syllogisms, are not obviously so. According to the perfectibility thesis, there is, for any imperfect syllogism, a perfect proof that the argument in question is a syllogism. A perfect proof is one, all of whose rules are obviously good rules. There are two types of perfect rule. One, which Aristotle calls *common*, are rules such as conversion and *reductio ad absurdum*. The other type of perfect rule, for which we propose the name *syllogistic rule*, is the conditionalization of any perfect syllogism. Finally, in a proof of the perfectibility of an imperfect syllogism, the original argument's premisses serve as hypotheses of a conditional proof. To these hypotheses, perfect rules are applied to generate conclusions which may themselves serve as hypotheses to which perfect rules may also be applied. The conditional proof terminates with the derivation of the original conclusion of the imperfect syllogism. Thus a perfectibility proof is a conditional proof of the original argument's conclusion from the original argument's premisses by repeated application of perfect rules.

According to the perfectibility thesis, the conclusion of any imperfect syllogism is in the descendent class of the argument's premisses under the perfect rules. In this way, what Aristotle claims to have demonstrated is that for any syllogism that is not obviously a syllogism, there exists a perfectly perspicacious way of *making it obvious* that the argument in question is a syllogism.

The perfectibility thesis is discussed in greater detail in the *Prior Analytics*. We mention it here to make the point that since it was not something that Aristotle could draw on in his earlier writings, the issue of fallacies remains a serious difficulty for the earlier logic. This makes it all the more curious that Aristotle's treatment of the fallacies is, for the most part, rather thin and fragmentary. As long as it remained the case that fallacies could not be recognized in a principled way, then the invention of logic itself would exacerbate the very problem it was designed to solve. For as long as we lack a principled grasp of the distinction between syllogism and fallacy, syllogisms can play only an uncertain role in distinguishing between good and bad arguments. When we return to a discussion of the fallacies in section 12, it will be advisable to keep this point in mind. It helps in attaining an understanding of Aristotle's problem-solving methodology.

Given that Aristotle's problem is to distinguish between syllogisms and fallacies, it is clear that Aristotle has two general strategies to consider. One is to produce what in fact he never got around to producing, namely, a full account of each of the fallacies in the original taxonomy of thirteen, and of those other fallacies (such as *ad hominem*) mentioned elsewhere.[26] But a second possibility is that Aristotle would hit upon a way of making syllogisms effectively recognizable, which would not require an account of any fallacy, whether fragmentary or full. As we have remarked, some writers (*e.g.*, [Boger, 1998a]; *cf.* chapter 3 of this volume) are drawn to the view that the logic of the *Prior Analytics* was already available to Aristotle when he was writing about the fallacies in *On Sophistical Refutations*. We ourselves tend to demur from this opinion largely for reasons set forth in [Hitchcock, 2000a]. But it is grist for the mill of this controversy that in what we take to be his earlier writings on logic, Aristotle expressly avails himself of neither strategy. We could go so far as to say that in *On Sophistical Refutations* the fallacies elude Aristotle's theoretical grasp and, indeed, his theoretical *interest*. If we were to take this latter possibility seriously, we would be left with the necessity of trying to explain how it came to pass that having exposed a gaping wound in the theory of syllogisms Aristotle had no interest in following this up in a theoretically determined way. One possibility is that he was stymied, and did not yet know how to proceed with the requisite theoretical articulation. The other is that he already had a conception of how he would proceed in the *Prior Analytics*. If Boger is right, he had this conception of how he would proceed in the *Analytics* because that way of proceeding was already an accomplished fact during the writing of *Topics* and *On Sophistical Refutations*.

[26]Care needs be taken in attributing to Aristotle the view that *ad hominem* arguments are fallacies. In one sense of "proof" they are proofs of no kind; but in another sense of "proof," they are proofs of *that kind*. See, below, section 11 and [Woods, 2003, ch. 1].

Whichever explanation is favoured, it is worth noting that from the point of view of the syllogism, Aristotle's *examples* of sophistical refutations very often do not even *appear* to contain syllogisms. In some places, for example, quantifiers are conspicuous by their absence (*On Sophistical Refutations* 165^b, 34 35 and 166^a, 10–12). In others the number of premisses is wrong (*e.g.*, 166^b, 37; 168^a, 12–16; 168^b, 11; 180^a, 33-34). In still others, premisses and conclusions are not in strict propositional form (*e.g.*, 165^b, 38–166^a, 2; 166^a, 9-10; 167^a, 7–9, 29 30; 167^b, 13-17; 177^a, 36–38; 177^b, 37 178^a, 2, 11–16; 178^b, 24–27; 179^a, 33; 180^a, 34–35; 180^b, 9–10, 11 12, 21-23 and 23–26).[27] What these deviations suggest to us is that Aristotle's interest in these examples is a good deal more everyday than theoretical. He is more interested in getting across the main ideas of his taxonomy of fallacies that ruin refutations than showing in strict detail that they instantiate non-syllogisms that really do appear to be syllogisms.

6 A LOGIC OF GENERIC SYLLOGISMS

In saying that Aristotle's notion of syllogisms in the broad sense is a contending contribution to logical theory, it is necessary to have in mind some fixed star with which to box our compass. We shall need to have in mind a conception of what a core logic is. It is widely assumed by present-day theorists that the core of logic is the study of deducibility relations and that these relations display three jointly sufficient structural properties that capture the essentials of the deductive transmission of information. These three properties are reflexivity, transitivity (also called *cut*),[28] and monotonicity (also called *dilution*).[29]

By reflexivity any statement is derivable from itself, or (to have an expositionally handy converse) yields itself. By transitivity any statement yielding a statement which itself yields another also yields that other. By monotonicity any statement derivable from a statement is also derivable when that statement is supplemented by any others in any finite number. On the view we are examining, a core logic is a theory of deducibility in which the deducibility relation satisfies these three structural conditions. We call such a logic a *Gentzen logic.*

As will become apparent, Aristotle's conception of the syllogism fails the Gentzen conditions hands down. This indicates that Aristotle would be sympathetic to a distinction which the expression "deducible from" all but obliterates. This is the distinction between *implication* and *inference.* Concerning inference, it appears to be Aristotle's position that while inference may obey (perhaps a restricted form of) transitivity, it certainly does not obey either reflexivity or monotonicity. In con-

[27]These and other syllogistic deviations are well canvassed in [Hitchcock, 2000a].

[28]This is not quite accurate. Transitivity requires that the subordinate argument have only one premiss, which is identical to the conclusion of the superordinate argument. Cut permits multi-premissed subordinate arguments, where one of the premisses is identical to the conclusion of the superordinate argument.

[29]These properties are proclaimed in the three structural rules of the same name of Gentzen's sequent calculus. See [Gentzen, 1935]; [Szabo, 1969]. See also [Scott, 1971, p. 133].

trast, Aristotle's idea of implication is given by his notion of necessitation, which is an unanalyzed primitive in his writings [Lear, 1980, pp. 2, 8]. We will suggest in due course that necessitation should be understood as fulfilling the Gentzen conditions, hence that Aristotle has, implicitly, a core logic for the implication relation.

Part of what may be truly original about Aristotle's thinking is its apparent openness to a twofold fact: first, that inference is not (the converse of) implication but, second, that inference can be modelled in a restriction of the core logic of implication. So conceived, the inference relation is the converse of the implication relation under certain rather powerful constraints. What is more, a valid deduction in a Gentzen logic, when subjected to those same constraints, yields a structure of a sort that Aristotle called *syllogisms*. This suggests that what Aristotle wanted to do with the concept of syllogism was to "inferentialize" the validity rules of a given core logic. That is, he wanted to make rules such as Gentzen's deducibility rules more *like* rules of inference.

We trust that we will not have to apologize for anachronisms so blatant as to be self-announcing. Charity, if nothing else, provides that brazenness alone cancels any idea of express attribution to Aristotle. But Aristotle does have an implication relation rolling around in his theory of syllogisms, and we should want to know what it is. We are saying that if it is the implication relation of a Gentzen logic, then we get the result just noted. Of course, it may strike us as obvious that getting this result is nowhere close to showing that Aristotle's implication is the converse of Gentzen-deducibility and that Aristotle's validity is Gentzen-validity. We should think again. Gentzen-deducibility and Gentzen-validity are structures of a *core* logic; *i.e.*, they satisfy the three structural rules of reflexivity, transitivity and monotonicity. Of course, in Gentzen's own calculi the structural rules are supplemented by what Gentzen called "operational rules," and these are rules which, under certain assumptions, characterize the logical constants. What we are saying here has nothing to do with operational rules. We are not saying that Aristotle's validity is the validity of the full sequent calculus. We are saying only that a case can be made for supposing that Aristotle's validity is validity according to the core properties, that is, validity as characterized by these three structural rules. Here is the case. Whatever its details, the property of being a syllogism, or "syllogisity," is *some* kind of validity, minus the properties of reflexivity and monotonicity.

There is a crucial difference between syllogisity and Aristotle's validity, whatever it is in detail. Syllogisms are irreflexive and nonmonotonic. Let V be a property of arguments that results from the syllogisity property by reimposing the conditions of reflexivity and monotonicity. Then V is (core) Gentzen-validity if V is also transitive. But it is plausible to suppose that nothing qualifies as validity unless it obeys transitivity; so if V *is* transitive, then since it is also reflexive and monotonic it is core Gentzen-validity. And since Aristotle's definition of syllogisity implies that syllogisms are valid arguments, Aristotle's validity, whatever its details, is on this plausible assumption transitive. So there is *Aristotle's* validity

property call it V^a—which, being a validity property, is transitive on our present supposition. And there is Aristotle's *syllogisity* property to which, when reflexivity and monotonicity are restored, gives V. But V just is V^a. So *Aristotle's* validity is core Gentzen-validity. Thus, syllogisms are Gentzen-valid arguments for which the conditions of reflexivity and monotonicity are stipulated to fail; and, equivalently, syllogistic implication is Gentzen-implication failing those some conditions. If this is right, it is important. For in one good sense of the word, syllogisms have an underlying core logic. Let us look to this possibility in greater detail. Let us attend to syllogisms.

Aristotle says that "a refutation is a *syllogismos*" (*On Sophistical Refutations* 1, 165^a, 3).[30] This is a view in which he clearly persists, for it is repeated in the *Prior Analytics*: "Both the demonstrator and the dialectician argue syllogistically after assuming that something does or does not belong to something" (A1, 24^a, 26–27), and

> it is altogether absurd to discuss refutation without first discussing *syllogismos*; for a refutation is a *syllogismos*, so that one ought to discuss *syllogismos* before describing false refutation; for a refutation of that kind is a merely apparent *syllogismos* of the contradictory of a thesis (*On Sophistical Refutations* 10, 171^a, 1–5).

Let us, then, "first discuss syllogisms":

> A *syllogismos* rests on certain statements [*i.e.*, propositions] such that they involve necessarily the assertion of something other than what has been stated, through what has been stated (*On Sophistical Refutations* 1, 165^a, 1–3).

This is very much Aristotle's long-held and settled view. The same conditions are laid down at *Topics* 1, 100^a, 25–27,[31] and repeated in the *Prior Analytics* A1, 24^b, 19 20:

> A *syllogismos* is a discourse in which, certain things being stated, something other than what is satated follows of necessity from their being so.

Further, says Aristotle,

> I mean by the last phrase that it follows because of them, and by this, that no further term is required from without in order to make the consequence necessary (*Prior Analytics*, 24^b, 20–22).

[30] Unless otherwise noted all translations are from [Barnes, 1984]. An exception is συλλογιμös translated by Barnes as *deduction*, but for which we have used the transliteration *syllogismos*.

[31] "A *syllogismos* is an argument in which certain things being laid down, something other than these necessarily comes about through them."

Syllogisms, here, are what Aristotle calls "direct." They contrast with "hypothetical syllogisms",[32] which we shall not be much concerned with in these pages. It suffices to remark *en passant* upon an interesting feature of the distinction between direct and non-direct syllogisms. Hypothetical syllogisms, in contradistinction to those of the direct variety, are arguments construable as indirect proofs in modern systems of natural deduction. In typical cases, they are *per impossibile* arguments, that is, arguments such as the following:

Given:

(1) All A are B	Premiss
(2) Some A are not C	Premiss

To prove: (K) Some B are not C.

(3) All B are C	Hypothesis, contradicting K
(4) All A are C	From (1), (3)
(5) (4) contradicts (2)	

Thus: (K) Some B are not C.

A key difference between direct and indirect proofs is reflected in the different roles played by propositions introduced as premisses and propoistions introduced as hypotheses. Premisses are permanent in all arguments in which they occur. Hypotheses have a fugitive role. They are introduced, they perform their intended functions, then they are cancelled. A simple way, therefore, of marking the distinction between direct and hypothetical syllogisms is to notice that in direct syllogisms all propositions other than the conclusion must be premisses, whereas in hypothetical syllogisms at least one such line must be a non-premiss, that is, a hypothesis.

Aristotle concedes that the perfectibility proof of the *Prior Analytics* applies only to direct syllogisms (*Prior Analytics*, 41^a, 37–41^b, 1). If so, hypothetical syllogisms are truly a breed apart. In a way, this is an ironic twist. As we saw, Aristotle's project is to perfect all syllogistical reasoning. Perfection is achieved by reduction to the first figure. Aristotle recognizes that in some cases, the reductions can be indirect, by way of arguments *per impossibile*; and he says further that all reductions whatever are achievable in this way (*Prior Analytics*, 62^b, 29–31; 41^a, 23–24). But *per impossibile* arguments are hypothetical syllogisms. Thus some of the syllogisms used by Aristotle to show that all syllogisms reduce to the first figure are themselves syllogisms which do not reduce to the first figure.[33]

[32][Lear, 1980, ch. 3; pp. 34–35]. The last chapter of the *Topics* is a struggle to get clear about hypothetical syllogisms, and the need to do so is evident in the discussion of refutations in *On Sophistical Refutations*. The task is taken up again in the *Prior Analytics* at 40^b, 22–26; 41^a, 23–26, 32–37; A44, 50^a, 16–28.

[33]Ironic though the twist may be, neither it nor its irony is lost on Aristotle in the *Prior Analytics*. At A2, 25^a, 14–17, there occurs a proof of *e*-conversion concerning which "all scholars

In one respect the analysis of direct syllogisms is a matter of lively controversy. Some writers hold that they are irreducibly conditional in form, hence that they are a kind of statement.[34] Others are of the view that they are argumental structures, hence sequences of statements.[35] Others, still, favour the ecumenical suggestion that they can be taken either way and that the two approaches are interderivable without significant loss.[36] Not wanting to re-open this debate, let us simply declare for the second alternative. Aristotle's syllogisms are structures of a sort that a modern reader would identify as derivations in a system of natural deduction.[37,38]

A *syllogismos*, then, "rests on certain statements such that they involve necessarily the assertion of something other than what has been stated, through what has been stated." As we see, syllogisms are thus valid sequences of *propositions*, distinguished as to premiss and conclusion, which satisfy the following two conditions:

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agree that Aristotle's argument is ecthetic [*i.e.*, not narrowly syllogistic]" [Mignucci, 1991, p. 11]. An argument is ecthetic if Darapti is proved ecthetically (see A6, 28^a, 22-26); and at A8, 30^a, 6–14, ecthetic arguments are advanced for Baroco NNN and Bocardo NNN, each a modal syllogism. Moreover, the proof of Darapti requires *modus ponens* [Mignucci, 1991, p. 23] and the proof of Baroco NNN requires *modus tollens*. Aristotle expressly recognizes that neither *modus ponens* nor *modus tollens* is reducible to syllogisms in the narrow sense (*Prior Analytics* A23, 41^a, 23 ff.; A44, 50^a, 16 ff.). On the other hand, Aristotle also claims that any conclusion sanctioned by a *per impossibile* syllogism can also be derived by a direct syllogism employing the same premisses.

[34]See, for example, [Lukasiewicz, 1957, pp. 20–34] and [Patzig, 1968, pp. 3–4].

[35]For example, [Smiley, 1973]; [Corcoran, 1972]; [Lear, 1980, pp. 8–9]; and [Frede, 1987, pp. 100-116].

[36]*Cf.* [Thom, 1981, p. 23]: "Aristotle's syllogistic can ... be presented, either as a system of deductions [arguments] (a natural deduction system) or as a system of implicative theses [conditionals] (an axiomatic system). [Smiley, 1973] has carried out the former task admirably well; we shall attempt the latter. But, for those who remain unconvinced that the syllogism can be treated as an implication, we shall provide a way of re-interpreting our system as a natural deduction system."

[37]Here is Corcoran on the point: "My opinion is this: *if* the Łukasiewicz view [that Aristotle's logic is an axiom system] is correct *then* Aristotle cannot be regarded as the founder of logic. Aristotle would merit the title no more than Euclid, Peano or Zermelo insofar as these men are regarded as founders, respectively, of axiomatic geometry, axiomatic arithmetic and axiomatic set theory. (Aristotle would merely have been the founder of "the axiomatic theory of universals')" [Corcoran, 1974b, p. 98].

We note in this connection that Gentzen's structural rules are not by any means exclusive to the Gentzen calculi. They hold in Frege's system and in virtually every other logic published subsequently. Why do we invoke the name of Gentzen? Why is the core theory of validity not a Frege-logic or a Whitehead–Russell-logic? Our answer is that Gentzen was the first (along with Jáskowski, independently) to break with the axiomatic tradition in modern logic and to show that natural deduction systems have all the power of axiomatic set-ups. Because we hold, with Corcoran, that Aristotle conceived of logic in natural deduction terms, it is seemly to use the honorific "Gentzen" in reconstructing Aristotle's conception of validity.

[38]*Terminological Note*: We are using the expressions "deducible from," "consequence of" and "follows from" without due regard for what logicians have come to admire in a distinction between logical syntax and semantics. Even in the absence of a theoretically weighty divide between syntax and semantics, there is an intuitive distinction between deriving a conclusion from certain premisses and that conclusion following from them. We return to this point at the conclusion of the present section. For now we shall only say that any looseness in our usage will be tightened by context.

Min: They are *minimal*; that is, they contain premisses needed for their validity and none other.

Non-Circ: They are *elementarily non-circular*; that is, their conclusions repeat no premiss.

Condition *Non-Circ* comes directly from this characterization of syllogisms. The conclusion of a syllogism is something other than what has been stated, that is, its premisses. There are two ways in which an argument might violate *Non-Circ*. Its conclusion might repeat a premiss exactly as formulated, "word for word," or its conclusion might be a form of words syntactically different from a preceding line but synonymous with it. Assuming the reflexivity of synonymity, the two cases sum to one in the requirement that the conclusion of a syllogism not be synonymous with any premiss. It is sometimes supposed that circularity is a species of question-begging. We believe this is not Aristotle's own view. But whether it is or not, it is not Aristotle's intention to impose on syllogisms the requirement that they not beg questions. Whether an argument begs a question or not arises only in the context of further conditions which a syllogism might fulfill, as when it is used as a refutation of an opponent's thesis. When it is so *used*, it is held to a non-question-beggingness constraint, but it is a constraint not on syllogisms as such but on refutations, *i.e.*, on syllogisms in their use as refutations. We employ in *Non-Circ* the qualification "elementarily" to mark this point. An argument that fails to be a syllogism because of its violation of *Non-Circ* is one in which the conclusion is synonymous with some premiss, and hence repeats it. It is clear that *Non-Circ* denies syllogisms the property of reflexivity, as witness the argument $A \Vdash A$. Even if it is allowed that A necessitates A, it could not be true that $A \Vdash A$ is a syllogism, since the conclusion A repeats the premiss A. So, *syllogistic* implication is not reflexive.

Premisses will "involve necessarily" propositions other than what has been stated by the premisses. "Involve necessarily" has the sense of "*following of necessity*" (*Prior Analytics* A1, 24^b, 20 22). From this it can be seen that Aristotle requires that syllogisms be protaseic arguments, that is, that their premisses entail their conclusions. This is given in the *basic* condition that a syllogism is a protaseic argument, that is, a valid argument all of whose statements are propositions. *Min* makes the additional point that if a valid argument is a syllogism it cannot contain superfluous premisses. For an argument to be a syllogism it is not enough that its conclusion results of necessity from other propositions but, rather, that the conclusion results of necessity *because* of them.[39] It is open to question whether premiss-minimality captures all there is of the "because-of-them" requirement. We shall not pursue the matter here, but will return to it later. We see that the property of being a syllogism (again, "syllogisity" for short) is not a monotonic property. It is consistent to suppose that necessitation is monotonic, but syllogisity is a *restriction* on necessitation (or validity). If syllogisity were monotonic

[39] *Cf. Posterior Analytics* 71^b, 22: Premisses must be "causes of the conclusion" [Ross, 1949] or "explanatory of the conclusion" [Barnes, 1984].

then, if $A, B \Vdash C$ were a syllogism, so too would be $A, B, D_1, ..., D_n \Vdash C$, for any D_i. But $A, B, D_1, ..., D_n \Vdash C$ offends against *Min*. It is not a syllogism even if it is a valid argument. Since validity is reflexive, every statement is validly deducible from just one premiss, namely, itself. *Non-Circ* denies this property to syllogisms.

Various commentators have read off further conditions from our present two. Mindful of syntactic niceties, it has been proposed that the plural form "certain things being supposed" precludes single-premissed syllogisms,[40] and that the singular form of "thing which results" rules out multiple conclusions. Corcoran's opinion is that Aristotle did not require of syllogisms as such that they have just two premisses. That he did not impose this restriction

> is suggested by the form of his definition of syllogism ([*Prior Analytics*] 24^b, 19–21), by his statement that every demonstration is a syllogism (25^b, 27–31; *cf.* 71^b, 17; 72^b, 28; 85^b, 23), by the context of chapter 23 of *Prior Analytics I* and by several other circumstances .... Unmistakable evidence that Aristotle applied the term in cases of more than two premises is found in *Prior Analytics I*, 23 (especially 41^a, 17) and in *Prior Analytics II*, 17, 18 and 19 (esp. 65^b, 17; 66^a, 18 and 66^b, 2) [Corcoran, 1974b, p. 90].

Still, it is clear that Aristotle often does reserve the term "syllogism" for two-premiss arguments. We follow Corcoran in supposing that such a restriction is explicable by the fact that Aristotle thought if all two-premiss syllogisms are deducible in the logic of the *Prior Analytics*, then all direct syllogisms whatever are also deducible.

On the other hand, evidence from the *Topics* plainly indicates Aristotle's willingness to countenance syllogisms of just one premiss containing two terms not occurring in the conclusion [Allen, 1995, p. 1]. There is, in any case, little doubt that the settled opinion is that syllogisms require at least two premisses. We shall tentatively record the consensus in a further condition, *viz.*,

Prem+: They are *multi-premissed.*[41]

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[40] See John Maynard Smith, "Notes to Book A," [Smith, 1989, p. 110]. See also [Frede, 1987, p. 114]: "The Greek commentators all agree that the plural of 'certain things being laid down' has to be taken seriously as referring to a plurality of premises ... and everybody in antiquity (except for Anipaster, *cf.* Sextus Empiricus *P.H.* II, 167) agreed that arguments have to have at least two premises."

However, as Barnes points out, there is textual evidence that Aristotle plumped for premisses greater than two [Barnes, 1975, p. 68]. See *Prior Analytics* A14, 34^a, 17-18; 23, 40^b, 35; and *Posterior Analytics* I 3, 73^a, 7–11. But see *Prior Analytics* 42^a, 30 34: "So it is clear that every demonstration and every deduction will proceed through three terms only. This being evident, it is clear that a conclusion follows from two propositions and not from more than two...."

[41] Against *Prem+*, Robin Smith writes: "Aristotle thinks this is worth arguing for; but if, as the ancient commentators thought, it is simply part of the definition implicit in the plural 'certain *things* being supposed'—then the point is trivial and the argument redundant" [Smith, 1995, p. 30]. But this overlooks the fact that it is an open question for Aristotle whether indeed definitions *can* be argued for. This he discusses in the *Topics* and *Prior Analytics* (and comes up with contradictory answers).

Also advanced is the view, which we share, that a premiss for a syllogism is

> the result of a distillation from all those contexts [of conversational use] of a fundamental core meaning, excluding any epistemic [and semantic] properties ... [Smith, 1995, p. 108].[42]

Thus the premisses of a syllogism are the propositional contents of cognate speech acts (*e.g.*, asserting that P, asking whether P, *etc.*) independently of how they are spoken, independently of whether they are true, and independently of whether they are known in a certain way, or at all.[43] Syllogistic premisses are, in Aristotle's technical sense, propositions. In their turn, syllogisms are sequences of these, themselves independent of the pragmatic, semantic and epistemic conditions of their production in day-to-day social congress. If this is right, let us again note that syllogisms are not inherently dialectical structures; they are inherently logical structures.[44] They are protaseic arguments satisfying the restriction-conditions *Min*, *Non-Circ* and *Prem+*.

We are now launched on the task of discerning an account of syllogisms in the broad sense in Aristotle's dialectical treatises. The core conception is set by conditions *Min*, *Non-Circ* and *Prem+*, with *Prem+* admitted on sufferance. They seem to follow fairly immediately from a characterization of syllogisms sufficiently recurrent in the *Organon*, and beyond, to warrant the status of a definition. If these conditions are taken as our theoretical basis, then the theory of syllogisms will be roughly the least class satisfying *Min*, *Non-Circ* and *Prem+*, supplemented here and there by various other textual implications. We must proceed to the further specification of the theory, and we will do so with an eye on the collateral obligation to judge it as a *contending* theory, *i.e.*, a theory that merits our consideration in its own right, apart from its antiquarian importance. But there are other things to do first.

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[42] *Cf.* [Lear, 1980, p. 51]: "A direct syllogism may be described in an epistemic vacuum. One may or may not know the premisses and one may or may not use a knowledge of the premisses to gain knowledge of the conclusion." See also [Frede, 1987, p. 110]: "But later Peripatetic authors, and even Aristotle in the *Analytics*, no longer thought of the definition [of 'syllogism'] as dependent on dialectical context." *Cf. Prior Analytics* A32, 47^a, 33–35. It is well to note that the semantic independence of syllogisms is independence from the *truth* of their premisses; it is not independence from the *entailment* of their conclusions.

[43] If this sounds too Fregean a view for the likes of Aristotle, it suffices to characterize premisses as declarative *sentences* considered in isolation of contexts of their use.

[44] The point is sharpened by contrasting Aristotle's definition of "syllogism" with, say, Boethius'. Boethius says that a syllogism is an expression (*oratio*) in which when some things have been laid down (*positis*) and agreed to (*concessis*), some different things follow necessarily by virtue of the things which were agreed to (*De Differentus Topicis*, ed. *Patrologia Latina*, vol. 64, coll. 1173–1216). "[M]ediaeval commentators explain this divergence from Aristotle by saying that Boethius defines the *dialectical* syllogism, and Aristotle the syllogism *as such*. This cannot be historically correct, however, since Boethius makes the same 'addition' in his *De Categoricis Syllogisms*, and there it is certainly the syllogism as such that he defines. Thus it seems that Boethius demands that the premises of a syllogism are accepted as true" [Green-Pederson, 1984, pp. 44 45], emphasis added.

For example, we want here to revisit briefly the claim that syllogisms are not inherently dialectical structures. If this is so, then Aristotle's *fallacies* are not inherently dialectical either. In its most general sense, Aristotle thinks of a fallacy as something which appears to be a good argument of a certain kind, but which is not in fact a good argument of that kind. So understood, there are several ways in which an argument could be a fallacy. Its premisses might not necessitate its conclusion, though they appear to. It might contain an inapparent redundancy in its premiss set. Its conclusion may be identical to one of its premisses in camouflage. It may be an argument that appears, but fails, to be a demonstration, *i.e.*, a syllogism from first principles; and so on.

If a fallacy is something which appears to be a good argument of a certain kind, but is not, there might be (and are) fallacies which merely appear to be good *dialectical* arguments. Aristotle understands dialectical arguments to be arguments from dialectical premisses, and he understands dialectical premisses to express opinions either widely held, or supported by experts, or endorsed by "the wise" (*i.e., endoxa*). One way, then, for an argument to be a fallacy is by being a syllogism from premisses which appear to be *endoxa* but are not. Were it the case that fallacies as such are dialectical, it would have to be true that all arguments that merely appear to be good do so because they contain premisses that merely appear to be *reputable*. But this was never Aristotle's own view (*cf.* *Topics* 101^a ff.). By this same reasoning we should also resist the idea that the thirteen types of sophistical refutation listed in the treatise of the same name are inherently dialectical. In strictness, the thirteen are types of fallacies, hence types of ways in which arguments can be sophistical refutations; but an argument is a *sophistical refutation* when it merely appears to be a refutation, and this clearly can happen even when its premisses appear to be, and are, expressions of reputable opinion.

There is value in having made this aside, for it highlights a touchy ambiguity in the concept of dialectical argument. The idea of dialectic is in fact multiply ambiguous in Aristotle's thought, and in Greek philosophy generally. What matters here is an ambiguity that straddles Aristotle's thinking and that of his present-day successors. For writers such as Hintikka and Hamblin, an argument is a multi-agent interchange of speech acts[45] over which, however inchoately, the idea of a *challenge* is definable. In some sense, parties to a dialectical exchange are each other's opponents. Typical is the question-answer dialogue in which one party seeks to refute a claim of another party. Here the notion of challenge is overtly applicable, but we may also find it in muted form in the interrogative exchange between teacher and pupil, in which there are presumptive challenges to teach and to learn. Even a specific enquiry could be seen as an interrogative engagement between an enquirer and Mother Nature herself [Hintikka, 1989; Hintikka, 1987]. Suffice it to say that for our purposes, an argument can be said to be dialectical when it is an interchange of speech acts under conditions of challenge or test. Let us think of such arguments as dialectical in a generic sense. When it is recalled that our word "dialectic" comes from the verb *dialegestha*, meaning

[45]See also [van Eemeren and Grootendorst, 1992] and [Walton, 1989]. *Cf.* [Hansen, undated].

"argue" or "discuss," then generically dialectical arguments are a natural fit and certainly were recognized by Aristotle.

From this it is apparent that there are at least two senses of "dialectical argument" and that they do not coincide. An argument from reputable premisses can be transacted solo, on a lonely speech-writer's lap-top late at night, but if it is dialectical in the generic sense it cannot. For it to be true, as some modern commentators suggest, that a fallacy is an inherently dialectical structure, it must be true that its false apparent goodness inheres in the conditions in virtue of which the argument is an interchange of speech acts under conditions of challenge or trial. Yet this would imply that the argument

All men are mortal
Some dogs are not men

Therefore, some dogs are not mortal

could not be a fallacy unless it arose in some actual give-and-take between real-life disputants.

Some critics may not like our example of the lonely speech writer working the solitary night shift in Ottawa or Hong Kong. They will say, and it will be true, that in crafting the speech, the writer will be mindful of how it will play in Parliament or at the next day's news conference. He will attempt to marshal premisses attractive to those whom the speech is meant to convince. This is very often true, but it is also the sheerest nonsense that he could not be crafting an argument which simply records the position of his beleaguered boss who is intent on resigning honorably the next morning and intent only on declaring himself honestly from premisses which others will not accept, and not caring whether they do. Arguments composed solo are always the sorts of things that could be transacted interpersonally, but that makes them no more intrinsically dialectical in the generic sense than the fact that a boat can always be used for fishing makes a corvette intrinsically a trawler. It is not even true that when *spoken* a solo argument is dialectical in the present sense, for it might be spoken into a tape recorder. It is, we suppose, perfectly open to the would-be theorist to constrain the world "fallacy" in this generically dialectical way, but this is not Aristotle's way.

Necessitation, as we have said, is primitive in Aristotle's logic. If we adopt the idea that an argument is valid if and only if its premisses necessitate, or imply, its conclusion, we put ourselves in a position to wonder whether there are properties of validity that Aristotle might be brought to acknowledge, if only we could ask him. It is clear that Aristotle understands the class of syllogisms to be a restriction of the class of valid arguments. We have already said that it is consistent with the constraints on syllogisms that validity itself could satisfy the conditions that qualify it as having a Gentzen-logic. If Aristotle had views about this possibility, he did not state them. But we are left, all the same, with two things to theorize about: validity and syllogisity, which are linked, latter to former, by the relation "is a restriction of." It is hardly wayward, therefore, to think of there

being two theories awaiting proclamation, a theory Θ_ν of validity, together with its restriction, Θ_s, a theory of syllogisity, and that it is only concerning the latter that Aristotle offers an account.

Concerning Θ_ν, we shall proceed conservatively. We adopt, on Aristotle's behalf, the common lexical stipulation that an argument is *valid* just when its premisses necessitate, or entail, its conclusion. The task of Θ_s is to give an account of syllogisms. Validity abets the task. Aristotle requires a conception of validity that enables syllogisms to be specifiable as a proper subset of protaseic arguments. A methodological principle of interpretation now drops out:

> *Meth*: Keep the account of validity as simple as is consistent with its obligations in the theory of syllogisms.

We take *Meth* to commit us to what we might, with some looseness, call the received contemporary view of validity. It is nearly enough the conception of validity fashioned in the metatheory of first-order logic, or for those whose tastes run that way, in systems of strict implication such as S4 or S5 and their quantificational extensions.

It might be wondered whether there is any need to be speaking of Θ_ν at all. Why do we need a subtheory of validity if it has already been decided that Aristotle's validity is core Gentzen-validity? The answer is that a core Gentzen-logic gives rise to different and incompatible full logics. If, for example, we restrict Gentzen's structural rules in such a way that deductions can have only single conclusions (or only unit sets as conclusion), then such a logic is *intuitionistic* (a point we return to later). Similarly, Gentzen's structural rules (which define the core) lay no constraints other than consistency on the theorist's choice of operational rules (which complete the specification of the full logic). So it is left open that a given theorist, including Aristotle (if only we could consult him), might plump for operational rules that make the full logic significantly non-classical.

In speaking, just now, of the subtheory, Θ_ν, we were speaking not of the core logic but rather of a full logic of validity. There are several obvious questions to ask about this full logic. One of them is whether it is classical. We have done nothing so far to prove that it is classical. Perhaps no such attempt will succeed, given that Aristotle himself has so little to say about validity. Perhaps the saner course is to abandon any serious effort to pin a particular Θ_ν on Aristotle himself, but it is still open to us to wonder whether some Θ_ν might be assumed on Aristotle's behalf. We ourselves think that this is a reasonable thing to try to do. For this we want the comforts of *Meth*. It encourages the attribution, however tentative, of what we think is the best full logic of validity. Others will disagree about "best," and as we proceed it is possible that we will unearth textual reasons for revising the classical attribution which *Meth* defensibly calls for. Both these points are taken up in what follows, but as a point of departure, we allow *Meth* to tell us that Θ_ν is a classical theory of validity.[46]

[46] *Terminological Note*: We follow the convention by which *classical* logic is, of course, the dominant part of modern logic. Who says that logicians have no sense of humour?

There is a further reason to want *Meth*. If we examine the case for saying that Aristotle's validity is (at least) *core* Gentzen-validity, we see that it depends upon the assumption that syllogisms are non-vacuous, or proper restrictions of valid arguments. But we have not shown that this is Aristotle's express view. Were it not Aristotle's view, or were it not attributable to him on the basis of considerations which are at least textually *based*, our case would be badly damaged. We want some reason, in the absence of textual evidence to the contrary, to persist in our attribution of a core Gentzen-logic; and this is what *Meth* gives us.

Meth bids us to attribute to Aristotle such a conception of validity so long as doing so does not impede the development of Θ_s, and provided also that the attribution is not contradicted or contra-indicated by Aristotle's text. Our three conditions warm us to this task in a particular way. If we propose as a plausible assumption that Aristotle intended conditions *Min*, *Non-Circ*, and *Prem+* to be independent of one another, and of the necessitation condition, it follows immediately that validity is not syllogisity and that it cannot be that an argument is valid if and only if it satisfies conditions *Min, Non-Circ* and *Prem+*. We thus detach ourselves from the view that "Aristotle's conception of 'following necessarily' is very different from the classical one."[47]

How plausible is the independence assumption? Suppose it were not true. Then necessitation (or entailment) would satisfy all or some of the other conditions. If it satisfied some but not others, this would leave the question of why. About this Aristotle has nothing whatsoever to say. However, if entailment itself were required to satisfy all the remaining conditions, there would be no difference between entailment and, as we might say, syllogistic entailment, or between validity and syllogistic validity. This is possible, but it cannot be Aristotle's view. In as much as syllogisms in the broad sense are only a proper subset of logically correct deductions which Aristotle clearly recognizes to be so (*e.g.*, impossibility proofs, ecthetic proofs, conjunctive *modus ponens* arguments, immediate inferences), it must be said that Aristotle acknowledges a conception of entailment other than syllogistic entailment. It does not follow that non-syllogistic entailment is entailment in the modern sense (although it is just this that *Meth* suggests).

If we go with *Meth*, the independence of *Min*, *Non-Circ* and *Prem+* from the necessitation condition is more directly established. It suffices to specify a valid argument that fails all three conditions. Such an argument exists:

$$\frac{\text{All } A \text{ are } A}{\text{Therefore, all } A \text{ are } A.}$$

Valid by modern lights, this argument's conclusion repeats a premiss, of which

[47] Normore supports this claim by observing that Aristotle would not accept as a syllogism the argument {"No animal is a human," "Every human is an animal"} $\Vdash$ "No human is a human", presumably "because it does not contain three terms" [Normore, 1993, p. 447–448]. This is a correct assessment by the formal lights of the *Prior Analytics*; but there is a chronologically and conceptually prior reason to complain. It is that the argument contains inconsistent premisses, which for Aristotle also rules out its syllogisity. It does not follow, of course, that the argument is made invalid by a condition that denies it the status of syllogism.

there is only one, and being a logical truth, its conclusion also follows from a proper subset of its premisses, namely, the empty set.

This leaves the question of whether *Min*, *Non-Circ* and *Prem+* are independent of one another in their application to valid arguments. We proceed by constructing a satisfaction-failure matrix for *Min*, *Non-Circ* and *Prem+*. The plus sign "+" in the matrix denotes satisfaction of the condition, and "–" its failure. Given that satisfaction-failure is a bimodal pair, the number of total satisfaction-failure combinations with respect to the three-membered set *Min*, *Non-Circ*, *Prem+* is $2^3 = 8$. The full matrix is given in Table 1.

Table 1.

	Min	***Non-Circ***	***Prem+***
Row 1	+	+	+
Row 2	–	+	+
Row 3	+	–	+
Row 4	–	–	+
Row 5	+	+	–
Row 6	–	+	
Row 7	+	–	–
Row 8	–	–	–

Now recall that we are seeking an answer to the question whether conditions *Min*, *Non-Circ* and *Prem+* are independent. To show that they are, we need to find a valid argument that satisfies each condition, but that also fails to satisfy the other two, or a valid argument that fails to satisfy each condition, even though it satisfies the other two. In other words, we need to find a series of valid arguments corresponding to rows four, six and seven, and rows two, three and five. We attempt to do so as follows:

Row 4: What is required for this row is the specification of a valid argument which satisfies *Prem+* but fails *Min* and *Non-Circ*. There is such an argument, *viz.*,

All men are mortal
All ducks are quackers
———
Therefore, all men are mortal.

It is multi-premissed and hence satisfies *Prem+*. But its conclusion repeats a premiss and its second premiss is superfluous. Hence *Non-Circ* and *Min* fail.

Row 6: Here we require a valid argument whose conclusion does not repeat a premiss, but which both has a superfluous premiss and does not have more than one premiss. It is plain that such an argument does not exist, *unless* we allow as valid those arguments whose conclusions are logical truths, and hence follow from the empty set of premisses. Suppose that Σ is an argument of this kind. Add to

Σ any premiss other than its conclusion. We call the resulting argument Σ^*. Now Σ^* satisfies *Non-Circ* and fails *Min* and *Prem+*. It fails *Min* because Σ^* is valid without its premiss, and it fails *Prem+* because it has only one premiss.

The issue of logical truths as the possible conclusions of syllogisms is an interesting one to which we shall return shortly. For the present we simply remark that the independence question motivates our attention with respect to the status of logical truths as conclusions of the empty set of premisses.

Row 7: What is needed here is a valid argument free of superfluous premisses but which repeats a premiss as conclusion and has only one premiss. There are such arguments, for example, the argument $A \Vdash A$. Let Σ be a valid argument without superfluous premisses. If Σ's conclusion both repeated a premiss and had more than one premiss, Σ would have a valid proper sub-argument, which is contrary to the original assumption.

Row 2: Here we require a valid argument that satisfies both *Non-Circ* and *Prem+* but that fails to satisfy *Min*. Such arguments exist, and can be easily constructed simply by adding a redundant premiss to any valid, non-circular, multi-premissed argument, for example as follows:

All men are mortal
All Greeks are men
All Romans are Europeans
―――――――――――――――
Therefore, all Greeks are mortal.

Row 5: In this case, we need to find a valid argument that satisfies both *Min* and *Non-Circ* but that fails to satisfy *Prem+*, and again, such arguments are easy to find:

All men are mortal
―――――――――――――――
Therefore, some men are mortal.

Row 3: Finally, we come to Row 3, the case in which we need to find a valid argument that satisfies both *Min* and *Prem+*, but that fails to satisfy *Non-Circ*. However, such arguments cannot be constructed. The reason is that any argument that fails to satisfy *Non-Circ* will be one in which a premiss is repeated as conclusion.

At the same time, if the premiss is repeated as the conclusion, and if the argument contains premisses in addition to that premiss, it follows that those additional premisses will be reduandant and that *Min* will not be satisfied, contrary to our requirement.

What this shows is that although the set of *Min*, *Non-Circ* and *Prem+* is independent of the validity condition, *Min*, *Non-Circ* and *Prem+* are not independent of each other.

Why should we be interested in whether a syllogism's defining conditions are independent? To answer this question, let us call a definition *clean* if its defining conditions are independent of one another, and *muddy* otherwise. Muddiness is evidently a matter of degree. By the proofs just above, we see that the proferred definition of syllogisity is somewhat muddy. Why should we care about this? In

a quite general way, muddy definitions obscure the net impact of their defining conditions. A muddy definition presents us with two tasks instead of one. The first task is the more important one. It is the task of producing conditions necessary and sufficient for the definiendum. Though less important, the second task imposed by a muddy definition is no mere call upon the theorist's discretion. It is the task of elucidating the interconnections among the defining conditions in virtue of which indepedence is lost.

If simplicity were allowed to rule in such cases, we should be prepared to consider dropping a condition in favour of the independency of those that remain. Supose that we dropped *Prem+*. Then, as it turns out, the other two are indeed independent. What is more, this independence requires that validity be classical enough to permit valid arguments from the empty set of premisses. But since this is what *Meth* already bids us say about validity, the independence of *Min* and *Non-Circ* is an attractive bonus.

The independence of *Min* and *Non-Circ* is exhibited by the satisfaction-failure matrix in Table 2:

Table 2.

	Min	*Non-Circ*
Row 1	+	+
Row 2	−	+
Row 3	+	−
Row 4	−	−

Independence is established by specification of valid arguments, one or more, displaying the satisfaction-failure profiles of rows two and three.

Row 2: It suffices to find a valid argument in which the conclusion repeats no premiss but which contains a superfluous premiss. Such an argument exists:

All men are mortal
All ducks are quackers
Therefore, some men are mortal.

Row 3: It suffices to specify a valid argument which contains no superfluous premiss but whose conclusion repeats a premiss. Such an argument exists:

All men are Greeks
Therefore, all men are Greeks.

We have it then, that *Min* and *Non-Circ* are independent conditions. Independence is lost by the addition of *Prem+*. So why add it? One reason for doing so is that *Prem+* is a condition expressly proclaimed in the *Prior Analytics*. This leaves us with two options to think about. *Option one*: Withhold *Prem+* as a

condition on syllogisms in the broad sense and reserve it for syllogisms in the narrow sense. By these lights, it may be that Aristotle changed his mind about syllogisms and came to apply *Prem+* to facilitate the reductive programme in the *Prior Analytics*, even at the cost of definitional muddiness. *Option two*: Impose *Prem+* as a condition on all direct syllogisms, broad or narrow, and deal with the ensuing muddiness. This matter is discussed in greater detail in [Woods, 2001].[48]

7 INFERENTIALIZING THE CONSEQUENCE RELATION

It is interesting to note similarities between Aristotle's account of syllogisms and Bolzano's logic of deducibility (*Ableitbarkeit*), which in turn is widely seen as a precursor of Tarski's account of logical consequence [Corcoran, 1975; Thompson, 1981].

Bolzano requires deductions to have mutually consistent premisses [Bolzano, 1973]. Thus where, for Tarski, every sentence is a logical consequence of an inconsistent set of sentences, for Bolzano inconsistent premisses bear the *Ableitbarkeit*-relation to nothing whatsoever. Bolzano also requires that if a sentence is deducible from a given set of sentences, it not also be deducible from any proper subset of that set. Tarski, on the other hand, imposes no such constraint. Bolzano's condition makes his deducibility relation nonmonotonic. Tarksi's consequence relation, of course, is monotonic.

Bolzano's two constraints on deducibility have exact counterparts in Aristotle's logic of the syllogism. Bolzano's requirement that no proper subset of a set proving a sentence prove that same sentence is simply Aristotle's condition *Min*, adjusted from a condition on syllogisity to a condition on deducibility. Bolzano's requirement that there be no deductions from inconsistent premisses can also be found in Aristotle. It is directly provable from Aristotle's condition *Non-Circ* with the aid of the principle of (argumental) conversion, which is one of Aristotle's common rules of logic. Let

$$\begin{array}{l} A \\ B \\ \hline \text{Therefore, } C \end{array}$$

be any syllogism. Then, by argumental conversion, the following is also a syllogism:

[48]It is noteworthy that *On Sophistical Refutations* contains several examples of fallacies as single-premiss arguments that are valid without premissory supplementation, *i.e.* examples which are not enthymemes. David Hitchcock points out that this form of argument is prominently on display in Aristotle's fifteen examples of what he would later call the *secundum quid* fallacy. See, for example, 166^b, 37; 167^a, 1, 7 9; 168^b, 11; 180^a, 23–24, 31–32, 33–34, 34-35, 35-36; 180^b, 9–10, 11–12, 14–16, 18–19, 20–21, 21–23. Other one-premiss arguments instantiate the fallacies of equivocation (165^b, 31 32), illicit conversion of an A proposition (168^b, 35–169^a, 3), and so on. As Hitchcock says, "... twenty-six of Aristotle's sixty-five fully detailed examples consist wholly or partly of one-premiss arguments" [Hitchcock, 2000a, p. 214].

A
$\neg C$
Therefore, $\neg B$.

Thus the rule of argumental conversion is syllogisity-preserving and non-syllogisity-preserving. Consider now the non-syllogism

A
B
Therefore, A.

This argument fails to be a syllogism because it violates *Non-Circ*, the rule that forbids circular syllogisms. Applying the rule of argumental conversion to this non-syllogism gives us

A
$\neg A$
Therefore, $\neg B$

an argument in which the premiss set is expressly inconsistent. Since argumental conversion is non-syllogisty-preserving, no argument of this form is a syllogism.

The minimality requirement makes for the nonmonotonicity of Aristotle's syllogisity just as it does for Bolzano's *Ableitbarkeit*. Nonmonotonicity is not a nineteenth-century discovery, still less one of twentieth-century computer science. It was imposed at the very beginning of logic, by the discipline's founder. In making for the nonmonotonicity of syllogisity, *Min* also endows syllogisms with two other modern-looking features. A logic is *linear* when each premiss is used exactly once. It is easy to see that a linear logic is thus also a *relevant* logic for that sense of relevance in which something follows relevantly from a set of premisses if there exists a deduction of that sentence in which all those premisses are used. We have it, then, that although *Min* imposes additional constraints as well, in imposing *Min* Aristotle is producing the first linear, hence relevant, logic in this subject's long history.

The consistent-premisses condition is also consequential. It expressly provides for an implication relation (syllogistic implication, as we might say) which fails the condition *ex falso quodlibet*, according to which everything is deducible from an inconsistent set of sentences. Aristotle constrains syllogisms in such a way that nothing syllogistically follows from an inconsistency. In so doing Aristotle provides that the logic of syllogisms is the first paraconsistent logic.

The requirement that syllogisms be constructed solely of propositions also bears thinking about, quite apart from the audacity of the claim that anything stateable is stateable without relevant loss in the language of propositions. Recall that a proposition in Aristotle's sense is a statement in which one thing is said of one thing. With the exception of negation, they are also statements free of connectives. Syllogisms are sequences of propositions. Each line of a syllogism is occupied by one and only one proposition. Were it otherwise, were it the case that a line was

occupied by more than one proposition, then it would be a line in which more than one thing is said of more than one thing. To see how the propositional constraint works as a construction rule for syllogisms, it is essential that we see syllogisms as sequences in which *at each line* one thing (only) is said of one thing (only). This being so, syllogisms cannot have multiple conclusions. It might here be noted that any logic in which dilution fails and in which the standard operational rules are upheld (*e.g.*, the introduction rules) is an intuitionistic logic. To generate from such conditions a classical logic, multiple conclusions must be admitted.[49] Thus the first logic was at least in the spirit of an intuitionistic logic.

The theory of syllogisms was the first linear (hence relevant and nonmonotonic), paraconsistent and intuitionistic-like logic ever known. Syllogisms are classically valid arguments constrained in ways that make them *very* different from the classical validities that they would be if left unconstrained. There are worlds of difference between consequence relations that do (and do not) permit closure under the proper subset relation on premiss sets, that do (and do not) permit multiple conclusions. The very fact that Aristotle did not labour to bring forth a theory of unadorned validity indicates that, while essential to his purposes, validity delivers none of the special goods for which these truly encumbering constraints were needed. As we have seen, corresponding to any syllogism is its conditional statement, and corresponding to it is a rule of inference (so-called). Aristotle wanted a logic whose rules of inference—or at least the syllogistic rules—were not given the free flight of classical rules.

Why did Aristotle think it so important to constrain his rules? The answer appears to be that he wanted the rules of his logic of syllogisms to be usable in systematic accounts of real-life argument and thinking. Left without all this baggage, the validity rules regulate a content-free notion of consequence, which is defined simply for sequences of linguistic, truth-valuable entities, and whose primary function is *truth preservation.* But no rules that deliver on these objectives and none other are at all realistic as rules of real-life thinking and debate. It is one thing to say that inconsistent statements imply any statement; it is another thing entirely to say that when real-life reasoners are faced (in real time) with an inconsistency in their belief sets or commitment sets they do (or should) accept or commit to every statement whatever. Similarly, it is one thing to say that any truth preserving argument remains truth-preserving under arbitrary supplementation of premisses arbitrarily many times; but it is another thing altogether to say that when one has a truth-preserving argument at hand, it is always good argumentational strategy to make any supplementation of it that preserves truth. Such would be the course of risk aversion taken to ludicrous extremes.

Aristotle is essaying a bold experiment. He is taking seriously the idea that usable real-life rules for the conduct of argument and thinking can be got from context-free truth conditions on a purely propositional relation, provided the right constraints are imposed. In their unconstrained form, whether one proposition logically implies another tells us virtually nothing about whether it would be ap-

[49] [Shoesmith and Smiley, 1978, p. 4]

propriate, helpful, realistic or possible to conform one's argumentative or cognitive strategies to that bare fact of logical consequence. Aristotle's gamble is that facts about logical consequence do give the requisite guidance for argument and reasoning when constrained in the right ways.

Aristotle's example serves to remind us that the kind of logic a logician thinks up is, unless he is simply being playful, more or less directly the product of what the logic is wanted *for*. In a celebrated quip, W.V. Quine proposed that logic is an ancient discipline, and that since 1879 it has been a great one. The year 1879 marks a momentous event in the history of logic: the publication of Frege's *Begriffsschrifft*. It takes no disparagement of Frege's great accomplishment to make the point that Quine has judged Aristotle and Frege on the wrong basis. Frege needed a streamlined second-order predicate logic, coupled with something resembling set theory, as the analytical home for arithmetic. Frege was a logicist. He thought that it was possible to prove that the truths of arithmetic were analytic. This he sought to do by finding an analytic discipline to which the truths of arithmetic could be reduced without relevant loss. This host theory for arithmetic was second-order logic plus set theory (nearly enough), to which Frege himself made utterly seminal contributions. But it is absurd to abjure *Aristotle's* logic for failing *Frege's* objectives. Given Frege's purposes there was nothing to be said for making the consequence relation linear, nonmonotonic, paraconsistent and intuitionisitic-like. Doing so would not have advanced Frege's logicist ambitions one jot and would, in fact, have impeded their realization in various ways. In contrast, Aristotle had very different objectives, none of them bearing on the epistemology of arithmetic. Given the objectives that he had, the constraints imposed by Aristotle on the consequence relation were very much in the right direction. Aristotle's project, then, was to "inferentialize" truth conditions on a consequence relation purpose built for service in a realistic account of human cognitive and argumentative practice. Much of the so-called nonstandard logic of the present day is in various ways a continuation of this project to inferentialize the consequence relation, to retrofit it for work that is psychologically real or some approximation thereto. Opinion is divided as to whether the desired goods in theories of cognition and argument can in fact be delivered by inferentializing the consequence relation (see *e.g.*, [Woods, 1994] and [Woods, 2003]). It is a question for those nonstandard logics, no less than for Aristotle's original logic.

8 ARISTOTLE'S VALIDITY

We have proposed that Aristotle holds what modern logicians call a classical conception of validity and, correspondingly, a classical conception of logical implication or entailment. We make this proposal in the face of the fact that Aristotle gives no account of these things anywhere in his writings. We have grounded our proposal on two basic facts. One is that Aristotle's logic does not require that validity be non-classical and is in no discernible way improved by assuming so. The other is that, if we assume that the syllogisity conditions are non-redundant,

then Aristotle's validity must fail to be everything that syllogisity is required to be, validity itself excepted. So there is good reason to believe that Aristotle's validity is not nonmonotonic, not relevant, not paraconsistent and not intuitionistic. Of course, it does not strictly follow from these facts that validity is not non-classical; but it does make the nonclassicality assumption highly implausible.

Against this is Aristotle's Thesis, so-called by Storrs McCall. Aristotle advances this thesis at *Prior Analytics* B4 57^b, 4–7; so strictly speaking it does not fall within the ambit of Aristotle's earlier logic. Even so, Aristotle's Thesis matters for certain things we wish to say about this logic. In particular, it matters for the claim that in his early writings Aristotle's concept of validity is classical. Whether it is or not is complicated by the fact that different scholars read the thesis in different ways. A further complication is the uncertainty that attends the question as to what Aristotle's Thesis is a thesis *about*. Aristotle writes,

> But it is impossible that the same thing should be necessitated by the being and by the not-being of the same thing. I mean, for example, that it is impossible that B should necessarily be great if A is white and that B should necessarily be great if A is not white.

McCall takes this passage to assert that

$$\neg(\Phi \Vdash \neg\Phi)$$

i.e., that no proposition entails its own negation (see [McCall, 1996]). This is also the interpretation of [Routley *et al.*, 1982, pp. 132, 343]. McCall also interprets the passage as denying the validity of each of the arguments $\Phi \Vdash \ulcorner\neg\Phi\urcorner$ and $\ulcorner\neg\Phi\urcorner \Vdash \Phi$. Woods, on the other hand, reads the passage as asserting that of the pair of arguments

$$\frac{\Phi}{\text{Therefore, } \Psi} \qquad\qquad \frac{\neg\Phi}{\text{Therefore, } \Psi}$$

at most one can be valid [Woods, 2001, pp. 55 ff]. Despite their important differences, these conflicting interpretations have common features of consequence for our claim that Aristotle's validity is classical. On the McCall-Routley interpretation it cannot be true either that a necessary truth is entailed by any set of premisses or that an inconsistency entails any consequent whatever. On the Woods interpretation, the same is true. Necessary propositions are not consequences of arbitrary premisses; and if transposition holds true, neither is it the case that inconsistencies entail any consequent whatever. Note, however, that these consequences follow only if Aristotle's Thesis is a thesis about *validity*. Also required is a strong interpretation of invalidity, which we shall call counter-validity. An argument form is counter-valid when all its instantiations are invalid.

We will not here attempt to settle the question of how best to interpret Aristotle's words. It suffices for our purposes that if the thesis is understood to be attributing counter-validity, then on either interpretation, whether that of McCall

and Routley or of Woods, there will arise difficulties for the claim that Aristotle has a classical conception of validity. In what remains of this section we will thus concentrate on showing that if Aristotle's validity accepts just three classical principles, *viz.*, $\wedge$-elimination, $\vee$-introduction and transitivity, it is easily shown that Aristotle's Thesis is false or, if true, that at least it cannot be a theory about validity. Bearing in mind (we say) that the thesis is for *no* Φ and Ψ is it the case that *both* $\Phi \Vdash \Psi$ and $\ulcorner \neg\Phi \Vdash \Psi \urcorner$ are valid, it suffices to instantiate Φ and Ψ to opposite effect. Let $\ulcorner \Phi = C \vee \neg C \urcorner$ and $\ulcorner \Psi = C \vee \neg C \vee D \urcorner$, for arbitrary C and D. Then $\Phi \Vdash \Psi$, by $\vee$-introduction. Since $\ulcorner \neg\Phi \urcorner$ is $\neg C \wedge C$, then $\ulcorner \neg\Phi \Vdash C \urcorner$, by $\wedge$-elimination. But $C \Vdash C \vee \neg C \vee D$, by $\vee$-introduction. Hence $\ulcorner \neg\Phi \Vdash C \vee \neg C \vee D \urcorner$ (*i.e.*, Ψ), by transitivity. So $\Phi \Vdash \Psi$ and $\ulcorner \neg\Phi \Vdash \Psi \urcorner$: for some Φ and Ψ, both $\Phi \Vdash \Psi$ and $\ulcorner \neg\Phi \Vdash \Psi \urcorner$ are valid arguments. (A similarly strong argument can be given in the case of the McCall/Routley interptation.)

Thus Aristotle's Thesis is false if these three principles hold for validity. If Aristotle's Thesis is indeed incompatible with *ex falso*, the incompatibility is now a technicality. No claim is over turned by false propositions incompatible with it. Perhaps we should try to imagine whether Aristotle would himself have acquiesced to our three rules. We think it exceedingly likely that he would have. But right or wrong, it can also be shown that *ex falso* is true using only the following principles: reflexivity, monotonicity, transitivity, "conversion," and *modus ponens*. We do so as follows: for all Φ, Ψ, and χ,

(1) $\Phi \Vdash \Phi$ is valid	Reflexivity
(2) $\Psi, \Phi \Vdash \Phi$ is valid	Monotonicity, 1
(3) $\neg\Phi, \Phi \Vdash \neg\Psi$ is valid	Conversion, 2
(4) If $\Phi \Vdash \Phi$ is valid, so is $\neg\Phi, \Phi \Vdash \neg\Psi$	Transivitity, 1, 2, 3
(5) $\neg\Phi, \Phi \Vdash \neg\Psi$ is valid	*Modus ponens*, 1, 4.

Since Ψ is arbitrary, it covers all negations $\ulcorner \neg\chi \urcorner$. Hence all χ are validly deducible from any $\{\ulcorner \neg\Phi \urcorner, \Phi\}$.

If the proof is good, it suffices to topple Aristotle's Thesis if it is indeed incompatible with *ex falso* (or more directly, with its dual, which sanctions the derivation of logical truths from arbitrary premiss-sets, including the null set).[50]

Aristotle himself endorses *modus ponens* and conversion, and he allows transitivity for hypothetical reasoning and, apparently, for chains of syllogisms.[51] It remains to wonder what he may have thought about reflexivity and monotonicity considered not as conditions on syllogisity (in which case they both fail), but as conditions on validity. It is well to note in passing our all but complete surrender to *Meth*: given that the syllogisity conditions are independent of the validity condition, make it your point of departure concerning any property not a property

[50] Also toppled is McCall's *Aristotle's Thesis*, as witness the validity of $\ulcorner \neg(\Phi \vee \neg\Phi) \Vdash \Phi \vee \neg\Phi \urcorner$. The importance of this turns on whether we have independent reason to attribute to Aristotle the derivation principles which sanction such arguments.

[51] There is a problem with chains. Although championed in the Scholastic tradition and beyond, it is hard to find passages in which chains of syllogisms are given syllogistic recognition in the *Prior Analytics*. However, *Topics* 100^a, 27 offers some encouragement.

of syllogisms but which could be a property of valid arguments to ascribe to it validity until you have found good reason not to.

What, then, about reflexivity? If reflexivity holds in Θ_ν, we must say that every statement necessitates itself. On one interpretation this is nonsense. It is nonsense if self-necessitation is so understood that every statement makes itself true. We take it without further ado that there is nothing to be said for the view in which every statement is its own verification. What makes the self-necessitation claim sound wrong is a misinterpretation of "necessitation." Aristotle means by $\ulcorner\Phi$ necessitates $\Psi\urcorner$ that it is *guaranteed* that Ψ is the case if Φ is. Reflexivity or self-necessitation is just a special case of this: Φ is the case if Φ is the case. That this is so is encouraged by remarks at *Posterior Analytics* 73^a, 4–6, where Aristotle derides an imaginary opponent for complaining that all demonstrations are circular. Aristotle claims that the complainant is saying "nothing but that if A is the case A is the case," and he adds *not* that this is untrue but, rather, that "it is easy to prove everything in this way." For our present purposes it is enough that Aristotle does not here disallow reflexivity, but it is interesting to note that his objection against the complainant is that the purported proof that demonstrations are circular is an argument grossly in the form

(1) Demonstrations are circular

(2) Therefore, demonstrations are circular.

He adds, ironically, that of course it is easy to prove everything in this way. This is irony twice over. Aristotle means that whereas (1) $\Vdash$ (2) is a valid argument, the last thing it is, is a proof (for if it were, everything could be proved). Aristotle is also lampooning his critic by so representing the critic's own argument as to make it a case of the thing he is objecting to. So we conclude that we lack sufficient cause to make Aristotle's validity irreflexive.

What of monotonicity? It is helpful to bear in mind the sort of thing that condition *Min* is supposed to provide. In syllogisms, conclusions follow not only from their premisses but also *because of them*. In introducing *Min*, Aristotle takes pains to mark a contrast. It is a contrast between *necessitation from* and *necessitation because of*. It is important that Aristotle does not say that there are no necessitations-from. In fact, *every* syllogism is a necessitation-from. A syllogism is also something more; it is a necessitation-because-of. If the distinction is to have a point, there must be properties of necessitations-because-of that necessitations-from do not have. One such is the property of being causative of conclusions of syllogisms. In mere necessitations-from there will be premisses that are not causative of conclusions. These fairly enough can be said to be irrelevant to those conclusions. But if necessitations-because-of banish irrelevant premisses, it can only be expected that, in contrast, necessitations-from *allow* them. That is what monotonicity allows, too. Let $\Phi_1, ..., \Phi_n \Vdash \Psi$ be any valid argument with relevant premisses. Let χ be any statement irrelevant to Ψ (and to all the Φ_i, for that matter). Monotonicity nevertheless sanctions the validity of $\chi, \Phi_1, ..., \Phi_n \Vdash \Psi$. It

sanctions what Aristotle himself appears also to sanction. So we conclude that Aristotle would have no occasion to refuse the monotonicity principle for validity.

Against this it might be argued that monotonicity goes further than anything portended by Aristotle's distinction between necessitation-from and necessitation-because-of, and that it is this additional feature that Aristotle might well have been minded not to accept. Monotonicity expressly allows what the contrast between "from" and "because of" certainly does not expressly allow, *viz.*, that it is always all right to supplement the premisses of a valid argument in such a way that the resulting argument is valid and *inconsistently premissed.*[52] So let us turn to inconsistency. Inconsistency is not much discussed by Aristotle. It is difficult to see a stable policy on inconsistency and difficult therefore to see why the present point should persuade us to make Aristotle's validity nonmonotonic. We have already argued that Aristotle's validity is captured by the core Gentzen conditions, one of which is monotonicity. It will take the heft of substantial evidence to shift us from this view.

9 NECESSITIES

The theorems of the earlier logic and of the *Prior Analytics* register essential truths about direct syllogisms. These are Aristotle's "truths of logic." Either these truths of logic are themselves logical truths or they are not. If they are, the reasoning which underwrites them cannot be the reasoning which they themselves describe. If they are not, then presumably they are nonlogical *necessary* truths, and the same conclusion follows. No truth about direct syllogisms is the conclusion of a direct syllogism. As we saw earlier, this comes as no shock to Aristotle. Hypothetical syllogisms, such as *reductio per impossibile* arguments and ecthetic proofs, are not direct, and yet they are indispensable to the story that Aristotle wishes to tell about those that are direct. Even so, it is somewhat unsettling that in direct syllogisms no logical or necessary truths may appear as conclusions. This excludes too many cases that would appear to be paradigms of *perfect* syllogisms, as witness

All squares are rectangles
All rectangles are four-sided
———
Therefore, all squares are four-sided.

The exclusion of syllogisms such as these is so implausible that, *Meth* aside, we might consider rethinking our decision to attribute to Aristotle a classical notion of validity.

[52]Let it be noted that monotonicity does not give *ex falso*. It provides only that whenever there is a valid argument there is a valid *superargument* of it with inconsistent premisses, and whose conclusion is the conclusion of the original. *Ex falso* is stronger. It provides that an inconsistent set of premisses endorses everything as conclusion. An equivalent difference is this. Let Ψ be valid from inconsistent premisses by monotonicity alone; then it follows that Ψ is valid from a proper subset of those premisses. On the other hand if, by *ex falso* alone, Ψ is valid from inconsistent premisses it does not follow that Ψ is valid from a proper subset of them.

Necessary truths are a disaster for syllogisms. They are a disaster, that is, if counter-*ex falso* is true. We have said why counter-*ex falso* appears to *be* true and we have said why it is likely that Aristotle himself would have acknowledged its truth. If this is right, then the disaster that necessary truths produce is that, for reasons that Aristotle would not dispute, they wreck the project of laying a deductive substructure for the sciences. It is true that purely formal necessities such as "All A are A" are spared the embarrassment of failing the *Min* condition on syllogisms, but they are spared only by being victimized by a different embarrassment; for they cannot even be *expressed*, never mind proved, in Aristotle's theory of syllogisms.

It might be said that no logical theory has ever had success in dealing with nonlogical semantic necessities such as "All red things are coloured." So why should we impose on Aristotle's theory an expectation that no one else has met? There is little to recommend this leniency. It overlooks semantic necessities of precisely the sort that an Aristotelian apparatus is designed to capture, namely, statements such as "All bachelors are unmarried" and "All squares are rectangles" that are true *by definition*. What makes "All red things are coloured" a problem for logicians is that "red" appears not to have a definition. Its problem, at least in part, is that "All red things are coloured" is necessarily true, but not a truth of logic and not a definitional truth.[53]

A related difficulty attaches to semantically valid but formally invalid arguments such as

All Granny Smiths are apples
All apples are red
―――――――――――――――――
Therefore, all Granny Smiths are coloured.

Formally invalid, this argument commits the fallacy of four terms. Yet it is also true that given the meanings of "red" and "coloured," the argument cannot have a false conclusion if its premisses are true. Perhaps we could remedy the situation by relaxing the prohibition on extra terms. We might say that an argument

$Q\ (A, B)$
$Q\ (B, C)$
―――――――――
Therefore, $Q\ (A, D)$

commits the fallacy of four terms *except* where the Qs are, in this order: All-All-All, or Some-All-Some, or All-All-Some *and* C semantically entails D; or All-No-No, or No-All-No, or Some-No-Some ... not, *and* C is semantically entailed by D.

But even if this is a complete rule, covering all the right cases, it still leaves the fact that there is no rule that tells us how to determine in the general case whether π semantically entails π' or is semantically entailed by it.

Whatever we decide to say about semantic necessities, the necessary truths of mathematics cry out for rescue. In such an extremity it is permissible to clutch

[53] See [Searle, 1959]; *cf.* [Woods, 1967] and [Woods, 1974].

at straws. Aristotle recognizes different grades of necessity. Corcoran says that in Aristotle's modal logic there are up to five different and apparently incompatible systems competing for theoretical disclosure.[54] A straw presents itself: suppose that we granted to mathematical truths an attenuated necessity, a necessity of lesser grade than such full-blown semantic truths as "All bachelors are unmarried" and "Everything red is coloured." To this end, we could appropriate the expression "mathematically necessary" and contrast it with semantic necessity by stipulating that it is a grade (perhaps the strongest grade) of non-formal necessity for which *counter-ex falso* fails. In saying so, we would be adopting for the truths of mathematics a kind of *nomological* necessity that lacks full generality. Scientific laws are sometimes held to be nomologically necessary. By this is meant that they are unfalsifiable in their own domains, *i.e.*, their necessity is discipline specific; but they fail, perhaps vacuously, in other domains. This suggests that mathematical necessities might be reconciled to this conception. It cannot be said that their not being true is in no sense possible, but rather that their falsehood is in no sense possible within the domain of plane figures, or natural numbers, or topological spaces, or whatever else. In contrast, we might expect to find full-blown semantic necessities to be true in all domains. Unlike "Every triangle has the 2R property," whose failure is not possible in plane geometry, we would have "Every bachelor is unmarried" whose failure is not possible in any domain.

Yet this will still not work. If "Every triangle has the 2R property" fails outside of geometry, say in metallurgy, it does so vacuously. It is not a statement formulable in metallurgy, so its negation also fails. Another way of saying this is that the terms "triangle" and "has the 2R property" carry no metallurgical reference; they are empty terms in metallurgy. It is the same way with necessities of full-blown purport. "Every bachelor is unmarried" also fails in metallurgy, since "bachelor" and "unmarried" are empty terms there. Aristotle requires terms to be non-empty —and this is the source of his infamous doctrine of existential import. There is little doubt that Aristotle would welcome the suggestion that mathematical necessities fail in domains in which their embedded terms are empty. Admittedly, saying so leaves open the problem of how the quantitative sciences are to be understood; but Aristotle has this problem anyway. It instantiates the general prohibition of statements from one discipline serving as premisses in another discipline. So the present suggestion does not create a new problem for Aristotle; it simply exemplifies a problem that was already there. So, for good or ill, Aristotle would welcome the suggestion that "Every triangle has the 2R property" fails in metallurgy; but he would not welcome, nor should he, the suggestion that full-blown necessities fare any differently.

It seems best to give up the notion of full-blown semantic necessities; that is, nonformal truths of a grade of necessity strong enough to satisfy counter-*ex falso*. On this suggestion, we *evade* the problematic provisions of counter-*ex falso* by pleading that there is no grade of necessity attaching to nonformal truths suffi-

[54]See again [Corcoran, 1974c, p. 202]. This is also McCall's view. See [McCall, 1963] and [Patterson, 1995].

ciently strong to trigger the metatheorem. For let Ψ be any nonformal necessity. Then its failure is guaranteed in all alien domains. Thus there will be *some* statements $\Phi_1, ..., \Phi_n$ from which Ψ does not follow. These will be statements true or false in any domain in which Ψ fails on account of alienation, and these will be precisely the domains in which the necessity of Ψ fails to satisfy counter-*ex falso*.

Still, this is too much to hope for. In the shady glades of watered-down necessity, one hand washes the other. Our current speculation provides that if Ψ is a nonformal necessary truth, there will be a domain in which its negation is impossible and which, in alien domains, it is neither possible nor impossible. (In fact, it turns out not to be a proposition there.) Correspondingly, $\ulcorner \neg\Psi \urcorner$ will have a necessary negation in the home discipline and in all others it will be neither possible nor impossible. Consider, then, the argument

$$\begin{array}{l} \Phi_1 \\ \Phi_2 \\ \hline \text{Therefore, } \Psi \end{array}$$

in which, for some discipline D, Ψ is a necessary truth. Then $\ulcorner \Phi_1 \wedge \Phi_2 \wedge \neg\Psi \urcorner$ will be impossible in D. Beyond D, the argument vanishes, owing to what might now be called reference failure. So we will say that our argument is *D-valid*; and if it meets the other conditions on syllogisity, it is a *D-syllogism*. Let us suppose that it does meet these conditions. Let $\emptyset^D$ be the empty set of premisses from D. Then

$$\begin{array}{l} \emptyset^D \\ \hline \text{Therefore, } \Psi \end{array}$$

is a D-valid argument since $\ulcorner \neg\Psi \urcorner$ is impossible in D and $\ulcorner \neg\Psi \urcorner$ is already conjoined with the putative membership of $\emptyset^D$. But $\emptyset^D \Vdash \Psi$ is a D-valid sub-argument of $\Phi_1, \Phi_2 \Vdash \Psi$, which is itself D-valid. Hence $\Phi_1, \Phi_2 \Vdash \Psi$ is not a syllogism, contrary to our hypothesis.

We have not found a way of attenuating mathematical and other nonformal necessities in ways that avert the problem they pose for syllogisms. The disaster they occasion recurs. Aristotle thinks that every science is (or contains) the demonstrative closure of first principles, that first principles are necessary, and that their necessity is preserved in their closures.[55] Either those necessities are full-blown, *i.e.*, they hold in every domain including the null domain, or they are domain- or discipline-relative. That is, they hold in their own domains but fail for want of reference in every other, including the null domain. If the first possibility holds, there will be *no* sciences since there will be no demonstrative syllogisms. In every putative science for which there are full-blown necessary truths, they will follow from the null set of premisses. If the second possibility holds, the same unwelcome result awaits: once again there will be no sciences since there will be no scientific syllogisms. For, again, let Ψ be a proposition necessary in D and only in D. Then Ψ follows from $\emptyset^D$, the empty set of premisses in D. $\emptyset^D \Vdash \Psi$ is a

[55]Thus to have scientific knowledge of something is to know the cause or reason why it must be as it is and why it cannot be otherwise (*Posterior Analytics* 71^b, 17–33).

valid proper sub-argument of any D-valid *premissed* argument for Ψ. Yet no such argument is a syllogism. It follows that there are no demonstrations in D. So D is not a science.

We have been taking *Min* to preclude syllogisms with valid sub-arguments. As things have developed, *Min* and counter-*ex falso* collide with one another momentously. For together they make science impossible. This is a consequence sufficiently disagreeable to call *Non-Circ* and counter-*ex falso* both into question. There are plenty of logicians, *e.g.*, those of the Anderson-Belnap persuasion, who would think that counter-*ex falso* is the obvious choice for rejection. Counting for this, in an indirect sort of way, is that there is something that obviously counts against the rejection of *Min*. What counts against rejecting it is that Aristotle seems expressly to proclaim it. His commitment, if such exists at all, to counter-*ex falso* is nothing that Aristotle ever expressed; and given the seriousness of its conflict with a principle Aristotle does express, there appears to be nothing to be said for its retention.

We lack the space here to reflect further on how best to adjudicate the tension between *Min* and *counter-ex falso*, since doing so is highly conjuctural. However, interested readers may consult [Woods, 2001, ch. 7].

10 REFUTATIONS

An important precursor of the Aristotelian refutation argument is the *eristic* argument, prominently on display in Plato's *Euthydemus*. Eristic argument, in turn, is a refinement of the Socratic *elenchus*, found in such dialogues as the *Euthyphro*, *Laches*, *Charmides* and *Lysis*. Bonitz identifies twenty-one different eristic arguments in the *Euthydemus*.[56] Each begins with the assertion of a thesis. A second party (either Euthydemus or Dionysodorus, depending upon the particular case) presses the first with questions. Most of these are Yes-No questions, to which the expected answer is nearly always in the affirmative. In some cases, the questions have an Either-Or structure, and the answerer responds by picking one of the disjuncts.[57] The questioner attempts to draw conclusions from the answerer's responses. Usually this is done deductively. "The refutation is successful when the questioner is able to draw from his interlocutor's admissions either some conclusion incompatible with the original thesis (not necessarily its direct contractory) or some absurdity whose derivation used the thesis as a premiss."[58]

The form of the interplay between Euthydemus and his brother Dionysodorus is very similar to that of the Socratic *elenchus*, except that tougher constraints are imposed on what the answerer is permitted to say. Thus an eristic argument is an *elenchus* with stiffer rules.

[56] See [Bonitz, 1968]

[57] In argument 19 there are three examples of information-solicitation questions, *e.g.*, "Can you name three types of craftsmen by the work that they do?" Again, see [Bonitz, 1968].

[58] [Hitchcock, 2000b, p. 60].

Eristic arguments are not problem free, something that Aristotle would attend to in his *On Sophistical Refutations.* As Hitchcock says,

> The most probable origin of professional eristic ... is Socrates himself. This is not to say that the brothers got their repertoire of fallacious tricks from Socrates, but that they practised the type of refutation in which Socrates engaged, and inserted into it the trickery which subsequently earned the name 'sophistry' [Hitchcock, 2000b, p. 63].

The distinction between syllogisms-as-such and syllogisms-in-use affects Aristotle's conception of refutation in an interesting way. The distinction is exemplified by those arguments (syllogisms-as-such) that are the core of refutations (syllogisms-in-use). How, then, do refutations work? As with eristic arguments, Aristotle provides that there are two participants, Q, a questioner, and A, an answerer. A proposes a thesis T.[59] Q's role is to question (*erotan*) A, putting questions to him, answers to which are formatted as simple (non-compound) declarative sentences, or propositions (*protaseis*) in Aristotle's technical sense of the term. A's answers are thus available to Q as premisses of a syllogism, (let's call it *Ref*), which it is Q's role to construct. If *Ref is* constructed and if its conclusion is the contradictory, $\neg T$, of A's original thesis, then Q's argument is a refutation of T. In this we see the pure form of Locke's *ad hominem*, for *Ref*'s premisses are A's own principles and concessions (and nothing else); and *Ref*'s conclusion, the contradictory of A's thesis, is got by pressing those concessions with their consequences. For concreteness, we now imagine a simple case of refutation. A's thesis is T. Q's refutation is

$$\begin{array}{l} C \\ B \\ \hline \text{Therefore, } \neg T \end{array}$$

in which C and B are A's concessions and $\neg T$ is syllogistically derived from these. It is a noteworthy feature of our case—a feature which generalizes to all refutations — that A's thesis T cannot be a premiss of *Ref.* Here is why. Suppose that

$$\begin{array}{l} T \\ A \\ \hline \neg T \end{array}$$

were a valid argument. As it stands, it has the appearance of a *reductio*, provided that we understand T as a hypothesis rather than as a premiss. But *reductios* are not refutations. If our present argument is to make the grade as a refutation, it must be a syllogism. Since it is a valid argument, the set of its premisses, together with the negation of its conclusion, is inconsistent. This is the set $\{T, A, T\}$, which is the same set as the set of the argument's premisses alone. But syllogisms cannot

[59]A word of caution: in Aristotle's usage a thesis is a paradoxical claim. This is not here its intended sense. Aristotle's word is *problema*.

have inconsistent premiss sets. Hence our argument is not a syllogism, and not a refutation.

Since *Ref* is a syllogism, $\{C, B, T\}$ is an inconsistent set. Hence at least one sentence of the three is false. Because *Ref* is a refutation *of* T, it is attractive to suppose that *Ref* establishes that it is T that is false (for what else would *Ref*'s refutation of T consist in but showing that T is false?) Yet this will not do. Saying so is fallacious. It is the fallacy of distributing negation through conjunction, said by some to be Aristotle's fallacy of Noncause as Cause. (It is not, but let that pass; see [Woods and Hansen, 2003].) What we require is some principled reason to pick out T, *rather than* C or B, as the unique proposition refuted by *Ref*. How are we to do this?[60] There are two possible answers to consider. The first will prove attractive to people who favour a broadly dialectical conception of fallacies. The second will commend itself to those who think of fallacies as having a rather more logical character. We examine these two possibilities in turn.

First Answer (Dialectical): The first answer proposes that question-and-answer games of the sort played by Q and A are subject to the following pair of linked dialectical rules:

> *Premiss-Selection Rule:* In any dispute between Q and A in which Q constructs a refutation of A's thesis T, Q may use as a premiss of his refutation any *affirmation* of A provided that it is subject to the *no-retraction rule.*
>
> *No-Retraction Rule:* In any such dispute as above, no answer given by A to *a question of* Q may be given up by A.

It is entirely straightforward that among A's affirmations, germane to his defence of T, T itself is uniquely placed in *not* being subject to the No-Retraction rule. If it were, then once affirmed it could not be retracted. But if it couldn't be retracted, it couldn't be refuted (or, more carefully, couldn't be given up for having been refuted). This would leave refutations oddly positioned. The rules of the game being what they are, a refuted proposition would be precisely what the refutation could not make (or allow) A to abandon. Thus it may be said that, according to our first answer, if our refutation game is a procedurally coherent enterprize, then, of all of A's relevant affirmations, T alone stands out. It is the only affirmation that A *can* retract and it thus qualifies for the proposition which Q's refutation, *Ref*, refutes.

It may be thought, even so, that it is unrealistically harsh to hold all of A's other affirmations to the satisfaction of the No-Retraction rule. A little reflection will discourage the complaint. In the give-and-take of real life argument some latitude is given to A. He is allowed to retract some affirmations some of the time. There are limits to such sufferance. If there were not, then any thesis would be made immune to any refutation of it. If A always had the option of cancelling one

[60] The same problem visits the idea of counterevidence to a scientific theory construed *holistically*.

of Q's premisses, rather than giving up his own T, then T would be made strictly irrefutable by any Q whose opponent were prepared to exercise the option. Thus the No-Retraction rule can be considered an idealization of this limit.

Second Answer (Logical): According to our second answer, that T is the uniquely positioned proposition that Q's *Ref* refutes can be explained with greater economy as follows. Looking again at *Ref*,

$$\begin{array}{l} C \\ B \\ \hline \text{Therefore, } \neg T \end{array}$$

we see that T is distinguished as the proposition refuted by *Ref* precisely because it satisfies a certain condition. Before stating the condition, it is necessary to introduce a further fact about syllogisms. Aristotle requires that syllogisms always have consistent premiss sets. For suppose that they did not, and let

$$\begin{array}{l} A \\ \neg A \\ \hline \text{Therefore, } B \end{array}$$

be a syllogism. Then since syllogisms obey the argument-conversion operation, and since conversion preserves syllogisity, our imaginary syllogism converts to

$$\begin{array}{l} A \\ \neg B \\ \hline \text{Therefore, } A \end{array}$$

in which the conclusion repeats a premiss. This is explicitly forbidden by Aristotle's definition:

> A *syllogismos* rests on certain statements such that they involve necessarily the assertion of something other than what has been stated (*On Sophistical Refutations*, 1, 165^a,1–3).

Whereupon, since the second argument is not a syllogism and yet is an argument converse of the first, neither is the first a syllogism. (This ends our current aside.)

The condition proposed by the second answer to our question is now given as follows:

> *C1:* Since *Ref* is a syllogism, its premiss set is consistent. Let *Aff* be the set of A's affirmations with respect to *Ref*. Thus *Aff* is the set $\{T, C, B\}$. Since *Ref* is valid *Aff* is inconsistent. It is easy to see that *Aff* possesses exactly three maximal consistent subsets: $\{C, B\}$, $\{C, T\}$, $\{B, T\}$. We will say that a maximal consistent subset of *Aff* is *excluded* by *Ref* if and only if it does not *syllogistically* imply *Ref*s conclusion, $\neg T$. Thus $\{C, T\}$ and $\{B, T\}$ are excluded by *Ref*, and $\{T\}$ is their intersection. Thus T is *the* proposition refuted by *Ref* precisely because it is the sole member of the intersection of all maximal consistent subsets of *Aff* excluded by *Ref*.

It is worth noting that the requirement that syllogisms be consistently premissed bears directly on our present question. T is dignified as the proposition which *Ref* refutes by virtue also of the requirement that *Ref* have consistent premisses. Since *Ref* is a syllogism then $\{C, B, T\}$ is inconsistent and $\{C, B\}$ is consistent. So, too, are $\{C, T\}$ and $\{B, T\}$, since if they were not, C would entail $\neg T$ and B would entail $\neg T$, a happenstance precluded, each time, by *Ref*'s syllogisity. Thus three ideas cohere: (1) the idea that T is unique in *Aff* precisely because it is T that cannot be a premiss of *Ref*; (2) the idea that syllogisms must be consistently-premissed; and (3) the idea that $\{T\}$ is the intersection of all maximal consistent subsets of *Aff* excluded by *Ref*.

Just how narrowly "logical" is this characterization? Or, more carefully, how non-dialectical is it? Suppose that we said that the idea of the proposition that a refutation refutes could be analyzed without any reference to dialectical procedure. Then any syllogism whatever would count as the refutation of the contradictory of its own conclusion. Every syllogism would be a refutation, and a great many refutations would be refutations of propositions no one has ever proposed or will. There is no great harm (in fact, there is considerable economy) in speaking this way. But it is not Aristotle's way. Taken his way, refutations can arise only in question-and-answer games of the sort that we have been considering. We may say, then, that refutations have a *dialectically minimal* characterization. That is, (a) their premisses must be answers given by A to Q, and (b) what they refute must be theses advanced by A. Nothing further is needed beyond this dialectical minimum. In particular, there is no need to invoke the dialectical rules, Premiss-Selection and No-Retraction. Premiss-Selection is unneeded in as much as there is an entirely non-dialectical reason for excluding T as a premiss of any refutation of T. As we have seen, no argument of the form

$$\begin{array}{l} A_1 \\ \vdots \\ A_i \\ \vdots \\ A_n \\ \hline \text{Therefore, } \neg A_i \end{array}$$

is a syllogism.

Nor, as we have also seen, is the No-Retraction Rule needed to enable the specification of T as the unique proposition refuted by a refutation. Thus the idea of refutations as refutations of some unique T can be specified without exceeding what we have been calling the dialectical minimum. It is in precisely this sense that our target notion is a "broadly logical" matter.

Whether we find ourselves drifting toward a dialectical explication of that which a refutation refutes or to a more narrowly syllogistic specification of it, it is fundamentally important that on neither construal is a refutation of T *definitively probative* for T. Refutations do not in the general case falsify what they refute. In

contrast to this, *reductio ad impossible* arguments are probative, but they are not refutations in the present sense. They lack the requisite forms.[61] It is not that refutations fail to falsify something. They always falsify the conjunction of the members of *Aff*. And it is not that refutations are indeterminate. They always refute some unique single proposition. What refutations do not manage to do in the general case is to bring what they falsify into *alignment* with what they refute. What they falsify and what they refute are different propositions. We have said that refutations do not falsify what they refute, in the general case. We must ask whether there might be exceptions to this. There are. Sometimes the premisses of a refutation, Ref^t, are (known to be) true. If so, Ref^t constitutes a proof of the falsity of T, when T is the proposition that Ref^t refutes. Does it not follow from this that although refutations do not in general demonstrate the falsity of what they refute, sometimes they do?

Suppose that Ref^t is

$$\begin{array}{l} X \\ Y \\ \hline \text{Therefore, } \neg T \end{array}$$

in which X and Y are (known to be) true. As before, $\{X, Y, T\}$ is an inconsistent set, but there is a difference. Ref^t *falsifies* T. This makes for a curious dialectical symmetry between A and Q. A's obligation to answer honestly is the obligation to offer to Q premisses which he (*i.e.*, A) believes to be true. Thus for any rule-compliant answerer against whom there is a successful refutation, Ref^t, given what A is required to believe, he must also believe that Ref^t falsifies his own thesis T. Though this is what, in all consistency, A is required to believe, it does not follow that it is true, nor need A himself believe that its truth follows from what he is required to believe.

Q, on the other hand, is differently positioned. Q has no role to play in the semantic characterization of A's answers, hence in judging the truth values of his (*i.e.*, Q's) premisses. Of course, Q will often have his own opinions about the truth or falsity of A's replies, hence about the truth or falsity of his (Q's) own premisses. Whatever such opinions are, they have no role to play in the construction of Q's refutation. There is in this a strategic point. Since the success or failure of Q's refutation of A's thesis, T, is independent of the truth values of the refutation's premisses, it might be thought that the most appropriate stance for Q to take toward those premisses is the one that shows A to best advantage. In fact, there is *no* semantic stance which Q can take towards A's answers, because there is none which shows A to best advantage.

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[61] For one thing, *reductio* arguments allow for a special class of non-premisses as lines in the proof, *i.e.*, *assumptions*. For another, the conclusion of a *reductio* is a pair $\{\Phi, \ulcorner\neg\Phi\urcorner\}$, which, though a contradic*tion*, is not the contradic*tory* of the proof's assumption.

11 *AD HOMINEM* PROOF

In several passages in *On Sophistical Refutations*, Aristotle seems to think that refutations are proofs, but in a looser sense of "proof" than *reductio* arguments.[62] In other places, refutations appear to be proofs in no sense of the word.[63] For

> I mean, 'proving by way of refutation' to differ from 'proving' in that, in proving, one might seem to beg the question, but where someone else is responsible for this, there will be a refutation, not a proof (*Metaphysics*, 1006^a, 15-18).[64]

Thus

> In such matters there is no proof simply, but against a particular person, there is (*Metaphysics*, K5, 1062^a, 2–3).[65]

This is Ross. In Barnes' version we have it that

> About such matters there is no proof in the full sense, though there is proof *ad hominem*.[66]

It is hardly imaginable that there should be *any* contention about the origins of the phrase "*ad hominem*" to characterize a particular class of arguments. We owe the concept not to Locke (as Locke himself expressly said), not to Galileo, not to Boethius, but to Aristotle himself. It is clear that Aristotle is of two minds about the *ad hominem*. He is tempted to think of *ad hominem* arguments both as proofs of no kind and as proofs of some kind. Aristotle expressly contrasts arguments "against the man" with arguments against the man's position, and the former are considered substandard in *some* way, as witness *On Sophistical Refutations* 20, 177^b, 31–34, and 22, 178^b, 16–23.

At times (see *e.g.*, *On Sophistical Refutations*, 8, 170^a, 12–19), Aristotle tries to draw a distinction within the class of refutations between those that turn on *ad hominem* moves and those that do not. The former he condemns outright as sophistical refutations.[67]

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[62] *On Sophistical Refutations* 167^b, 8–9 ff. *Cf. Prior Analytics* B27, 70^a, 6–7 and *Rhetoric* Γ13, 1414^a, 31–37.

[63] See *On Interpretation* 11, 21^a, 5 ff. *Cf. Metaphysics* Δ5, 1015^b, 8, and *Posterior Analytics* A9, 76^a, 13 15.

[64] This is the Ross translation [Ross, 1984]. [Barnes, 1984] has it this way: "Now negative demonstration I distinguish from demonstration proper, because in a demonstration one might be thought to be assuming what is at issue, but if another person is responsible for the assumption we shall have negative-proof, not demonstration."

[65] *Cf. Metaphysics* K5, 1062^a, 30 31 and Γ4, 1006^a, 25 26.

[66] *Cf. On Sophistical Refutations*, 170^a, 13, 17 18, 20; 177^b, 33 34; 178^b, 8-17; 183^a, 22, 24; and *Topics*, Φ11, 161^a, 21.

[67] *Cf. On Sophistical Refutations*, 1, 164^a, 20ff.

Thus we have two contrasts to keep track of, and whose confusion is ruinous for a correct understanding of Aristotle's position. There is the contrast,[68] first, between a proof "simply" and a refutation or argument *ad hominem*. In relation to this contrast, the following things can be said: first, *ad hominem* arguments are not sophistical or fallacious; and, second, *ad hominem* arguments are refutations, hence not proofs "simply." What is it to fail to be a proof simply? There are two possibilities, and Aristotle anticipates them both. One way of not being a proof simply is being what we have called a *non-falsifying* refutation. Non-falsifying refutations are in *no* sense proofs against the propositions they refute. Another way of not being a proof simply is being what we have called a *falsifying* refutation. Falsifying refutations are proofs in some sense, but they are not proofs in every sense. For example, they are not demonstrations in which there is a strict epistemic priority rating on the premisses of the refuting syllogism.

That is the story of the first contrast. The second contrast is another matter. It is a contrast between *ad hominems* in two separate senses. In the first sense, an argument is *ad hominem* just in case it qualifies as a refutation. Arguments that are *ad hominem* in the second sense are defective would-be *ad hominems* in the first sense and, as such, reasonably can be regarded as fallacious. Aristotle does not list the *ad hominem* in his catalogue of thirteen fallacies, as set out in *On Sophistical Refutations*; at least he does not give any of the thirteen the name "*ad hominem*." Even so, the account that he does give, such as it is, leaves plenty of room to accommodate *ad hominems* in the second sense in the category of *ignoratio elenchi*. Also possible in principle is accommodation of *ad hominem* fallacies known to a much later tradition as "circumstantial," "abusive" and "*tu quoque*."

This does not change the fact that, for Aristotle, the dominant notion of *ad hominem* argument is Lockean (if the anachornism may be forgiven). It is a concept of *ad hominem* that is nicely captured by the structure of refutation. With refutations as such there is no question of fallaciousness. The problem rather is how closely refutations resemble proofs. Aristotle has two answers: refutations resemble proofs not at all; and refutations resemble proofs loosely. As we see, this is a distinction nicely preserved by the distinction between non-falsifying and falsifying refutations.

12 SOPHISTICAL REFUTATIONS

We have already made the point that there is nothing in *On Sophistical Refutations* that would qualify as a full and mature theory of any fallacy there discussed. We have conjectured that Aristotle's apparent *theoretical* indifference to the fallacies might be an adumbration of the perfectibility thesis of *Prior Analytics*. For recall that if the perfectibility thesis is true, then syllogisms turn out to be effectively recognizable. If this is so, the distinction between genuine and only apparent

[68] *Metaphysics* K5.

syllogisms (*i.e.*, fallacies) becomes wholly transparent, thus making it unnecessary to have accounts of the thirteen fallacies in which *they* are effectively recognizable.

Even so, Aristotle does write at some length about the fallacies. He does so in the context of a particular kind of argument, of which refutations are a notable subcase. (We note in passing that fallacies are also discussed in the *Analytics* and *Rhetoric* in the context of different kinds of arguments than those discussed in *On Sophistical Refutations*, *viz.*, *demonstrations* and *enthymemes*, respectively.) In *On Sophistical Refutations* Aristotle gives fairly full accounts of sixty four[69] examples of sophistical refutations which are only apparently syllogisms. Of these, forty-nine have, by the lights of *Prior Analytics*, the wrong number of premisses, or premisses or conclusions of the wrong sort.

Aristotle's account of sophistical refutations begins with a discussion of "contentious arguments," a sort of intellectual contest commonly performed in the Greek academies, courts and councils. It is quite clear that at one level *On Sophistical Refutations* is a practical manual in which types of manoeuvres that result in unsatisfactory resolutions of contentious arguments are identified, and methods for spotting and blocking them are suggested; but at another level Aristotle is less interested in the practical question of how to train people to win argumentative contests than he is in developing a theory of objectively good reasoning.

At *On Sophistical Refutations* 16, 175^a, 5–17, Aristotle explains the importance of a theory of contentious argument:

> The use of [contentious arguments], then, is for philosophy, two-fold. For in the first place, since for the most part they depend upon the expression, they put us in a better condition for seeing in how many ways any term is used, and what kind of resemblances and what kind of differences occur between things and between their names. In the second place they are useful for one's own personal researches; for the man who is easily committed to a fallacy by someone else, and does not perceive it, is likely to incur this fate himself also on many occasions. Thirdly [sic] and lastly, they further contribute to one's reputation, *viz.*, the reputation of being well trained in everything, and not experienced in anything: for that a party to arguments should find fault with them and yet cannot definitely point out their weakness, creates a suspicion, making it seem as though it were not the truth of the matter but inexperience that put him out of temper.

Aristotle is concerned to set out various ways in which a would-be refutation fails. He classifies failed refutations into those that depend on language and those that depend on factors external to language, although it may be closer to Aristotle's intentions here to understand the word "language" as "speech." Aristotle is aware that some fallacies arise because a given word may be used ambiguously. However, when the argument in question is *spoken*, the offending word is given a different pronunciation at different occurrences. Thus hearing the argument,

[69] A further fifty-five examples are alluded to more briefly; see [Dorion, 1995, p. 93].

rather than reading it, sometimes flags the ambiguous term and makes it easy for the arguer to avoid the ambiguity. For such fallacies, the mediaevals used the term *in dictione*, and it would appear that what is meant are fallacies whose commission is evident by speaking the argument. Not all fallacies can be identified just through speaking the arguments in which they occur. The mediaevals translated Aristotle's classification of these as *extra dictionem* fallacies, that is, as not being identifiable by speaking them. On the other hand, Aristotle also says that there are exactly six ways of producing a "false illusion in connection with *language*" (165^b, 26), (emphasis added), and his list includes precisely six cases. Further, Aristotle occasionally notices that some of his *extra dictionem* fallacies also qualify for consideration as language dependent, for example, *ignoratio elenchi* (167^a, 35) and *many questions* (175^b, 39). So the modern day practice of taking the *in dictione* fallacies to be language-dependent and the *extra dictionem* fallacies to be language-independent finds a certain justification in Aristotle's text. The following schema presents itself:

Table 3. *Sophistical Refutations*

In Dictione	***Extra Dictionem***
(1) equivocation	(7) accident
(2) amphiboly	(8) *secundum quid*
(3) combination of words	(9) *ignoratio elenchi*
(4) division of words	(10) consequent
(5) accent	(11) non-cause as cause
(6) form of expression	(12) begging the question
	(13) many questions

It is immediately evident that Aristotle's placement of these sophistries does not fit especially well with the "discernible in speech versus not discernible in speech" distinction. For example, *equivocation* involves the exploitation of a term's ambiguity, and can be illustrated by the following argument:

The end of life is death
Happiness is the end of life
———
Therefore, happiness is death.

But this is a mistake that is not necessarily made evident just by speaking the argument. It is interesting that in this example a certain logical form is discernible, *viz.*,

T is D
H is E
———
Therefore, H is D

in which H stands for "happiness," E for "the end of life" in the sense of the *goal* or *purpose* of life, T for "the end of life" in the sense of the *termination* of life, and D for "death." The form is certainly invalid; it commits what later writers would call the "fallacy of four terms." It does not however commit the fallacy of *ambiguity*, since in it the term "end" is fully disambiguated.

In contrast, *amphiboly* arises from what today is called *syntactic* (as opposed to lexical) ambiguity, as in the sentence "Visiting relatives can be boring," which is ambiguous between (1) "Relatives who visit can be bores" and (2) "It can be boring to visit relatives." To see how amphiboly can wreck an argument, consider,

Visiting relatives can be boring
Oscar Wilde is a visiting relative
Therefore, Oscar Wilde can be boring.

If the first premiss is taken to have the meaning of (1), the argument is a syllogism. If it is taken to have the meaning of (2), the argument is not a syllogism but a *paralogismos*, a piece of "false reasoning." Even so, our case seems to collapse into an ambiguity fallacy, with "visiting relatives" the offending term—ambiguous between "the visiting of relatives" and "relatives who visit." Here, too, an amphibolous argument seems not to be one that an arguer would be alerted to automatically just by speaking it, although with sufficient oral emphasis the appropriate distinctions might be made: "VISITING relatives can be boring" may mean something quite different than "visiting RELATIVES can be boring."

The next two types of sophistical refutation, *combination* and *division of words*, can be illustrated with the example of Socrates walking while sitting. Depending on whether the words "can walk while sitting" are taken in their combined or their divided sense, the following is true or not:

Socrates can walk while sitting.

Taken as combined, the claim is false, since it means that

Socrates has the power to walk-and-sit at the same time.

However, in their divided sense, these words express the true proposition that

Socrates, who is now sitting, has the power to stop sitting and to start walking.

This is a better example of an *in dictione* fallacy. "Socrates, while sitting, CAN WALK" sounds significantly different from "Socrates can WALK WHILE SITTING." In this context, it is worth noting that composition and division fallacies of the present day are not treated as fallacies *in dictione* [Copi and Cohen, 1990, pp. 17–20]. Rather they are understood to be fallacies that result from mismanaging the part-whole relationship. Thus the modern fallacy of composition is exemplified by

All the members of the Oakland *As* are excellent players
Therefore, the Oakland *As* are an excellent team.

Division fallacies make the same mistake, but in the reverse direction, so to speak:

The *As* are a top-ranked team
Therefore, all the *As* players are top-ranked players.

We see, then, that combination and division of words is an *in dictione* fallacy for Aristotle, whereas composition and division is an *extra dictionem* fallacy for later writers.

Accent and *form of expression*, are perhaps rather difficult for the reader of English to understand, since English is not accented in the way, for example, that French is. It is troublesome that the Greek of Aristotle's time was not accented either; that is, that there were no syntactic markers of accent such as "é" (acute); "è" (grave); and "â" (circumflex). Even so, Aristotle *introduces* accents in his discussion of Homer's poetry. Conceding that "an argument depending upon accent is not easy to construct in unwritten discussion; in written discussion and in poetry it is easier" (*Sophistical Refutation*, 166^b, 1–2), Aristotle notes that

> some people emend Homer against those who criticize as absurd his expression τὸ μὲν οὖ καταπύθεται ὄμβρῳ. For they solve the difficulty by a change of accent, pronouncing the *øυ* with an acute accent (166^b, 2-6).

The emendation changes the passage from "Part of which decays in the rain" to "It does not decay in the rain," a significant alteration to say the least.

Form of expression is meant in the sense of "*shape* of expression" and involves a kind of ambiguity. Explains Aristotle,

> Thus (*e.g.*,) 'flourishing' is a word which in the form of its expression is like 'cutting' or 'building'; yet the one denotes a certain quality—*i.e.*, a certain condition—while the other denotes a certain action (166^b, 16–19).

Hamblin avers with a certain pungency that "[i]t was given to J.S. Mill to make the greatest of modern contributions to this Fallacy by perpetrating a serious example of it himself .... He said ...

> The only proof capable of being given that an object is visible, is that people actually see it. The only proof that a sound is audible, is that people hear it; and so of the other sources of our experience. In like manner, I apprehend, the sole evidence it is possible to produce that anything is desirable, is that people do actually desire it.

But to say something is visible or audible is to say that people *can* see or hear it, whereas to say that something is desirable is to say that it is *worthy* of desire or,

plainly, a good thing. Mill is misled by the termination '-able' " [Hamblin, 1970, p. 26]. Unfortunately here, too, we seem not to have an especially convincing example of a fallacy discernible in speech.

Turning now to *extra dictionem* fallacies, *accident* also presents the present-day reader with a certain difficulty. The basic idea is that what can be predicated of a given subject may not be predicable of its attributes. Aristotle points out that although the individual named Coriscus is different from Socrates, and although Socrates is a man, it would be an error to conclude that Coriscus is different from a man. This hardly seems so, at least when in the conclusion "is different from a man" is taken to mean "is not a man." The clue to the example is given by the name of the fallacy, "accident." *Part* of what Aristotle wants to say is that when individual X is different from individual Y, and where Y has the accidental or non-essential property P (*e.g.*, being six feet tall), it does not follow that X is not six feet tall, too. But this insight is obscured by two details of Aristotle's example. The first is that "is a man" would seem to be an essential property of man. Here, however, Aristotle restricts the notion of an essential property to a *synonymous* property, such as "is a rational animal." The other obscuring feature of the case is that Aristotle also wants to emphasize that what is predicable of an individual is not necessarily predicable of its properties. If we take "Coriscus is different from" as predicable of Socrates, it does not follow that it is predicable of the property *man*, which Socrates has. But why should this be so if no individual (Coriscus included) is identical to any property (including the property of being a man)? Perhaps it is possible to clarify the case by differentiating two meanings of "is different from a man." In the one meaning, "different from a man" is the one-place negative predicate "is not a man"; and in its second meaning it is the negative relational predicate "is not identical to the property of being a man." Thus, from the fact that Coriscus and Socrates are different men, it does not follow that Coriscus is not a man. But it does follow that Coriscus is not identical to the property of being any man. Little of this treatment survives in present day accounts. For example, in [Carney and Scheer, 1980, p. 72], the fallacy of accident is just a matter of misapplying a general principle, that is, of applying it to cases "to which they are not meant to apply."

Secundum quid is easier to make out. "*Secundum quid*" means "in a certain respect." In this sense, the error that Aristotle is trying to identify involves confusing the sense of a term in a qualified sense with its use in its absolute, or unqualified, sense. Thus from the fact that this black man is a white-haired man, it does not follow that he is a white man. Similarly, from the fact that something exists in thought it does not follow that it exists in reality (Santa Claus, for example).

In its most general form, the *secundum quid* fallacy is the mistake of violating the rule that, when reasoning or arguing, our claims and counterclaims about things must honour significant similarities and must not over exploit differences. If one party asserts "All A are B," it is not enough for the second party to attempt to confuse this opponent with the statement "Some A are not B," even if it is true. Also required is that his use of the terms "A" and "B" must agree with those of his

opponents with regard to meaning, respects in which the term is applied, temporal factors, and so on. Thus if the one party's "All A are B" were "All bachelors are unmarried" and his opponent's "Some A are not B" were "Some holders of a first university degree are married (eventually)," it would be ludicrous to suppose that, even if true, it damages the first claim in any way.

Ignoratio elenchi, or "ignorance of what makes for a refutation," results from violating any of the conditions on what constitutes a proper refutation. As we have pointed out, a refutation is genuine when one party, the questioner, is able to fashion from the other party's (the answerer's) answers a syllogism whose conclusion is the contradictory, not-T, of the answerer's original thesis, T. There are thus two ways in which the questioner might be guilty of *ignoratio elenchi*. He might have made the *syllogistic*-mistake of supposing that not-T follows from the premisses when it does not or, although it does follow from those premisses, one or more of them is syllogistically impermissible. For example, Aristotle requires that all premisses of a syllogism be ***propositions*** and, as we have pointed out, propositions are statements in which just one thing is predicated of just one thing (*On Sophistical Refutations* 169^a, 8). Thus the statement, "Bob and Sally are going to the dance," is not a proposition, even though it clearly implies "Bob is going to the dance," which *is* a proposition. As we have seen, Aristotle has technical reasons for restricting the premisses of syllogisms to propositions; and it is clear that he thinks that if a questioner derives a conclusion from non-propositions which imply it, he has not constructed a syllogism. Accordingly, he has not constructed a refutation. In fact, he has committed the fallacy of many questions (see below).

The second way in which a questioner can be guilty of ignorance of what makes for a (genuine) refutation is when he constructs from his opponent's answers a faultless syllogism, but its conclusion is not the contradictory of his opponent's thesis, T. It thus is the mistake of supposing that a pair of propositions $\{P, Q\}$ are one another's contradictories when they are in fact not. If the first type of error can be called a *syllogistic error*, the second can be called a *contradiction error* [Hansen and Pinto, 1995, p. 321]. This has a bearing on how Aristotle understands the relationship of fallacies to sophistical refutations. Some commentators hold that fallacies and sophistical refutations are the same thing. Others are of the view that a refutation is sophistical just because it *contains* a fallacy, *i.e.*, when the would-be syllogism that constitutes the would-be refutation commits either a syllogistic error or a contradiction error—a *logical* error in each case. Aristotle even goes so far as to suggest a precise coincidence between the *in dictione—extra-dictionem* distinction and the distinction between contradiction errors and syllogistic errors:

> All the types of fallacy, then, fall under ignorance of what a refutation is, those dependent on language because the *contradiction*, which is the proper mark of a refutation, is merely apparent, and the rest because of the definition of *syllogism* (*On Sophistical Refutations* 6, 169^a, 19-21; emphasis added; *cf.* [Hansen and Pinto, 1995, p. 321].)

In present-day treatments (*e.g.*, [Copi and Cohen, 1990, pp. 105–107]), the *ignoratio elenchi* is the fallacy of an argument which appears to establish a certain conclusion, when in fact it is an argument for a different conclusion. There is some resemblance here to Aristotle's contradiction-error, which can be considered a special case.

Aristotle says (*On Sophistical Refutations* 168^a, 27; 169^b, 6) that the fallacy of consequent is an instance of the fallacy of accident. Bearing in mind that Aristotle thinks that consequent involves a conversion error, perhaps we can get a clearer picture of accident. As noted above, accident is exemplified by a confusing argument about Coriscus and Socrates. We might now represent that argument as follows:

(1) Socrates is a man
(2) Coriscus is non-identical to Socrates
(3) Therefore, Coriscus is non-identical to a man.

In line (1), the word "is" occurs as the is-of-predication. Suppose that line (1) were in fact *convertible*, that is, that (1) itself implied

(1′) A man is Socrates.

In that case, the "is" of (1) would be the is-of-identity, not the is-of-predication, and the argument in question would have the valid form

(1′) $S = M$
(2′) $C \neq S$
(3′) Therefore, $C \neq M$.

Thus the idea that (1) is convertible and the idea that its "is" is the is-of-identity come to the same thing and this is the source of the error. For the only interpretation under which

Socrates is a man

is true, is when "is" is taken non-convertibly, *i.e.*, not as the is-of-identity, but as the is-of-predication.

Consequent is an early version of what has come to be known as the fallacy of affirming the consequent [Copi and Cohen, 1990, pp. 211, 282]. In present-day treatments this is the mistake of concluding that P on the basis of the two premisses, "If P then Q," and Q. Where P is the *antecedent* and Q the *consequent* of the first premiss, the fallacy is that of accepting P on the basis of having affirmed the consequent Q. Aristotle seems to have this kind of case firmly in mind; but he also thinks of consequent as a conversion fallacy, that is, as the mistake of inferring "All P are S" ("All mortal things are men") from "All S are P" ("All men are mortal").

Non-cause as Cause also appears to have been given two different analyses. In the *Rhetoric* it is the error that later writers call the fallacy of *post hoc, ergo*

propter hoc, the error of inferring that event e is the *efficient cause* of event e' just because the occurrence of e' followed upon (temporally speaking) the occurrence of e. In *On Sophistical Refutations*, however, it is clear that Aristotle means by "cause" something like "reason for." In this case, the non-cause as cause error is exemplified by the following type of case. Suppose that

$$\begin{array}{l} P \\ \underline{Q \qquad\qquad\qquad} \\ \text{Therefore, } \neg T \end{array}$$

is a refutation of the thesis T. Then the argument in question is a syllogism, hence a valid argument. As any reader of modern logic knows, if the argument at hand is valid, then so too is the second argument,

$$\begin{array}{l} R \\ P \\ \underline{Q \qquad\qquad\qquad} \\ \text{Therefore, } \neg T \end{array}$$

no matter what premiss R expresses. But this second argument is not a syllogism since a proper subset of its premisses, namely, $\{P, Q\}$, also entails its conclusion. Hence our second argument cannot be a refutation. This matters in the following way: Aristotle thinks of the premisses of a refutation as reasons for ("causes of") its conclusion; but since our second argument is *not* a refutation of T, R cannot be a reason for not-T.

In the course of real-life contentions, an answerer will often supply the questioner with many more answers than the questioner can use as *premisses* of his would-be refutation. Aristotle requires that syllogisms have no idle premisses. Thus the questioner is obliged to select from the set of his opponents' answers just those propositions, no more and no fewer, than non-circularly necessitate the required conclusion.

In this sense, the fallacy of non-cause as cause is clearly the mistake of using an idle premiss, but it may not be clear as to why Aristotle speaks of this as the error in which a *non-cause* masquerades as a *cause*. Something of Aristotle's intention may be inferred from a passage in the *Physics* (195^a, 15), in which it is suggested that in syllogisms premisses are the *material causes* (the stuff) of their conclusions, *i.e.*, that premisses stand to conclusions as parts to wholes, and hence are causes of the whole. Idle premisses fail to qualify as material causes; they can be removed from an argument without damaging the residual sub-argument. Real premisses are different. Take any syllogism and remove from it any (real) premisses and the whole (*i.e.*, the syllogism itself) is destroyed. In other places, Aristotle suggests a less technical interpretation of the fallacy, in which the trouble with R would simply be its falsity, and the trouble with the argument accordingly would be the derivation of not-T from a falsehood—a false cause (*On Sophistical Refutations* 167^b, 21).[70]

[70] Non-cause is discussed in greater detail and is given a somewhat different emphasis in [Woods and Hansen, 2003].

Aristotle provides several different treatments of *begging the question*, or *petitio principii.* In *On Sophistical Refutations* it is a flat-out violation of the definition of "syllogism" (hence of "refutation"). If what is to be proved is also assumed as a premiss, then that premiss is repeated as the conclusion, and the argument in question fails to be a syllogism. Hence it cannot be a refutation. On the other hand, in the *Posterior Analytics* 86^a, 21, begging the question is a *demonstration* error. Demonstrations are deductions from first principles. First principles are themselves indemonstrable, and in any demonstration every succeeding step is less certain than preceding steps; but if one inserts the proposition to be demonstrated among the premisses, it cannot be the case that all premisses are more certain than the conclusion. Hence the argument in question is a failed demonstration.

Aristotle recognizes five ways in which a question can be begged (*Topics* 162^b, 34-163^a, 2):

> People appear to beg their original question in five ways: the first and most obvious being if anyone begs the actual point requiring to be proved: this is easily detected when put in so many words; but it is more apt to escape detection in the case of synonyms, and where a name and an account mean the same thing. A second way occurs whenever anyone begs universally something which he has to demonstrate in a particular case ...

The third way is to beg a particular case of what should be shown universally; the fourth is begging "a conjunctive conclusion piecemeal" [Hamblin, 1970, p. 74]; and the fifth is begging a proposition from a proposition equivalent to it.

Contemporary readers are likely to find the expression "begging the question" rather obscure (and oddly dramatic). There is no beggary to begging the question, other than the questioner's soliciting a premiss for his evolving syllogism by putting to his opponent a Yes-No question. Further, Aristotle actually collapses the intuitive distinction between the question that produces the answer and the premiss that that answer is eligible to be. That is, what Aristotle calls a question is in this context the premiss produced by the answer to it. Strictly speaking, then, begging such a question is using it as a premiss in a would-be syllogism. The "original question" of the two parties is the answerer's thesis, T. An opponent would beg the question in the first way if he begged the point required to be shown, *i.e.*, not-T. But if not-T were indeed a premiss of any deduction whose conclusion were also not-T, the deduction would be circular, as we have said; hence it would be neither a syllogism nor a refutation. Aristotle rightly notices that such cases are unlikely to fool actual reasoners, but he reminds us that if a synonym of not-T were used as a premiss, then the premiss would *look* different from the conclusion, and the circularity might go undetected. In fact, this seems also to be what Aristotle has against the fifth way, *i.e.*, deducing a proposition from one equivalent to it.

In the second way of begging the question, Aristotle has in mind a certain form of what came to be called "immediate inference." It is exemplified by the subalternation argument

> Since all A are B, some A are B.

Aside from the fact that single-premiss arguments seem not to qualify as syllogisms, it is difficult to make out a logical fault here. Bearing in mind that the fault is the questioner's (*i.e.*, premiss selector's) fault, not the answerer's, one might wonder what is wrong with a questioner's asking a question which, if answered affirmatively, would give him a desired conclusion in just one step. Evidently Aristotle thinks that a refutation is worth having only if every premiss (individually) is consistent with the answerer's thesis. Such is Ross' view of the matter:

> And syllogism is distinguished from *petitio principii* in this, that while in the former both premises together imply the conclusion in the latter one premise alone does so [Ross, 1953, p. 38].

Thus in a good refutation, the thesis is refuted, never mind that the thesis is consistent with each separate answer given. (See [Woods, 2001, ch. 9].)

The third way of begging seems quite straightforward. It is illustrated by the plainly invalid form of argument

> Since some A are B, all A are B.

Even so, as the example makes clear, begging the question is fundamentally an error of *premiss selection*, given that the answerer's job is to elicit premisses that syllogistically imply its contradictory, "All A are B." Again, begging the question is selecting a premiss, and in the present case, the questioner has begged the *wrong* question, *i.e.*, he has selected the wrong premiss. It is a premiss which does the answerer's thesis no damage; and in any extension of this continuing argument in which damage were to be done, this premiss "Some A are B," would prove idle.

Genuinely perplexing is the fourth way of begging the question. By the requirement that syllogisms be constructed from propositions, it would appear that no competent syllogizer would ever wish to conclude his argument with a conjunctive statement (since these are not propositions). However, consider the argument

$$\frac{\begin{array}{l} P \\ Q \end{array}}{\text{Therefore, } P \text{ and } Q.}$$

It is clearly valid. What Aristotle appears to have in mind is that it is useless for the questioner first to beg for P, then for Q, if his intention is to conclude "P and Q." The manifest validity of the argument might deceive someone into thinking he had produced a syllogism, but the fault lies less with his premiss selection than

with his choice of target conclusion. Once begged, those questions will assuredly "get" him that conclusion, but it is a statement of a type that guarantees that his argument nevertheless is not a syllogism.

The problem of would-be refutations that derive their targets in one step from a single premiss is, according to Aristotle, the problem that the argument

All A are B

Therefore, some A are B

begs the question. The contradictory of its conclusion is inconsistent with each of the premisses (of which there happens to be only one). If it is correct to say that the contradictory of a syllogism's conclusion must be consistent with each premiss, then our argument is not a syllogism. On the other hand, Aristotle in places appears to accept subalternative arguments (*e.g.*, at *Topics* 119^a, 32–36). Some writers interpret Aristotle in a different way, as claiming an epistemic fallacy: one could not know the premiss to be true without knowing the conclusion to be true.

This raises a further matter as to whether, *e.g.*,

All A are B
All C are A

Therefore, all C are B

does not also beg the question. Mill is often said to have held that this is precisely the case with all syllogisms. This was not, in fact, Mill's view, but Aristotle seems to consider (and reject) the possibility (*Posterior Analytics* 72^b, 5–73^a, 20). If the first interpretation is correct, then were syllogisms fallaciously question-begging as such, it could not be for the reason that affects subalternation arguments. For

All A are B
All C are A

Therefore, all C are B

cannot be a syllogism unless its premisses fail individually to derive its conclusion; and it cannot commit a *petitio* of the subalternation variety unless one of its premisses does indeed yield the conclusion on its own.

As was said in the above discussion of *ignoratio elenchi*, Aristotle's fallacy of *many questions* is a very different thing from that presented in present day logic textbooks *e.g.*, [Copi and Cohen, 1990, pp. 96–97]. In such treatments, the fallacy is typified by such questions as, "Have you stopped beating your dog?," in which there is a unconceded presupposition, namely, "The addressee has been a beater of his dog in the past." Aristotle intended something quite different by the many questions fallacy. It is the error of admitting to the premiss set of a would-be syllogism a statement that is not a proposition, in Aristotle's technical sense of "one thing predicated of one thing." It was mentioned above that Aristotle had technical reasons for requiring syllogisms to be made up of propositions. This can

be explained as follows. In the *Topics* (100^a, 18–21) and *On Sophistical Refutations* (183^a, 37–36), Aristotle declares that his aim is to discover a method (or faculty of reasoning) from which we will be able to reason [syllogistically] about every issue from *endoxa*, *i.e.*, reputable premisses, and when compelled to defend a position, we say nothing to contradict ourselves.

In other places, his aims are forwarded more ambitiously. At *On Sophistical Refutations* 170^a, 38 and 171^b, 6–7, Aristotle says that the strategies he has worked out will enable a person to reason correctly about anything *whatever*, independently of knowledge of its subject matter. This is precisely what the Sophists also claimed to be able to do. Aristotle scorns their claim, not because it is unrealizable, but because the Sophists lack the theoretical wherewithal to bring it off. The requisite theoretical wherewithal Aristotle took to be the *logic of syllogisms.* In various respects Aristotle's boast seems incredible. For one thing, are there not far too many arguments, some of considerable complexity, for any one theory to capture in their totality? Part of Aristotle's answer lies in a claim advanced in *On Interpretation* (*e.g.*, 16^a, 19–26; 16^b, 6–10, 19–25; 16^b, 26–17^a, 2). There he asserts that anything stateable in any natural language such as Greek, can be expressed without relevant loss in a proper sublanguage made up exclusively of *propositions*, *i.e.*, statements that say one thing of one thing. Propositions may be said to be statements whose only logical particles are at most the quantifier expressions "all," "some," "no," "some—not," and the connective "not." In particular, compound statements held together by connectives such as "and," "or" and "if ... then," fail to qualify. Aristotle in effect is confining the class of propositions to the class of *categorical* statements: "All *A* are *B*," "Some *A* are *B*," "Some *A* are not *B*," and "No *A* are *B*." Suppose that a theorist wanted to produce a complete grammar for all the declarative sentences of Greek. If Aristotle's claim in *On Interpretation* is true, the theorist would succeed in his task if he could produce a complete theory of these four categorical forms. This very striking economy is also passed on to the claim that a theory of all good deductive reasoning is possible. By deductive reasoning, Aristotle means reasoning expressible in *syllogisms*, and syllogisms are made up of just three propositions, two premisses and a conclusion. Further, in every syllogism, there occurs exactly one more term than there are premisses. Thus any would-be syllogism will be made up of just two premisses and just three terms, say, "*A*," "*B*," and "*C*." There are just four propositional forms, and for each of the three terms only a low finite number of distributions of them in triples of those forms. Hence a complete theory of good reasoning (*syllogismos*) is possible because its domain is, as we now see, low finite, *i.e.*, it permits exhaustive examination.

In present-day treatments, the many questions fallacy is committed by asking a certain type of question, *e.g.*, "Have you stopped beating your dog?" In Aristotle's view, the fallacy is that of using an answer to the question as a *premiss.* Suppose that answer is "No." This is equivalent to

> It is not the case that (I have beaten my dog in the past and I do so at present)

or

Either I have not beaten my dog in the past or I do not do so at present.

In each case, the answer contains a connective other than "not"—"and" in the first instance, and "or" in the second. In neither case, then is the answer a proposition in which "one thing is said of one thing," so it is inadmissible as a premiss of a syllogism. Where the modern theorist sees the fallacy as an interrogative fallacy [Hintikka, 1987], for Aristotle it is a syllogistic error.

Although *On Sophistical Refutations* is the primary source of what people have come to call Aristotle's fallacies, Aristotle gives them a somewhat different characterization in his other writings. In the *Topics*, we read that an argument (not necessarily a *refutation*) is fallacious in four different ways: (1) when it appears valid but is invalid in fact; (2) when it is valid but reaches "the wrong conclusion"; (3) when it is valid but the conclusion is derived from "inappropriate" premisses; and (4) when, although valid, the conclusion is reached from false premisses. Case (1) might well be exemplified by the fallacy of affirming the consequent. Case (2) might be thought of in this way: let the premisses all be drawn from the discipline of economics, and let the conclusion be the logical truth, "Either it will rain today or it will not." Although that conclusion does follow validly from those premisses—at least by modern lights—it might be objected that it is the "wrong thing" to conclude from those premisses. Case (3) is similar. Aristotle's own example is one in which it is concluded that walking after a meal is not good for one's health (a conclusion from the art or discipline of medicine) from the premiss, proposed by Zeno, that motion (hence walking after a meal) is impossible. Even if Zeno's paradoxical proposition were true, Aristotle would claim it to be an inappropriate premiss for a medical argument, since it is not a *medical* premiss. Case (4) is obvious: though true conclusions are often compatible with false premisses, no true conclusion can be *established* by false premisses.

The following is a list, taken from [Hansen and Pinto, 1995, p. 9] of where in Aristotle's writings the individual fallacies are discussed:

- Equivocation: *Soph. Ref.*, 4 (165^b, 31–166^a, 7); 6 (168^a, 24); 7 (169^a, 22–25); 17 (175^a, 36–175^b, 8); 19; 23 (179^a, 15–19); *Rhetoric II*, 24 (1401^a, 13–23).

- Amphiboly: *Soph. Ref.*, 4 (166^a, 7–22); 7 (169^a, 22–25); 17 (175^a, 36–175^b, 19); 23 (179^a, 19–25).

- Combination of words: *Soph. Ref.*, 4 (166^a, 23–32); 6 (168^a, 22–25); 7 (169^a, 25–27); 20; 23 (179^a, 12–13); *Rhet. II* 24 (1401^a, 24–1401^b, 3).

- Division of words: *Soph. Ref.*, 4 (166^a, 33–39); 6 (166^a, 27); 7 (169^a, 25–27); 20; 23 (179^a, 12–13); *Rhet. II*, 24 (1401^a, 24–1401^b, 3).

- Accent: *Soph. Ref.*, 4 (166^b, 1–9); 6 (168^a, 27); 7 (169^a, 27–29); 21; 23 (179^a, 13–14).

- Forms of expression: *Soph. Ref.*, 4 (166^b, 10–19); 6 (168^a, 25); 7 (169^a, 30–169^b, 3), 22; 23 (179^a, 20–25).

- Accident: *Soph. Ref.*, 5 (166^b, 28–37); 6 (168^a, 34–168^b, 10; 168^b, 26–169^a, 5); 7 (169^b, 3–6); 24; *Rhet. II*, 24 (1401^b, 5–19).

- *Secundum Quid: Soph. Ref.*, 5 (166^b, 38-167^a, 20); 6 (168^b, 11–16); 7 (169^b, 9-13); 25; *Rhet. II*, 24 (1401^b, 35–1402^a, 28).

- *Ignoratio Elenchi: Soph. Ref.* 5 (167^a, 21–36); 6 (168^b, 17–21); 7 (169^b, 9–13); 26.

- Consequent: *Soph. Ref.*, 5 (167^b, 1–20); 6 (168^b, 26–169^a, 5); 7 (169^b, 3–9); *Pr. Anal. B*, 16 (64^b, 33); *Rhet. II* 24 (1401^b, 10–14, 20–29).

- Non-cause: *Soph. Ref.*, 5 (167^b, 21–37); 6 (168^b, 22–26); 7 (169^b, 13–17); 29; *Pr. Anal. B II* 17; *Rhet. II* 24, (1401^b, 30–34).

- Begging the Question: *Soph. Ref.*, 5 (167^a, 37–40); 6 (168^b, 25-27); 7 (169^b, 13-17); 17 (176^a, 27–32); 27; *Topics*, 8 (161^b, 11–18); (162^b, 34-163^a, 13); 13 (162^b, 34-163^a, 28); *Pr. Anal.*, 24 (41^b, 9); *Pr. Anal. B*, 16 (64^b, 28–65^a, 37).

- Many Questions: *Soph. Ref.*, 5 (167^b, 38–168^a, 17); 6 (169^a, 6–18); 7 (169^b, 13-17); 17 (175^b, 39–176^a, 19); 30.

In bringing this chapter to a close, we revisit Hamblin's harsh remarks on what he calls the Standard Treatment of the fallacies in which

> a writer throws away all logic and keeps the reader's attention, if at all, only by retailing the traditional puns, anecdotes, and witless examples of his forebears. 'Everything that runs has feet; the river runs; therefore, the river has feet'—this is a medieval example, but the modern ones are no better [Hamblin, 1970, p. 12].

Such treatments, says Hamblin, are useless and they leave us in a situation in which "[w]e have no *theory* of fallacies at all ...." [Hamblin, 1970, p. 11]. Whatever one thinks of these complaints, it seems that Hamblin would have been entirely happy to put Aristotle at the very top of the list of those whose views disappoint him so. In fact, this is not what Hamblin does. Instead he excoriates many a later writer for failing to pay due attention to Aristotle. In fairness, Hamblin does not think uniformly well of Aristotle's analyses. Certainly there is in these writings no wholly developed theory of fallacies, and some of the examples are rather silly. Even so, Hamblin thinks that Aristotle was on to something genuinely important in two respects. First, Aristotle was, in *On Sophistical Refutations*, attempting to work his way through the old subject of dialectics to the genuinely new discipline of logic. But, secondly and rather strangely, Hamblin commends

us to Aristotle's example in treating the fallacies as inherently dialectical entities (although Hamblin also thinks that on this point Aristotle showed signs of wavering [Hamblin, 1970, pp. 65–66]). It is worth noting that Aristotle has an answer to the objection that his examples of the fallacies are silly, and that they would fool no one. For if a fallacy is an inapparently bad argument, then it is hard to see how one could give convincing examples of such things. Presumably a good or non-silly example would have to be one which the reader would not recognize as a fallacy, and so in one clear sense of the term would also be a bad example.

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ARISTOTLE'S UNDERLYING LOGIC

George Boger

1 INTRODUCTION

1.1 Aristotle's commitment to reason and scientific understanding

The enlightenment thinkers of the ancient fifth century were natural heirs to the earlier thinkers who had aimed to replace mythopoeic and religious explanation of material phenomena with extensive observation and naturalistic explanation. With the advent of democracy and an increasing market economy there came a new spirit of inquiry, a new reliance on reasoned argumentation, and a new commitment to understanding nature. The aspirations of the enlightenment activists blossomed in the humanist ideals of scientific rationalism and political liberty, in profound philosophical inquiry into metaphysics, epistemology and ethics, and in artistic and cultural works of enduring value. These ancients condemned superstition in a way that recalls Hume's exhortation to employ philosophy as "the sovereign antidote ... against that pestilent distemper ... to restore men to their native liberty". We might recall Heraclitus's own exhortation to listen to the *logos*, the aims of the Pythagorean *mathêmatikoi*, the *nous* of Anaxagoras, and the condemnation of superstition by Hippocrates in *The Sacred Disease* and *Ancient Medicine*. Sophocles especially captured this spirit in *Antigone* (332–375) where he has the Chorus sing that "there are many wonders, but nothing is more wonderful than a human being". Indeed, Prometheus might well have been their patron saint, because, in providing humans with various *technai*, he affirmed optimism about their future. A principle underlying this optimism holds that human beings can understand themselves and nature sufficiently to govern their own destinies without the external and apparently capricious interventions of supernatural beings. These ancients embraced Kant's dictum, expressed many centuries later, "*Sapere aude!*" - "*Dare to know!*". Their steady strides in the second half of the fifth century toward consolidating and rationally organizing the sciences helped to bring the earlier inquiries to fruition and prepared the way for the enduring accomplishments of the later philosophers, scientists, and political theorists. Many high points mark the achievements of this enlightenment, but two stand out for their progressive humanist ambition. As Protagoras in relation to the social world developed a political *technê* that affirmed the teachability of virtue and citizenship and thus promoted an empowering democratic activism, so Hippocrates in relation to the natural world developed a medical *technê* that affirmed the intelligence of

Handbook of the History of Logic. Volume 1
Dov M. Gabbay and John Woods (Editors)

human beings to intervene in the workings of nature to preserve health and to prevent disease.

Aristotle, interestingly himself the son of a physician, is an exemplary fourth century heir to enlightenment trends in science and philosophy. He affirmed the principle that nature in its diversity and human beings in their complexity are comprehensible. In *Metaphysics 1.2* he openly avows a humanist ideal kindred to that of Hippocrates in *Ancient Medicine.*[1]

> The acquisition of this knowledge [ἐπιστήμη (*epistêmê*)] ... [has been] regarded as not suited for man. ... God alone may have this prerogative, and it is fitting that a man should seek only such knowledge as becomes him [and not, as the poets say, arouse the gods' jealousy]. *But we should not believe in divine jealousy*; for it is proverbial that bards tell many lies, and we ought to regard nothing more worthy of honor than such knowledge.[2] (982b28–983a7)

Aristotle in the fourth century embraced the earlier enlightenment's daring to know and its optimistic confidence in reason's ability to establish objective knowledge. We can thus appreciate his exhortation in *Nicomachean Ethics 10.7* that "we not follow the proverb-writers to 'think mortal thoughts' ... Rather, as far as we can, we ought to strive to be immortal and to go to all lengths to live a life that expresses our supreme element" (1177b31–34). This supreme element consists precisely in the capacity of intellect by which human beings make both themselves and nature objects of contemplation.

Aristotle boldly began *Metaphysics* by affirming that "all men naturally desire to know". He then traced the acquisition of knowledge from sensation through memory of the same thing and finally to art and science (*epistêmê*), which are produced through extensive experience.

> Art [τέχνη] is born when out of many bits of information derived from experience there emerges a grasp of those similarities in view of which they are a unified whole. Thus, a man is experienced who knows that

[1]Consider a typical passage from *Ancient Medicine* (*AM*) on the scientific spirit and from *The Sacred Disease* (*SD*) against superstition. "Some physicians and scientists say that it would be impossible for anyone to know medicine who does not know what species-man consists of, this knowledge being essential for giving patients correct medical treatment. The question that they raise, however, is ... [wholly an abstract] matter [fit only] for [the likes of] 'philosophy' ... I consider, first, that all that has been said or written by scientist or physician about natural science has less to do with medicine than it has with the art of writing [or painting]. Next, I consider that clear knowledge of nature can be derived from no other source except from medicine" (*AM 20*). "[The so-called sacred disease] is not any more divine or more sacred than other diseases, but has a natural cause, and its supposed divine origin is due to men's inexperience, and to their wonder at its peculiar character. ... Men continue to believe in its divine origin because they are at a loss to understand it ... But if it is to be considered divine just because it is wonderful, there will be not one sacred disease but many ... other diseases are no less wonderful and portentous" (*SD 1*, 2–14). Similar passages in *The Art* express this same spirit.

[2]We use R. Hope's 1960 translation of *Metaphysics* with modification. All emphases in Aristotle's texts cited here and below are added unless otherwise indicated.

when Callias was ill of this disease he was helped by this medicine, and so for Socrates and for many others, one by one; but to have art is to grasp that all members of the group of those who are ill of this disease have been helped by this medicine.

Now experience [ἐμπειρία] seems in no respect inferior to art in a situation in which something is to be done. ... The reason is that experience, like action or production, deals with things severally as concrete individuals, whereas art deals with them generally. Thus, a physician does not cure species-man (except incidentally), but he cures Callias, Socrates, or some other individual with a proper name, each of whom happens to be a man. If, then, someone lacking experience, but knowing the general principles of the art, sizes up a situation as a whole, he will often, because he is ignorant of the individuals within that whole, miss the mark and fail to cure; for it is the individual that must be cured.

Nevertheless, we believe that knowing and understanding [τό γε εἰδέναι κάι τὸ ἐπαίειν] characterize art rather than experience. And so we take experts [τοὺς τεχνίτας] in an art to be wiser than men of mere experience; because wisdom presumably comes only with knowledge, and we believe that *the experts can analyze and explain*, whereas others cannot. Men of experience discern the fact "that", but not the reason "why". Hence we also hold master workmen [τοὺς ἀρχιτέκτονας] in each craft to be more valuable and discerning and wise than manual laborers [τῶν χειροτεχνῶν], because the former can discriminate the various factors relevant to the various effects produced; whereas the latter, like inanimate objects, produce effects, as fire burns, without knowing what they are doing. Inanimate objects produce their effects somehow by nature; and manual workers, by habit. Master workers are presumably wiser, then, not because they are practical, but *because they have their reasons and can explain what they are doing* [ἀλλὰ κατὰ τὸ λόγον ἔχειν αὐτοὺς καὶ τὰς αἰτίας γνωρίζειν]. (981a5–981b6)

Notwithstanding a class supremacy expressed here, it is evident that Aristotle was animated by a firm commitment to the centrality of reason in human life. Indeed, his many treatises on natural science, metaphysics, ethics and politics give expression to his commanding commitments to discovering truth and establishing knowledge. Thus, in spite of his frequent complaints about Socrates, Aristotle nevertheless embraced his teaching in *Phaedo* (89d) that "there is no worse sin than misology" and in *Apology* (38a) that "the unexamined life is not worth living for a human being". The lessons of *Nicomachean Ethics* require a life of reason for realizing one's humanity and achieving happiness. Human virtue consists in making excellent the soul's deliberative and scientific faculties: practical wisdom (φρόνησις) "is a state grasping the truth, involving reason [μετὰ λόγου], concerned with action about what is good and bad for a human being" (*NE 6.5*: 1140b4 6);

and wisdom (σοφία) "is understanding plus scientific knowledge [νοῦς καὶ ἐπιστήμη] of the most honorable things" (*NE 6.7*: 1141a18–20; cf. *Meta.* 982a4–6). Human happiness requires a philosophic life. When we consider Aristotle's bold statements in *On the Soul 3.5* and *3.7* that "actual knowledge is identical with its object [τὸ δ' αὐτό ἐστιν ἡ κατ' ἐνέργειαν ἐπιστήμη τῷ πράγματι]" (430a19–20 & 431a1–2), we can more fully appreciate his exhortation in *Nicomachean Ethics 10.7* that reaffirms the spirit of his inquiry in *Metaphysics 1.2* not to bow to ignorance and inability, but always "to live a life that expresses our supreme element". Prometheus may have stolen for us fire from the hearth of the Olympians, but Aristotle aimed to secure for us a place at their table. In this connection, then, Aristotle's logical investigations are among his enduring accomplishments toward realizing this end.

1.2 Previous interpretations

Until recently the difference between traditional or 'Aristotelian' logic and Aristotle's own ancient logic had been blurred. This is similar to the blurring of a similar distinction between Christian religion and the teachings of Jesus, or the difference between various 'Marxian' philosophies and the teachings of Karl Marx. It is remarkable, for example, that for Aristotle in every syllogism the conclusion follows logically from the premisses. This contrasts with the usage of traditional logicians, who continue to speak of invalid syllogisms. For Aristotle this is a contradiction in terms, an oxymoron. In addition, Aristotle would never have tested the validity or invalidity of a syllogism according to rules of quality, quantity, and distribution. He had his own methods for establishing validity and invalidity. However, it was really not possible meaningfully to distinguish the historical logic of Aristotle from its later accretions and compare the two until modern logicians examined Aristotle's syllogistic through the lens of mathematical logic — that is, until modern logicians turned their attention specifically to the formal aspects of deductive discourses apart from their subject matters. As a result, studies of Aristotle's logic since the early 20^{th} century have established his genius as a logician of considerable originality and insight. Indeed, we can now recognize many aspects of his logical investigations that are themselves modern, in the sense that modern logicians are making discoveries that Aristotle had already made or had anticipated. Perhaps the longevity of this oversight about the nature and accomplishments of his logical investigations is attributable to scholars not having recognized that Aristotle expressly treated the deduction process itself.

Jan Łukasiewicz initiated the reassessment of Aristotle's syllogistic in the 1920s. He was followed by James W. Miller, I. M. Bochenski, and Günther Patzig among others. This reassessment culminated in the 1970s and 1980s with the works of John Corcoran, Timothy Smiley, and Robin Smith. These modern logicians used mathematical logic to model Aristotle's logic and discovered a logical sophistication long overlooked by traditionalist logicians such as R. Whately, H. W. B. Joseph, J. N. Keynes, W. D. Ross, and R. M. Eaton. These traditionalists, whose modern origin can be traced to the *Port Royal Logic*, believe that Aristotle com-

posed *Prior Analytics* as a logic manual for studying categorical arguments or syllogisms. They take a syllogism to be a fully interpreted premiss-conclusion *argument* whose validity or invalidity is determined by applying rules of quality, quantity, and distribution, all of which really only help to define a syllogism. However, traditionalists tend to conflate this sense of a syllogism with another sense when they take a syllogism also to be a relatively uninterpreted argument *pattern* whose instances are valid or invalid arguments.

Now, in spite of their equally criticizing *traditionalist* interpreters, mathematical logicians themselves tend to fall into two camps concerning Aristotle's project in *Prior Analytics*. In fact, when modern logicians mathematically modeled Aristotle's logic, they tacitly distinguished two tendencies in the traditionalist interpretation, the one treating what it believed were Aristotle's axiomatic interests, the other treating Aristotle's argumental interests. The *axiomaticist* interpretation by Łukasiewicz, Bochenski, Miller, and Patzig takes a syllogism to be a single, logically true conditional *proposition*, some of which are taken to be axioms. On this interpretation *Prior Analytics* contains an axiomatized deductive system with an implicit underlying propositional logic. Euclid's *Elements* is an ancient analogue. The axiomaticists examine Aristotle's syllogistic mathematically from a Frege–Russell view of logic as formal ontology. On the other hand, deductionists examine Aristotle's logic mathematically from a Quinian view of logic as formal epistemology.[3] The *deductionist* interpretation of Corcoran, Smiley, and Smith takes a syllogism to be a *deduction*, that is, to be a fully interpreted argumentation having a cogent chain of reasoning in addition to premisses and a conclusion. On this interpretation the number of premisses is not restricted to two. This interpretation sees *Prior Analytics* as having proof-theoretic interests relating to a natural deduction system. Interpretive lines, then, are drawn along what each view considers a syllogism to be and what each takes to be Aristotle's accomplishment in *Prior Analytics*.

However, notwithstanding significant differences among modern interpretations, there are two striking similarities. (1) All three interpretations consider the process of reduction (ἀναγωγή; ἀνάγειν) treated in *Prior Analytics A7* in virtually the same way. The various interpreters hold that reduction amounts to deduction of some syllogisms, taken as derived, from others, taken as primitive, to form a deductive system. In addition, they do not distinguish reduction from analysis (ἀνάλυσις; ἀναλυεῖν). Aristotle, though, distinguished deduction from reduction and each of these from analysis. (2) The axiomaticists and deductionists equally consider Aristotle to have employed the method of counterargument to establish knowledge of invalidity in his treatment of syllogisms in *Prior Analytics A4–6*. However, Aristotle there used neither the method of counterargument nor the

[3] J. Corcoran (1994) clarifies two approaches to logic that have been the vantage points of modern interpretations using the theoretical apparatus of mathematical logic. From the viewpoint of **formal ontology**, logic investigates certain general aspects of reality, and so Aristotle is seen to deduce laws of logic from axiomatic origins; he is concerned with logical truths. From the viewpoint of **formal epistemology**, logic amounts to an investigation of deductive reasoning *per se*, and so Aristotle is seen to describe deductions and the process of deduction.

method of counterinterpretation. It is astonishing that such different interpretations of a syllogism could produce such similar views about the *logical relationships* among the syllogisms.

In great measure, interpretive problems are attributable to scholars not having sufficiently recognized Aristotle's acumen in distinguishing logical and metalogical discourses. Deductions are equally performed in different languages: (1) in an object language about a given subject matter and (2) in a metalanguage, which is used to model formal aspects of object language discourses relating to sentences, arguments, argumentations and deductions. Discourses in these categorially different languages may or may not use the same deduction system or the same logic terms with the same or different denotations. We distinguish an *object language deduction* from a *metalogical deduction.* Aristotle understood his syllogistic deduction system to function at both levels. In *Prior Analytics* he both studied his syllogistic logic and its applications and he used this logic in that study. Traditionalists, however, altogether missed Aristotle's making this distinction by their conflating two senses of a syllogism and, consequently, they overlooked a syllogistic deduction process. Axiomaticists mistook a conditional sentence corresponding to a syllogism for the syllogism itself to confuse the two levels of discourse and thereby they lost sight of Aristotle's principal concern with deduction. Still, they were correct to focus on his metalogical treatment of 'syllogistic forms', even if in their enthusiasm to apply mathematical logic to Aristotle's work they mistakenly saw an axiomatized deductive system in *Prior Analytics.* Deductionists correctly focused attention on Aristotle's concern with the process of deduction and a natural deduction system. However, in reacting to the axiomaticists, they did not take Aristotle as himself modeling object language discourses by means of a metalogical discourse. Nor, then, did they consider his metalogical discourse to be sufficiently formal for his having distinguished logical syntax from semantics. Deductionists modeled Aristotle's logic but did not recognize Aristotle as himself providing an ancient model of an underlying logic with a formal language.

1.3 Aristotle's project: to establish an underlying logic

Aristotle would have agreed with Alonzo Church that "(formal) logic is concerned with the analysis of sentences or of propositions and of proof with attention to the *form* in abstraction from the *matter*" (1956: 1; author's emphasis). Thus, for Church the science of logic is a metalogical study of *underlying logics* (1956: 57–58). The difference between logic and metalogic is drawn between *using* a logic to process information about a given subject matter with a given object language and *studying* a logic or an underlying logic, which involves a language, a semantics, and a deduction system. Logicians use a metalanguage to study the formal aspects of an object language apart from its subject matter, often to study an underlying logic's deduction system. Aristotle undertook just such a study in *Prior Analytics.* Indeed, part of Aristotle's philosophical genius is to

have established a formal logic, while at the same time making the study of logic scientific. He recognized that deductions about a given subject matter are topic specific and pertain to a given domain, say to geometry or to arithmetic or to biology, but that such deductions employ a topic neutral deduction system to establish knowledge of logical consequence.

In having a keen interest in epistemics, Aristotle shares with modern logicians the notion that central to the study of logic is examining the formal conditions for establishing knowledge of logical consequence — that logic, then, is a part of epistemology. He composed *Prior Analytics* and *Posterior Analytics* to establish a firm theoretical and methodological foundation for ἀποδεικτικὴ ἐπιστήμη (*apodeiktikê epistêmê*), or demonstrative knowledge (24a10–11). In *Nicomachean Ethics 6*, where he treated the intellectual excellences, Aristotle indicated the importance he attributed to demonstration (ἀπόδειξις [*apodeixis*]): "scientific knowledge, then, is a *demonstrative state* [ἕξις ἀποδεικτική]" that constitutes an appropriate confidence in the results of deductive reasoning (1139b18–36; cf. *Po.An.A2*: 71b18–22). He saw his purpose in *Prior Analytics* precisely to establish confidence in the deduction process and particularly in his syllogistic *deduction system.* To accomplish this project he especially studied the formal or syntactic matter of deducibility. Aristotle thought of deduction as a kind of computational process. Indeed, the verb 'συλλογίζεσθαι' (*sullogizesthai*) used by Aristotle to denote the special kind of deduction process treated in *Prior Analytics* derives from mathematical calculation. His special concern, then, was to develop a deduction apparatus by which someone could decide in a strictly *mechanical*, or *computational*, manner which sentences are logical consequences of other sentences.

Aristotle's promethean contribution to science and philosophy, then, concerns his study of the deduction process itself. He knew that a given sentence is either true or false; and he recognized this to be the case independent of a participant. He also knew from his familiarity with mathematical argumentation and dialectical reasoning that a given sentence either follows necessarily or does not follow necessarily from other given sentences; likewise, he recognized this to be the case independent of a participant. These are *ontic* matters having to do with *being.* In addition, Aristotle knew that the truth or falsity of a given sentence or the validity or invalidity of a given argument might not be known to one or another participant. Now, a given axiomatic science aims to establish knowledge about its proper subject matter (*Po. An. A1*: 71a1–11 & *A3*: 72b19–22). Since Aristotle took such a science to consist principally in the collection of sentences — definitions, axioms, theorems — of its extended discourse, the project of such a science is to decide which sentences pertaining to its subject matter are true, or theorems, and which sentences, for that matter, are false and not theorems. Procedures for deciding a sentence's truth or falsity are *epistemic* matters having to do with *knowing.*

In respect of epistemics Aristotle recognized two ways to establish the truth of a given sentence: (1) by induction (ἐπαγωγή) and (2) by deduction (*Po.An.A1–2*, *EN 6.3* & *Meta. 1.9*: 992b30–993a1). In respect of an axiomatic science, while

definitions and axioms, or first principles, are determined inductively and are not the result of a deductive process,[4] its theorems are decided deductively. In the works of the *Organon*, particularly in *Prior Analytics* and *Posterior Analytics*, Aristotle treated the deductive method for establishing knowledge that a given sentence is true. This project requires two steps (*Pr. An. A1*: 25b28–31), which he treated separately in *Prior Analytics* and *Posterior Analytics*. In *Posterior Analytics* Aristotle treated the requirements for demonstrative science, a constituent part of which is demonstration. He writes in *Posterior Analytics A2*:

> By a demonstration [ἀπόδειξιν] I mean a scientific deduction [συλλογισμὸν ἐπιστημονικόν]; and by scientific I mean a deduction by possessing which we understand something ... demonstrative understanding [τὴν ἀποδεικτικὴν ἐπιστήμην] in particular must proceed from items that are *true* and *primitive* and *immediate* and *more familiar than* and *prior to* and *explanatory of* the conclusions. There can be a deduction [συλλογισμὸς (*sullogismos*)]even if these conditions are not met, but there cannot be a demonstration [ἀπόδειξις] — for it will not bring about understanding [ἐπιστήμην]; in respect of a given subject matter]. (71b17–25; cf. *Top. A1*: 100a27–29)

Aristotle early distinguished deduction (*sullogismos*) from demonstration (*apodeixis*). In *Prior Analytics A4* he stated that he would treat deduction before demonstration because it is more universal: "for [every] demonstration is a kind of deduction, but not every deduction is a demonstration" (25b30–31). In *Posterior Analytics A2* (cf. *Pr. An. B2–4*) he determined this universality to consist in a deduction's being possible even when the premiss sentences are not antecedently known to be true or even when they are false. Thus, one can know that the conclusion sentence of a given demonstration is true because (1) its premiss sentences are all true and (2) it is a deduction.

A deduction *per se*, then, establishes knowledge, *not* that the sentence that is the conclusion of a given argument is true, but *only* that it follows necessarily, or logically, from the sentences in a premiss-set. Aristotle made an important distinction in his logical investigations between epistemic concerns and ontic concerns. This is especially evident in *Prior Analytics B1–4* where he treated the deducibility of true and false sentences from various combinations of true and false sentences taken as premisses. This distinction indicates an understanding of logical consequence that modern logicians will recognize. The confidence one acquires from a demonstration derives from knowing, as Aristotle often pointed out, that it is impossible for true sentences to imply a false sentence (*Pr. An. B2–4*). Given true

[4]A science's first principles are part of the premiss-set in an extended deductive discourse. Aristotle referred to these as "unmiddled" (ἄμεσον), immediate or indemonstrable, that is, not themselves products of demonstration; their truth is established independently. See *Po.An.A3* on the notion that not every categorical sentence is demonstrable. In an extended deductive discourse the derived theorems are added to the original principles as additional premisses for subsequent derivations. Euclid's *Elements* serves as an ancient example.

sentences as premisses, established (initially) by means independent of deduction, one can be certain that the conclusion sentence of a demonstration also is true[5] precisely because it is shown to be a logical consequence of other true sentences. In *Prior Analytics* Aristotle was especially concerned to determine which *formal patterns* of argumentation might be used to establish knowledge that a given sentence necessarily follows from other given sentences. In particular, he saw his project as determining "how every syllogism is generated" (25b26–31) by identifying which elementary argument patterns could serve as rules analogous to such patterns as *modus ponens*, *modus tollens*, and disjunctive syllogism for modern propositional logic.

Looking back, we see that mathematicians of the fourth century had been assiduously attending to axiomatizing geometry. This activity principally concerned condensing the entire wealth of geometric knowledge into small sets of definitions and axioms from which the theorems of geometry could be derived and set out as a long, extended discourse. Euclid's *Elements* is an extant fruit of this activity. Except for identifying a small set of common notions (κοιναὶ ἀρχαί or τὰ κοινά), these mathematicians were not concerned with studying the epistemic process underlying geometric discourse. They took geometry intuitively as an *informal axiomatic system* (Church 1956: 57) with an implicit underlying logic. The ancient mathematicians may have formalized the truths of geometry, but they hardly formalized the deductive method for processing the information already contained in its definitions and axioms.

Undoubtedly Aristotle had participated in discussions, in the Academy and elsewhere, about axiomatizing geometry. He may have asked about deduction rules used to establish geometric theorems. Indications that he did include his attention to various proofs such as that of the incommensurability of the diagonal with the side of a square and those related to properties of triangles, and his frequent attention to the common notions of the mathematical sciences; there is also his curious mention of the middle term and syllogistic reasoning in connection with geometric demonstration (*Po. An. A9*: 76a4–10; *A12*: 77b27–28; cf. *Pr. An. A35* & *A24*). Aristotle surely wondered *how* one could be assured in geometric demonstrations that a conclusion necessarily follows from premisses. This matter is all the more interesting in the case of longer, more involved demonstrations. Still, we cannot say that he undertook a metalogical study of geometric proof.

Perhaps it was Aristotle's own insight or an implicit part of the philosophical discussion of the time that the axiomatization of geometry could serve in some way as a model for formalizing the non-mathematical sciences such as botany and zoology. Some such notion seems to have animated his scientific and logical investigations.[6] Now, the actual project of establishing a given science's definitions

[5] Aristotle recognized two necessities in a demonstration when he remarked about that which "it is impossible for it to be otherwise" (*Po. An. A2*: 71b15–16): (1) that having to do with the subject matter of a demonstration — that about which; (2) that having to do with logical consequence — that following necessarily. See below Section 5.3 on Aristotle's notion of logical consequence.

[6] See, for example, on the matter of Aristotle's application, or lack of application, of his

and axioms and then its theorems, which would conform to the criteria set out in *Posterior Analytics A2*, did not concern Aristotle in the *Organon.* This project lies outside the scope of logic. Rather, his preeminent concern there was to study deduction and demonstration *per se*: not with, that is, one or another distinct subject matter, but with the formal deduction process that has no similar subject matter. Aristotle in his logical investigations subordinated a concern with the *what* or the *why* and *wherefore* to focus on the *that* and the *how.* Thus, presupposing various axiomatic sciences with distinct domains, he took up the narrower and more poignant questions about the epistemic process of deriving theorems from axioms. Aristotle especially examined the *deductive foundations* of demonstration, that is, of demonstrative knowledge or axiomatic science.

The first chapters of *Metaphysics 1* reveal Aristotle's intellectual disposition toward scientific knowledge and signal the importance he attributed to metalogical study of deduction. In fact, Aristotle identified this task as a province proper only to philosophy. In *Metaphysics 2.1* he writes that "philosophy is the science of truth [τὴν φιλοσοφίαν ἐπιστήμην τῆς ἀληθείας]" (993b20). And since "status in being governs status in truth [ὥσθ' ἕκαστον ὡς ἔχει τοῦ εἶναι, οὕτω καὶ τῆς ἀληθείας]" (*Meta. 2.1*: 993b30–31), the philosopher's project includes studying, not a particular part of being, but being-*qua*-being (τὸ ὄν ᾗ ὄν; *Meta. 4.1*: 1003a21-26). "The philosopher must have within his province the first principles [τὰς ἀρχὰς] and primary factors of primary beings" (*Meta. 4.2*: 1003b17–19) and "be able to view things in a total way" (1004a34–1004b1). Accordingly, "it is not the geometer's [nor any other specialist's] business to answer questions about what contrariety is, or perfection, or being, or unity, or sameness, or diversity [or even, for that matter, about deduction rules]; for him these remain postulates [ἐξ ὑποθέσεως]" (*Meta. 4.2*: 1005a11–13; cf. 1005a31). Later in *Metaphysics 4.3* Aristotle addressed the philosopher's responsibility to examine certain axioms precisely because they refer to *all* of being — being-*qua*-being — and not just a part of being (1005a21-22).

> But it is clear that the axioms *extend to all things as being* (since they all have being in common); hence the theory of axioms [περὶ τούτων ⟨ἀξἰώματων⟩ ἐστὶν ἡ θεωρία] also belongs to him who knows being as being. (1005a27 29)

He is not writing about axioms special to a particular science, but about ontic principles that apply alike to all domains.

After indicating the limitations of the special sciences for examining these axioms, Aristotle writes that

> the philosopher, who examines the most general features of primary being, must investigate also the *principles of deductive reasoning* [τῶν συλλογιστικῶν ἀρχῶν]. ...So that he who gets the best grasp of beings as beings must be able to discuss *the basic principles of all being*

syllogistic to the actual project of axiomatizing a natural science: J. Lennox 1987, R. Bolton 1987, J. Hintikka 1996, W. Wians 1996, D. K. W. Modrak 1996, J. J. Cleary 1996, and Graham 1987.

[τὰς πάντων βεβαιοτάτας], and he is the philosopher. (1005b5–6–11; cf. *Meta. 11.4*)

Immediately Aristotle cites the principle of non-contradiction as one of the principles "about which it is impossible to be mistaken" and writes, moreover, that such a principle that is "[necessary] in order to understand anything whatever cannot be an assumption [τοῦτο οὐχ ὑπόθεσις]" (1005b11 12 & 15–16): "It is impossible for the same thing at the same time to belong and not to belong [ὑπάρχειν τὲ χὰι μὴ ὑπάρχειν] to the same thing and in the same respect" (1005b19–22). He states the principle here as an *ontic* principle,[7] but he immediately relates it to demonstration, implicitly reminding us that *being governs truth.*

> Hence, if contraries cannot at the same time belong to the same thing ... and if an opinion [δόξα] stated in opposition to another opinion is directly contrary to it, then it is evidently absurd for the same man at the same time to believe the same thing to be and not to be; for whoever denies this would at the same time hold contrary opinions [τὰς ἐναντίας δόξας]. It is for this reason that all who carry out a demonstration rest it on this as on an ultimate belief [ἐσχάτην δόξαν]; for *this is naturally a foundation also of all other axioms* [φύσει γὰρ ἀρχὴ καὶ τῶν ἄλλων ἀξιώματων αὔτη πάντων]. (1005b26–34)

This passage ends *Metaphysics 4.3.* Much of the remainder of *Metaphysics 4* is devoted to establishing the absurdity of rejecting the principle of non-contradiction for intelligible discourse. It is evident that Aristotle understood a philosopher's responsibilities to include examining the *principles of deductive reasoning.* For Aristotle, studying the principles of being is simultaneously a study of the principles of thought. Logic, which he took to be a part of epistemology, is nevertheless grounded in the nature of being. Perhaps Aristotle appropriated Parmenides' dictum that "thought and being are the same [τὸ γὰρ αὐτὸ νοεῖν ἔστιν τὲ καὶ εἶναι]", taking this to mean that truth and logical consequence, as they appear in thought, follow being.

Now, Aristotle writes in *Metaphysics 11.4* that "taking equals from equals leaves equal remainders" is an example of a notion common to all quantitative being (1061b20–21). He had recognized that the common notions belonging to the mathematical sciences belonged equally to all and specially to none.[8] And, although

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[7]Aristotle provides both ontic and epistemic expressions of this principle in *Meta. 4*. Here he provides an ontic expression, but in *Meta. 4.4* he expresses this law in the following way: "If when an affirmation [φάσις] is true its denial [ἀπόφασις] is false, or when the denial is true its affirmation [κατάφασις] false, then it will not be possible at the same time to assert [φάναι] and to deny [ἀποφάναι] the same thing truly" (1008a34-1008b1). See §5.3.

[8]Consider *Po. An. A5*: 74a17 25. "Again, it might be thought that proportion alternates for items as numbers and as lines and as solids and as times. In the past this used to be proved separately, although it is possible to prove it of all cases by a single demonstration: because all these items numbers, lengths, times, solids — do not constitute a single named item and differ in form from one another, they used to be taken separately. Now, however, it is proved

he never articulated a complete list of common notions for the non-mathematical sciences, nor for that matter in any systematic way for the mathematical sciences, he stated in *Metaphysics 4* that the principles of contradiction and of the excluded middle are among the common notions applicable to all rational discourse. Aristotle noticed that the common notions relating to different branches of mathematics, while stipulative of magnitudes in general, do not stipulate any one domain, such as arithmetic or geometry, in particular. They generally do not specify any content however much they anticipate establishing relationships among magnitudes special to a mathematical science. They are topic neutral in this circumscribed sense, and, thus, they have a relative independence unlike lines, angles, or numbers. They are neither embedded in nor mentally inextricable from the objects of a particular science. This is not the case with a science's principles. The common notions, then, may be taken *in abstractum* and treated on their own account irrespective of the subject matter of a given quantitative science.

These common notions, moreover, generally express *formal relationships* among magnitudes within a quantitative science and accordingly apply equally to a variety of different, quantitative domains. Because of their relative formality and their special universality, the common notions were applied as inference rules across the mathematical sciences. Their use in this epistemic manner is evident in Euclid's *Elements.*[9] Aristotle's having recognized the common notions as principles of reasoning had profound consequences for the development of ancient logic. Understanding this and that Aristotle took a syllogism to fit an elementary argument pattern with only valid instances help to confirm the rule-nature of his statements in *Prior Analytics A4–6* relating to when a syllogism comes about. In the case of Euclid's common notions, two magnitudes remain incommensurable without there being a third, or middle, that unites them as extremes in a particular way — using a common notion makes this evident. An exactly analogous relationship applies in the case of the patterns of perfect or complete syllogisms, the τέλειοι συλλογισμόι (*teleioi sullogismoi*), in *Prior Analytics A4* in respect of relating substantive terms and making evident their connections. Aristotle's manner of expressing the patterns of the syllogisms in sentences beginning with 'ἔαν' and 'εἰ' comports exactly with Euclid's expressions of the common notions and suggests their similar rule-nature.[10] Aristotle went on to express his rules using schematic letters where Euclid did not. Scholars have overlooked Aristotle's written statements of the rules to see only their schematic representations. Consequently, they have not recognized this as part of his effort to model a logic. Accordingly, they

universally: what they suppose to hold of them universally does not hold of them as lines or as numbers but as *this*." We use J. Barnes's (1994) translation of *Posterior Analytics* with modification.

[9] I. Mueller (1981: 32–38), following T. Heath (1956: 117–124, 221–226), maintains that Euclid's common notions serve as deduction rules in his proofs. He believes this is the case even allowing that Euclid did not treat the matter of deducibility. Cf. B. Einarson 1936: 42–49 and H. D. P. Lee 1935: 114 115.

[10] See T. Heath 1956: 120–122 on ancient ideas of the common notions. Also see Heath 1956: 221–232 on Euclid's treatment of the common notions.

missed this link to mathematics and thus they missed an important part of his theory of deduction.

Finally in this connection, besides stating syllogistic deduction rules and his actually using the patterns of the *teleioi sullogismoi* as rules in *Prior Analytics*, Aristotle virtually stated his taking them formally as rules in *Prior Analytics A30* (46a10–12/15). There he used the expression 'the principles of deduction' — "αἱ τῶν συλλογισμῶν ἀρχαί" — that we also encounter in *Metaphysics 4.3* (1005b7). Aristotle did not refer here in *Prior Analytics* to the principles or axioms of a given science, but to the most general principles of all being as they are grasped in thought. And again, in this same connection, he used the expression 'the principles of demonstrations' — "αἱ τῶν ἀποδεικτικῶν ἀρχαί" — in *Metaphysics 3.2* (996b26). Indeed, throughout *Metaphysics* Aristotle used the following expressions as synonyms in referring to common notions, including the laws of non-contradiction and the excluded middle: 'τὰ κοινά' (1061b18), 'ἀρχαί' (996b26, 997a13), 'ἀξιώματα' (997a11, 13), and 'κοινὰι δόξαί' (996b28, 997a21; see esp. *Po. An. A10–11*). Thus, we can see that a *pattern* of a syllogism is a relatively uninterpreted object. In fact, Aristotle treated each pattern exactly as a topic neutral rule of deduction in *Prior Analytics A4–7* analogous to Euclid's use of common notions in *Elements*. Perhaps the patterns of the four *teleioi sullogismoi* are Aristotle's adaptation to the non-mathematical sciences of the common notions employed as deduction rules in the mathematical sciences.

1.4 The scope of this study

Our concern here is to present Aristotle's system of logic while also revealing the mathematical sophistication of his logical investigations. Modern logicians believe that the possibility of mathematical logic, an important part of which involves generating models, consists in making a clear distinction between syntax and semantics. They also believe that the clear distinction between syntax and semantics resulted from borrowing symbolic notations from mathematical practice and then applying them to studies of deductive logics, but that earlier thinkers, lacking such notations, could not have made such distinctions. However, mathematical logic, considered as a discipline in general, has a formal and a material aspect. Its formal aspect has principally to do with the symbolic notations that have helped to illuminate underlying structural, or logical, features of deductive discourses. Yet, the substance of mathematical logic does not consist in its sophisticated notations, but in the problems logicians consider when studying underlying logics — that is, in particular, when they distinguish a logic's syntax and its semantics and then ask questions about their relationships. Principal in this respect have been questions about a logic's consistency, soundness, and completeness, which involve determining relationships between deducibility and logical consequence. A distinguishing feature of mathematical logic, then, consists precisely in these substantive mat-

ters.[11] Remarkably, with only a rudimentary notation Aristotle considered just such mathematical matters in his concern to establish the practical, epistemic power of his logic for establishing scientific knowledge. In this connection we can grasp the revolution in the history and philosophy of logic — the "hypostatization of proof" — consolidated by Aristotle's works on logic. "*Prior Analytics* is the earliest known work which treats proofs as timeless abstractions amenable to investigation similar to the investigations already directed toward numbers and geometrical figures" (Corcoran: personal communication). Thus, *Prior Analytics* is a proof-theoretic treatise on the *deduction system* of an underlying logic. Aristotle recognized the epistemic efficacy of certain *elementary argument patterns*, to wit, those of the syllogisms, and he formulated them as rules of natural deduction. Having raised important metalogical questions about the properties of his syllogistic deduction system, he successfully established a set of formal, epistemic conditions for recognizing logical necessity, and in this way he became the founder of formal logic.

Below we set out Aristotle's underlying logic much as he himself did in the works of the *Organon*. We include *Metaphysics* among the treatises of his logical investigations. It is natural and not surprising that modern logicians and commentators, when treating Aristotle's logic, focus principally on *Prior Analytics*: *Prior Analytics* is the most 'logical' of the treatises. In truth, the attraction of *Prior Analytics* has consisted in a scholar's implicit recognition that Aristotle there treated the deduction system of an ancient underlying logic. We say 'implicit' because it was not until the studies of J. Corcoran and T. Smiley, and later those of R. Smith, that there is a growing explicit recognition that this is so. In any case, a deduction system is only one part of an underlying logic, which also contains a

[11] While the initial impetus of modern logic involved axiomatizing geometry and number theory and attempting to reduce mathematics to logic, it was David Hilbert in the 1920s who turned attention specifically to the formal deduction process and made it an object of mathematical investigation. He worked on devising an algorithm or decision procedure. Moreover, he noted that the semantic concepts of validity and satisfiability coincide with the syntactic concepts of derivability and consistency. Hilbert emphasized the study of such syntactic questions as those of consistency and completeness, which he considered to fall under what he called "metamathematics", or "proof theory". We take this attention on deduction systems to be a substantive concern of mathematical logic, whatever formal systems mathematical logicians may devise. Of course, modern mathematical logic far surpasses the accomplishments of Aristotle, particularly in respect of devising and studying uninterpreted formal systems. We need only mention here the developments of genuine variables, functions, quantification, set theory, highly formal languages, recursion theory, and model theory. Nevertheless, if we take logic principally to treat underlying logics as A. Church, then we see that mathematical logic fundamentally addresses foundational questions about deduction systems. In addition, the origin of modern mathematical logic involved axiomatic systems, both in respect of ontic subject matter and epistemic formal matters. This was just the kind of concern that occupied Aristotle himself, whom we detect as distinguishing the content of an axiomatic science from the deduction apparatus used to establish its theorems. Aristotle came to address similar foundational questions, although likely he did not come to them as modern logicians initially did by way of attempting to reduce mathematics to logic. On Aristotle's having proof-theoretic interests, see R. Smith 1984: 594–596, 1986: 55–61, and 1991: 48–50. J. Corcoran (1974, 1994) and R. Smith (1989) have generally made Aristotle's case in this respect.

grammar and a semantics. Our contribution takes this recognition a little farther to hold that Aristotle intentionally aimed to develop an underlying logic along the lines of modernist thinking. This means that Aristotle invented a formal language to model his logic. However, since Aristotle did not set out his underlying logic in as systematic a manner as a modern logician, while, nevertheless, accomplishing much the same result, we employ the theoretical apparatus of modern mathematical logic to structure his account. With the aid of this template we show in Aristotle's own words that he was concerned with exactly similar matters as a modern logician. We begin by presenting Aristotle's treatment of the syntax and semantics of natural language in *Categories*, *On Interpretation*, and *Metaphysics.* These studies laid a foundation for his developing the formal language found in *Prior Analytics* for modeling axiomatic discourse. We then proceed to extract the syntax of sentence transformations leading to his establishing a set of deduction rules in *Prior Analytics.* Next we treat the logical methodology by which Aristotle established his deduction rules. We conclude with a statement of his understandings of "formal deducibility" and "logical consequence" and with a final section that summarizes four proof-theoretic accomplishments of his logical investigations.

1.5 Logic terminology

The following terminology assists in our study of Aristotle's logic. We use Aristotle's own terminology wherever it exists, which, interestingly, often corresponds exactly to ours. An **argument** is a two part system consisting in a set of sentences in the role of premisses and a single sentence in the role of conclusion; an argument is either valid or invalid. A **sentence** is either true or false. We sometimes use 'conclusion' elliptically for 'sentence in the role of conclusion' or 'conclusion sentence', and similarly for 'premiss'. An **argumentation** is a three part system consisting in a chain of reasoning in addition to premisses and conclusion; an argumentation is either cogent, in which case it is a **deduction**, or fallacious, a **fallacy**. A sentence, an argument, and an argumentation are object language phenomena and domain specific. An **argument pattern** is a two part system consisting in a set of sentence patterns in the role of a premiss-set and a single sentence pattern in the role of a conclusion. A pattern is a metalinguistic object distinguishable from a **form** and is commonly represented schematically. An argument is said to fit, or to be an instance of, one or more argument patterns.[12] A given argument pattern may have all valid instances, all invalid instances, or some valid and some invalid instances. An argument pattern is not properly valid or invalid, although logicians have used 'valid' in this connection, but we distinguish these category differences. An argument pattern with all valid instances is **panvalid**, that with all invalid

[12]We use "pattern" and "form", following J. Corcoran (1993: xxxi xxxvii) as, e.g., Irving Copi (1986: 288 291) uses "form" and "specific form" and as Willard O. Quine (1982: 44) uses "general [logical] schema" and "special case [logical schema]". Cf. W. O. Quine 1970: 47–51. We can express this difference in the following way: while a given argument has only one form, it might fit more than one pattern. See also Corcoran 1989.

instances is **paninvalid**, and that having instances of both is **neutrovalid**. We add that an **elementary panvalid argument pattern** is one having a *simple* premiss-set pattern whose epistemic value consists, in many cases, in its being quickly evident, or 'evident through itself', that its conclusion follows necessarily. An elementary argument pattern may be formulated in a corresponding sentence to express a **rule of deduction**. In addition, we follow G. Patzig (1968; cf. Rose 1968) to distinguish in Aristotle's logic a concludent pattern of two sentences in the role of premisses, or a premiss-pair pattern, from an inconcludent **premiss-pair pattern**. A **concludent** pattern has a necessary result, that is, it results in a panvalid pattern all of whose instances are syllogisms, while an **inconcludent** pattern has no necessary result, that is, it cannot result in a panvalid pattern but only in a paninvalid pattern with only invalid argument instances.

In addition, we understand Aristotle to have considered a συλλογισμός (*sullogismos*), which we translate by 'syllogism', to be a valid argument with only two premiss sentences, having only three terms, in one of three figures.[13] A **syllogism**, then, is an elementary argument fitting a panvalid pattern. No syllogism is invalid. Aristotle saw his project in *Prior Analytics* to identify all such patterns, precisely because of their epistemic efficacy in the deduction process. We use the traditional names of the 'syllogisms' — 'Barbara', 'Celarent', etc.[14] — to name patterns of syllogisms, just as '*modus ponens*' names a kind of familiar pattern in propositional logic used in a deduction process. Still, these names do not signify instances of such patterns. Of course, 'Barbara' and '*modus ponens*' also name deduction rules. However, in some cases — especially those pertaining to the *teleioi sullogismoi*, those in *Sophistical Refutations*, and those in *Prior Analytics* when Aristotle refers to a *sullogismos* as proving something — we translate '*sullogismos*' by 'deduction'. In these cases Aristotle recognized an epistemic process to occur in the mind of a participant who grasps that a given sentence is a logical consequence of other given sentences. Still, when he writes, in relation to a deduction process, that a syllogism arises (γίνεται), we understand him not to mean that a syllogism *per se* is a deduction, but that one's arising during a deductive chain of reasoning signals making logical consequence evident, just as when a participant links given propositions and produces an instance of *modus ponens* signals logical consequence in propositional logic.

Finally, we take treating patterns of sentences, patterns of arguments, and patterns of argumentations to constitute a large part of **modeling** a logic. Thus, for

[13]In Boger 1998 I used '*sullogismos*' rather than translate 'συλλογισμός' better to objectify its meaning. I argued that a *sullogismos*, as Aristotle treats it in *Pr. An.* *A4–7, 23, & 45*, is a panvalid argument pattern and neither an argument nor a deduction. However, Prof. David Hitchcock of McMaster University, while agreeing that Aristotle treated patterns in *Pr. An.*, has convinced me that he did not call them '*sullogismoi*' but reserved the word for valid arguments fitting such patterns.

[14]The traditional names of the syllogisms in the first figure are Barbara, Celarent, Darii, and Ferio. The traditional names of second figure syllogisms are Camestres, Cesare, Festino, and Baroco and of the third figure Darapti, Datisi, Disamis, Felapton, Ferison, and Bocardo. See W. T. Parry (1991: 282) for a short explanation of their origin and meaning.

example, while there are numerous simple sentences in a given object language, each of them, nevertheless, consists in a subject and a predicate. Extracting this elementary pattern and representing it abstractly, or metalinguistically, is modeling a simple sentence — either by means of another sentence, using the language of the given object language (but, nevertheless, in the metalanguage), or by means of mathematical notation. In either case, a sentence is modeled and becomes an object of logical investigation. Thus, we take a formal language to be a **model** of one or another object language, with one or another degree of precision. In this way a logician can model arguments, deductions, and deduction systems better to study their respective properties and logical relationships.

2 ARISTOTLE'S ANCIENT MODEL OF AN UNDERLYING LOGIC

Aristotle knew that deductions about geometric objects are topic specific and that they employ a topic neutral deduction system, even if a participant uses that system implicitly. In *Prior Analytics* he turned his attention not to geometric or biological objects, nor even to geometric or biological discourses, but to the *deduction apparatus* used to make evident that a given categorical sentence necessarily follows from other given categorical sentences.[15] Aristotle had observed a number of elementary argument patterns used in various object language discourses, some of which he recognized in their use always to result in something following necessarily, others of which he recognized in their use never to result in something following necessarily. He subsequently extracted these patterns for *systematic* examination. In *Prior Analytics* Aristotle modeled his syllogistic logic and presented the results of his investigating these patterns. In this connection, then, *Prior Analytics* is a scientific study of the *syllogistic deduction system*, which, taken together with *Categories*, *On Interpretation*, and parts of *Metaphysics*, comprises Aristotle's treatment of an underlying logic.

The logic underlying cogent object language discourse accounts for that discourse's coherence. While this discourse is itself topic specific as it treats objects of a given domain, its underlying logic is topic neutral and not bound to any one subject matter. The *science of logic* is devoted in great measure to modeling these underlying logics and consists in their study. To accomplish this study, a logician must not only model the deduction system of such discourse, but he/she must also model the object language itself, often with an aim to make such a language more precise. A logician's principal concern is to extract and formalize (1) a grammar for the formation of sentences and their relationships and (2) a deduction system for sentence transformations. These are formal, syntactic concerns. A logician constructs a formal language to model one or more object language in respect of its structure. Such a formal language is taken to be uninterpreted, although

[15]While Aristotle named the parts of a categorical sentence — subject term and predicate terms, and the logical constants — he did not use the expression 'categorical sentence'. This expression is a later invention that nevertheless captures his meaning and distinguishes this kind of sentence from those of, for example, natural Greek, or natural language.

hardly is such a formal language purely uninterpreted — often its logical constants are interpreted or have an implicit intended interpretation, as are what count as a sentence and an argument, etc. In any case, the 'formulas' or patterns for constructing and transforming sentences are relatively uninterpreted, as evidenced by the impossibility of assigning them meanings and truth-values (save for logics with identity and tautology). Thus, in modeling an underlying logic a logician also treats the semantics of sentences — establishing meaning and truth conditions — and of sentence transformations — establishing conditions of logical consequence.

Aristotle, then, invented an artificial language for two closely related purposes that embrace a modernist concern for modeling a logic. *First*, he wanted to develop a language (1) that conformed to his ontology of substance, a core of which is presented in *Categories*, and (2) that promoted a precision in scientific knowledge, a concern that he forcefully expressed in *Metaphysics*. *Second*, he wanted to model the underlying logic he developed as an epistemic instrument for scientific discourse both (1) to facilitate determining the properties of his logic and (2) to represent his logic for instructing others in its use. It is doubtful that Aristotle developed this artificial language to model natural language and more likely that he aimed to standardize scientific discourse and to model his logic. Aristotle invented four categorical sentence patterns, and he treated them as formal objects in order to establish certain of their properties and logical relationships. And while he did not represent his logic with a modern rigor and system, we can easily organize his own discourses according to a mathematical template without distortion to his meaning and intention. In this section we first consider Aristotle's treatment of the grammars of natural language and his artificial language (§2.1), second, the semantics of his language (§2.2), and, third, the syntax of his deduction system (§2.3). While Aristotle treated the syntax of sentences in close relation to their semantics, he nevertheless sufficiently distinguished them so that we can treat them separately.

2.1 Aristotle's metalinguistic study of grammar

To extract and represent his deduction system for analysis in *Prior Analytics*, and to prepare for its application to the various axiomatic sciences as a science is construed in *Posterior Analytics*, Aristotle undertook a systematic study of language. While the Sophists are perhaps his more immediate predecessors in this connection, Aristotle's contributions firmly consolidated the early stages of linguistics as a special branch of learning. Efforts in this area are especially evident in *Categories*, *On Interpretation*, *Metaphysics*, *Topics*, *Sophistical Refutations*, and *Rhetoric*. In *On Interpretation* Aristotle treated the complexity of Greek grammar only generally as suited the purpose of his logical investigations. There he identified the simple sentence that predicates one thing of another thing as a proper object of logical analysis. By studying a natural language in these treatises Aristotle prepared the way to inventing the artificial language in *Prior Analytics*, perhaps the first

artificial, or formal, language in the history of philosophy. And he accomplished this task without the aid of a sophisticated system of symbolic or mathematical notation. With his treatment of predication in *Categories* and *Metaphysics* in the background,[16] we turn now to a part of the elementary grammar examined in *On Interpretation.*

The grammar of a natural language

In *On Interpretation 1–4* Aristotle writes about sentence formation in a natural language, in this case in his own natural language. There he uses Greek to mention and to illustrate his observations about intelligible discourse that might apply in principle to any language (16a5–6). In this connection he intuitively takes Greek to be what modern logicians call a fully interpreted language. Nevertheless, he carefully focuses attention on its structural aspects apart from any meanings, except for purposes of illustration, that sentences might express about a subject matter. Indeed, although he does not have expressions for 'natural language', 'object language', and 'metalanguage', it is evident that in *On Interpretation* Aristotle intentionally objectifies aspects of language in general and does not study only the Greek language in particular. *On Interpretation* is a metalinguistic treatise in which Aristotle consciously examines certain syntactic and semantic aspects of language.

Aristotle treats sentence formation in a natural language as essentially consisting in combining (σύνθεσις) a noun (ὄνομα) and a verb (ῥῆμα; i.e., a predicate [16a9–18]) so as to produce a meaningful expression (φωνὴ σημαντική), a complete thought. Every sentence necessarily has these two basic components, neither of which by itself is sufficient.

> Every affirmative sentence [πᾶσα κατάφασις] consists in a noun and a verb, whether [determinate or] indeterminate. Unless there is also a verb, there is neither an affirmation nor a denial [οὐδεμία κατάφασις οὐδ' ἀπόφασις]. (*On Int. 10*: 19b10–12; cf. *Cat. 2*: 1a16–19)[17]

In addition, Aristotle recognizes that the words making up a sentence must be concatenated or strung together in certain ways so as to bear meaning: "that

[16]Perhaps *Categories* (Κατηγορίαι) would better be named "On predication" or "On predicating properties of substance". *Categories* is a metalinguistic treatise on sentence formation, which, moreover, aims at precision and truth in the sciences. This treatise avows Aristotle's subscription to a correspondence notion of truth, as does *On Interpretation* as well as to a materialist theory of nature, or substance. While much has been said about *Categories*, we here only provide a brief reference to Aristotle's theory of substance. Of the ten categories (listed in *Cat. 4*) — viz. substance (οὐσία), quantity (ποσόν), quality (ποιόν), relation (πρός τι), place (πού), time (ποτέ), position (κεῖσθαι), condition (ἔχειν), action (ποιεῖν), and affection (πάσχειν) — substance is predicated of nothing but the others are predicated of a substance, a 'this'. Substance is treated in *Cat. 5*, quantity in *Cat. 6*, relation in *Cat. 7*, quality in *Cat. 8*, and the others in *Cat. 9*.

[17]For translations of *Categories* and *On Interpretation* we work with H. P. Cooke's (1938) and J. L. Ackrill's (1984) translations and make significant modifications, e.g., translating 'ἀπόφανσις' by 'sentence' and by neither 'proposition' as Cooke nor 'statement' as Ackrill.

the words are pronounced [merely] in succession [σύνεγγυς]] does not make them a unity [εἷς]" (17a14).[18] In *On Interpretation 4* Aristotle defines 'sentence' as follows:

> A sentence [λόγος] is meaningful speech [φωνὴ σημαντική] — the parts of which, as expressed [separately] mean something as an expression but not as an affirmation [ὡς φάσις, ἀλλ' οὐχ ὡς κατάφασις]. I mean, for example, that 'man' means something, but [by itself] not that *it is or is not*; there will be an affirmation or a denial [only] if something is added [τι προστεθῇ]. (16b26–28; cf. 10: 19b10–12)

A noun and a verb by themselves may possess meaning, but by themselves they do not constitute a sentence, nor do they constitute a sentence merely by being strung together arbitrarily. Thus, from his notion of sentence in *On Interpretation*, we can extract Aristotle's rule for the formation of a generic sentence in a natural language and express it as follows:

SFR1 A sentence in a given natural language consists in combining a noun and a verb (i.e., a predicate) in certain ways so as to produce a meaningful expression.

This rule identifies the broadest *pattern* of a sentence in a natural language. Aristotle's syntax language specifies, abstractly, only nouns and verbs as its vocabulary, which are combined to form sentences according to this elementary rule. We might wish that Aristotle had expressed this rule with at least the modest precision here. However, Aristotle has neither a complete nor a complex set of syntax rules of sentence formation in *On Interpretation*. Still, it is evident from his treatment of this topic in his logical investigations that his understanding of the grammar of a natural language is richer than his lack of rigorously stated rules would indicate. And while this syntax rule is mixed with semantic notions, he nevertheless has identified here the basic pattern of a sentence in a natural language.

Aristotle continues his discussion of grammar in *On Interpretation 4* by focusing his principal attention on the kinds of sentence that are subject to logical analysis. He excludes, for example, prayers; and we take him also to exclude imperatives, interrogatives, and exclamations (17a3-4). In *Metaphysics 9.10*, for example, he makes this point rather emphatically: "for an affirmation and a sentence are not the same [οὐ γὰρ ταὐτὸ κατάφασις καὶ φάσις]" (1051b24–25). Accordingly, Aristotle considers only those kinds of sentence that are either true (ἀληθής [*alethês*]) or false (ψευδής [*pseudês*]); or, as we express this nowadays, he considers only those sentences that have a truth-value (16a9-13). His explicit interest is only with the kind of sentence that expresses a proposition, namely, with the declarative sentence. He writes:

[18] Aristotle then drops this matter and remarks that it be treated at another place. He comments briefly on forming words in *On Int. 2–3*.

> While every sentence [λόγος] has meaning [σημαντικός], though not by nature but, as we observed, by convention [συνθήκην], *not every sentence is a declarative sentence* [ἀποφαντικὸς δὲ οὐ πᾶς], but [only those] to which being true or false belongs [ἀλλ' ἐν ᾧ τὸ ἀληθεύειν ἢ ψεύδεσθαι ὑπάρχει]. (16b33–17a3)

Aristotle uses 'ἀπόφανσις' (*apophansis*) or 'ἀποφαντικὸς λόγος' (*apophantikos logos*) to denote the declarative sentence. A little later in his discussion he uses 'κατάφασις' (*kataphasis*) and 'ἀπόφασις' (*apophasis*) to denote two species of declarative sentence, namely, affirmation and denial, respectively. He uses 'λόγος' (*logos*), and sometimes 'φάσις' (*phasis*), to denote the genus sentence, but he often uses '*logos*' (among its other uses) interchangeably with '*apophansis*'. Thus, while a sentence consists in a noun and a verb, both of which are themselves meaningful sounds or expressions established by convention and not by nature,[19] they do not necessarily express something true or false. Truth and falsity involve predication (16b7, 9–10), in particular for Aristotle, predicating one thing of another one thing by combining (σύνθεσις) or dividing (διαίρεσις). Thus, with his discussion in *On Interpretation* together with his fuller discussion of predication in *Categories*, Aristotle names as genuine objects of logical investigation only those sentences that involve predication so as to express a proposition.[20]

Aristotle also recognizes a natural language to consist in both simple and compound sentences. Again, his logical investigation focuses on the simple sentence, and in *On Interpretation 5* he anticipates his treatment of sentences in *Prior Analytics*.

> One kind of declarative sentence [ἀπόφανσις] is simple [ἁπλῆ], that is, it affirms or denies some one thing of another [τὶ κατὰ τινὸς ἢ τὶ ἀπὸ τινός], while the other is composite [συγκειμένη], that is, a sentence compounded [λόγος... σύνθετος] of [such] simple sentences. And [such] a simple sentence is a meaningful expression, concerning something belonging or not belonging [ἔστι δ' ἡ μὲν ἁπλῆ ἀπόφανσις φωνὴ σημαντικὴ περὶ τοῦ ἐι ὑπάρχει τι ἢ μὴ ὑπάρχει] in the different tenses. (17a20–24)

He states that "an affirmation and a denial are simple when they denote [σημαίνουσα] some one thing of one other, whether or not universally or of something universal [μία δέ ἐστι κατάφασις καὶ ἀπόφασις ἡ ἓν καθ' ἑνὸς σημαίνουσα, ἢ καθόλου ὄντος καθόλου ἢ μὴ ὁμοίως] (18a12–13). Again:

[19] That for Aristotle words and sentences have meaning by convention and not by nature see *On Int.* 16a5–8, 16a26–28, and 16b33–17a2. Aristotle takes combining (σύνθεσις) to be conventional. Moreover, since he was aware that Greek was one language among others, it seems likely that he understood his metalinguistic posture toward language.

[20] Aristotle in *Cat. 4*: 2a4–10 complements what he writes here in *On Int.* "None of these things mentioned [i.e., things said without combination] in itself is an affirmation; an affirmation comes about in combination with other things. Every affirmation, it seems, is either true or false, but of things said with no combination none is either true or false, for example, 'man', 'white', 'runs' or 'wins'." Cf. *On Int. 10*: 20a34.

> An affirmation [κατάφασις] is one that denotes [σημαίνουσα] something of something. The subject [τοῦτο] is either a *noun* or a something not having a name [an indefinite noun], and what is affirmed must be one thing about one thing. (19b5–7)

Thus, in *On Interpretation* Aristotle recognizes two kinds of sentence (*logos*), in particular, two species of declarative sentence, that express a proposition and that are, accordingly, proper subjects of logical analysis (Figure 1).

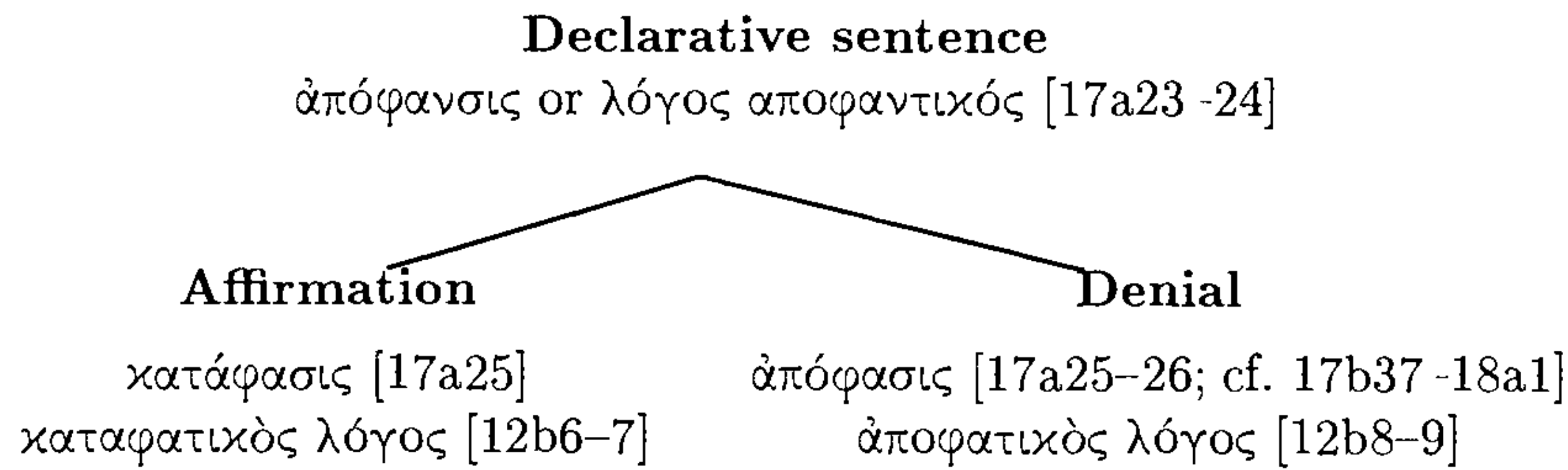


Figure 1.

Aristotle succinctly defines each of these in *On Interpretation 6*: "an affirmation is a sentence affirming one thing of another; a denial a sentence denying one thing of another [κατάφασις δέ ἐστιν ἀπόφανσις τινὸς, ἀπόφασις δέ ἐστιν ἀπόφανσις τινὸς ἀπὸ τινὸς]" (17a25–26). In this connection we can extract a second sentence formation rule for Aristotle, one pertaining especially to discursive discourse in a natural language and, following Aristotle, we restrict this rule to the simple sentence because it prepares us for his treatment of categorical sentences in *Prior Analytics*.

SFR2 A simple declarative sentence in a given natural language is a sentence that predicates one thing of another one thing, either attributively or privatively, so as to have a truth-value.

Attributive (κατηγορικός) predication produces an affirmation, privative (στερητικός) predication a denial, and such a denial always involves a negative operator (*Pr. An.* 51b31–35). We can represent the pattern of such a sentence graphically as follows (Figure 2).[21]

[21] In *On Int. 5*: 17a13–15 Aristotle remarks that 'two-footed land animal' is one thing and not many, but then refers this topic to another discussion. In any case, he here acknowledges that a term (or non-logical constant) need not be a single word. He briefly takes up this topic again in *On Int. 11*. There he claims that it is appropriate to combine predicates and subject terms when the predicates are not accidental to the subject. His example is "for two-footed and animal are contained in [ἐνυπάρχει γὰρ ἐν] man" (21a17–18). Aristotle always has his language follow his theory of substance.

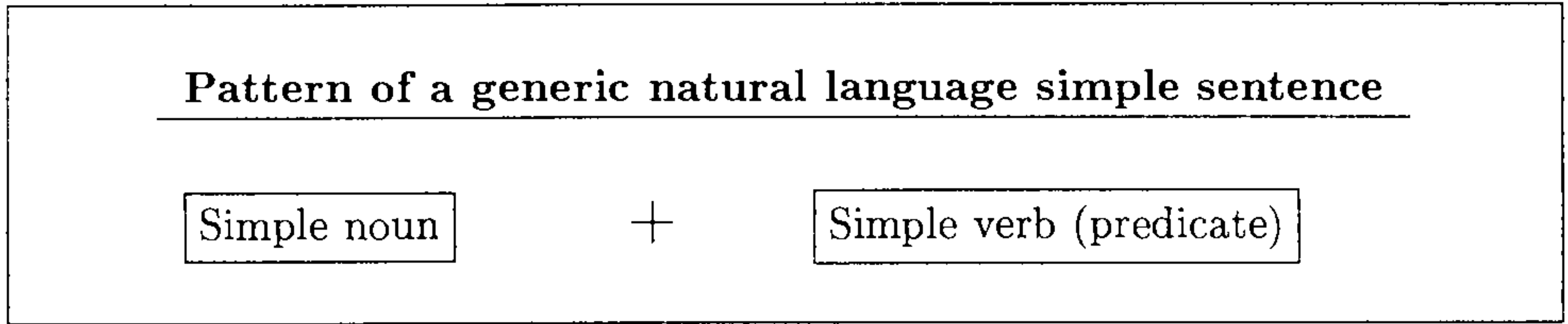


Figure 2.

Aristotle notes in *On Interpretation 10*: 20b1–12 that the general word order in natural Greek does not affect meaning: "nouns [subjects] and verbs [predicates] are interchangeable [μετατιθέμενα] and express the same meaning [ταὐτὸν σημάινει]" (20b1–2). He provides some examples to establish that interchanging the place of a noun and a verb does not generate two contradictions for a given sentence. He concludes by reaffirming his meaning: "Thus, by interchanging the noun and the verb an affirmation and a denial remain the same" (20b10–11), or, that is, express the same proposition.[22]

In *Sophistical Refutations* Aristotle poignantly emphasizes this point about the kind of predication specifically relating to the kind of discourse subject to logical analysis. There he focuses on sentences used in argumentation particularly as premisses. He writes in *Sophistical Refutations 6*, in connection with reducing all fallacies to *ignoratio elenchi*:

> And since deduction is based on [declarative] sentences [taken as premisses] [ἐπεὶ δ' ὁ συλλογισμὸς ἐκ προτάσεων], and refutation [ὁ δ' ἔλεγχος] is a deduction [συλλογισμὸς], refutation will also be based on [such] sentences [ἐκ προτάσεων]. If, therefore, [such] a sentence [ἡ πρότασις] is a single predication about a single thing [ἕν καθ' ἑνος], clearly this fallacy [viz., treating many questions as one] also depends on ignorance of the nature of refutation; for what is not [such] a sentence appears to be one [φάινεται γὰρ ἒιναι πρότασις ἡ οὐκ οὖσα πρότασις]. (169a12 16)

Of course, here Aristotle uses 'πρότασις' (*protasis*) to denote a sentence (*logos*, *apophansis*) used as the starting point for argumentation, that is, to denote a sentence in the role of a premiss. His discussion here comports exactly with his practice in *Prior Analytics* and with his definition of '*protasis*' there.

> A premiss, then, is a sentence affirming or denying something about something [Πρότασις μὲν οὖν ἐστὶ λόγος καταφατικός ἢ ἀποφατικός τινος κατά τινος].[23] (*Pr. An. A1*: 24a16 17; cf. *On Int. 11*: 20b22 25)

[22] All this, of course, is easier for classical Greek than for, say, modern English, because Greek is a highly inflected language.

[23] We use R. Smith's (1989) translation of *Prior Analytics* with some modifications, notably translating '*sullogismos*' by 'syllogism' and not by 'deduction' in all cases.

He also notes in *Prior Analytics A1* that "a syllogistic premiss without qualification will be either the affirmation or the denial of one thing about another [ἔσται συλλογιστικὴ μὲν πρότασις ἁπλῶς κατάφασις ἢ ἀπόφασις τινος κατά τινος]" (24a28 30). In *Posterior Analytics A22* he writes that "one thing is predicated of one thing [ἓν καθ' ἑνὸς κατηγορεῖσθαι]" (83b17 18; cf. *Po.An. A2*: 72a5–14).

While in *On Interpretation* Aristotle provides rules for sentence formation in a natural language in a rather intuitive and, by modern standards, non-rigorous manner, he nevertheless is especially concerned there with syntactic matters. He even provides definitions of denial and affirmation that have a syntactic character, although, again, they are mixed with semantic notions, and he cites examples to bear out his meaning. He writes:

> Whatever someone may affirm, it is possible as well to deny, and whatever someone may deny, it is possible as well to affirm. Thus, it is evident that each affirmative sentence has an opposite denial [ἀπόφασις ἀντιχειμενή], just as each denial has an [opposite] affirmative. (17a30 33)

Here Aristotle is particularly concerned with contradictories and not with contraries.

In this connection Aristotle provides in *On Interpretation* a rather syntactic rule for forming the negation, or contradictory, of a given affirmation. This rule is similar to that for forming an indefinite noun by prefixing 'οὐ' (*ou*, *ouk*, *oux*) to the common noun.[24] His example of negating a common noun is the following (16a29–30):

ἄνθρωπος (man) οὐκ ἄνθρωπος (non-man or not-man)

In *On Interpretation 10*: 20a7 9 he states that the '*ouk*' is attached to the *noun* and is not a part of the verb: "but the 'not' must be added to 'man' [ἀλλὰ τὸ οὐ, τὴν ἀπόφασιν, τῷ ἄνθρωπος προσθετέον]". Table 1 provides two sets of his examples of contradictories from *On Interpretation 10*, each having its one negation (contradictory) matched with its one given affirmation (19b14–19; cf. 19b27–29 & 19b38–20a1).

It is interesting to notice that Aristotle does not cite a partial privative sentence as the negation of a universal attributive sentence as is his customary practice when he treats contradictories in *Prior Analytics*. Here he prefixes *an entire sentence* with '*ou*' in a syntactic way, and he does this both with the verb 'to be' and with transitive and intransitive verbs. Thus, for "Πᾶς ἔστιν ἄνθρωπος δίκαιος" ("Every man is just") in the above examples he could have used "Τις ἄνθρωπος οὐκ ἔστι

[24]In *On Int. 2* (16a29–32; cf. *On Int. 10*: 19b7 10) Aristotle coined an expression for a noun prefixed by 'οὐ' (οὐχ, οὐκ): 'ὄνομα ἀόριστον. This is usually translated as 'indefinite noun' or 'indeterminate noun'. He also named indeterminate verbs (*On Int. 3*: 16b11 15) with a similar prefix: 'ῥῆμα ἀόριστον'.

Table 1.

Affirmation	**Negation**
Εστιν ἄνθρωπος. [Man is.]	Οὐκ ἔστιν ἄνθρωπος. [Not - man is.]
Εστιν οὐκ ἄνθρωπος. [Man is not.]	Οὐκ ἔστιν οὐκ ἄνθρωπος. [Not - man is not.]
Εστι πᾶς ἄνθρωπος. [Every man is.]	Οὐκ ἔστι πᾶς ἄνθρωπος. [Not - every man is.]
Εστι πᾶς οὐκ ἄνθρωπος. [Every man is not.]	Οὐκ ἔστι πᾶς οὐκ ἄνθρωπος. [Not - every man is not.]
Also consider the following from 19b32–35 and 20a5 7:	
Πᾶς ἔστιν ἄνθρωπος δίκαιος. [Every man is just.]	Οὐ πᾶς ἔστιν ἄνθρωπος δίκαιος. [Not - every man is just.]
Πᾶς ἔστιν ἄνθρωπος οὐ–δίκαιος. [Every man is non-just.]	Οὐ πᾶς ἔστιν ἄνθρωπος οὐ–δίκαιος. [Not - every man is non-just.]
Υγιάινει πᾶς ἄνθρωπος. [every man fares.]	Οὐχ ὑγιάινει πᾶς ἄνθρωπος. [Not - every man fares.]
Υγιάινει πᾶς οὐκ–ἄνθρωπος. [Every non-man fares.]	Οὐχ ὑγιάινει πᾶς οὐκ–ἄνθρωπος. [Not - every non-man fares.]

δίκαιος" ("Some man is not just") as its specific negation, but he did not.[25] In this connection, then, we can formulate another rule of sentence formation having to do with negation as follows (Table 1):

SFR3 The negation of a given affirmation in a natural language is formed by prefixing 'not' to the entire sentence.

Prefixing '*ou*' in this way to form the negation, or contradictory, of a given sentence does not comport with ordinary Greek syntax, and thus it indicates an artifice on Aristotle's part in his treatment of sentences in *On Interpretation*. We can take 'not' here to mean 'it is not the case that ...'.

[25] A strong test of Aristotle's having conceived of negation in a syntactic way would be to find instances of double negation, which we have yet to find. Another test would be to find instances of his prefixing a sentence with 'it is false that ...' or 'it is not the case that ...'. Here there seem to be ample cases. There is at least a kind of double negation of a term, i.e., of a simple noun, in *Po.An.A11*: 77a17 & 18. Aristotle writes "μὴ ζῷον δὲ μή" and "μὴ ζῷον δ' ὀύ", which J. Barnes translates as "not not an animal" (1994:17). Consider also *On Int. 9*: 18b11 14.

Although Aristotle only addressed sentence formation in a natural language in *On Interpretation*, we can see him there having already anticipated the formal language found in *Prior Analytics*.

Aristotle on proposition

Whether Aristotle had taken a philosophical position, *vis-à-vis* the modern discussion, on whether or not propositions exist as *ideal objects* need not concern us here. Given his anti-platonic, materialist tendency this seems unlikely. However, if we take 'proposition' more loosely to denote the *meaning* of a declarative sentence, we can easily see that Aristotle made and worked with a distinction between a sentence, which is a linguistic object, and the meaning or proposition it expresses, which is a non-linguistic object. Both of these, of course, he distinguished from what a sentence denotes, which is a state of affairs [πρᾶγμα], that is, something that obtains or does not obtain in the world. This is evident from his treatment of sentences in *On Interpretation* and in *Prior Analytics*, when he used '*apophansis*', and '*logos apophantikos*', as well as '*kataphasis*', '*apophasis*', and '*protasis*'. None of these words properly translates as 'proposition' *per se*; but each can be understood to convey what 'sentence expressing a proposition' means. Aristotle recognized that two or more different sentences, whether of one's own natural language or of different natural languages, might express the same proposition, as well as that the same sentence might express more than one proposition or have more than one meaning. In *On Interpretation 1* he writes that words are "symbols of affections in the soul" (τῶν ἐν τῇ ψυχῇ παθημάτων σύμβολα), that speech, and thus writing, is not the same for all peoples (16a3–6; cf. n19 above). He continues:

> But all the mental affections themselves [ταὐτὰ πᾶσι παθήματα τῆς ψυχῆς], of which these words are primarily signs [σημεῖα], are the same for everyone, [just] as are the objects [πράγματα] of which those affections are likenesses [ὁμοιώματα]. (16a6–8)

Perhaps his claim here about the whole of mankind is a bit sweeping. Nevertheless, he clearly indicates here his distinguishing very different linguistic objects as expressing the same meaning or expressing the same proposition — that peculiar, non-linguistic thing that is grasped by a human being in thought.[26]

[26] Aristotle makes an analogous remark in *Po.An.A10* about demonstration: "Deductions, and therefore demonstrations, are not addressed to external argument [οὐ γὰρ πρὸς τὸν ἔξω λόγον] but rather to argument in the soul [ἀλλὰ πρὸς τὸν ἐν τῇ ψυχῇ], since you can always object to external argument, but not always to internal argument" (76b24-27). There is another similar remark in *Meta. 4.5* in connection with his arguing against those who disparage the law of non-contradiction: "For those who hold such opinions because they are confused by real difficulties, can easily be cured of their ignorance by someone who addresses himself not to their arguments, but to their meaning [οὐ γὰρ πρὸς τὸν λόγον ἀλλὰ πρὸς τὴν διάνοιαν]; whereas those who argue for argument's sake can be cured only by refuting each of their explicit arguments verbally by other arguments" (1009a18-22). Also consider *On the Soul 3.3–8* and *SR 10* where Aristotle treats the distinction some make between an argument in thought and an argument in words.

Aristotle makes much the same point in *On Interpretation 14*, although there in relation to considering what count as genuine contraries.

> Is an affirmation [ἡ κατάφασις] contrary to a denial [τῇ ἀποφάσει] or contrary to another affirmation [ἡ κατάφασις τῇ καταφάσει]? Is the sentence [ὁ λόγος] "Every man is just" contrary to "No man is just"? or to "Every man is unjust"? For example, "Callias is just", "Not - Callias is just", "Callias is unjust". Which of these sentences are contraries?
>
> For if the expression corresponds with things in the mind, and it is the opinion of the contrary that is contrary, for example, that "Every man is just" [is contrary to] "Every man is unjust", then the same thing must hold of our expressed affirmations as well [ἐι γὰρ τὰ μὲν ἐν τῇ φωνῇ ἀκολουθεῖ τοῖς ἐν τῇ διανοίᾳ, εκεῖ δ' ἐναντία δόξα ἡ τοῦ ἐναντίου ... καὶ ἐπὶ τῶν ἐν τῇ φωνῇ καταφάσεων ανάγκη ὁμοίως ἔχειν]. But if it is not the case that the opinion of the contrary is not the contrary, then neither will one affirmation be the contrary to another [ἐι δὲ μηδὲ ἐχεῖ ἡ τοῦ ἐναντίου δόξα ἐναντία ἐστίν, οὐδ' ἡ κατάφασις τῇ καταφάσει ἔσται ἐναντία]; but the above mentioned denial will be the contrary [ἀλλ' ἡ ἐιρημένη ἀπόφασις]. And so, one must inquire which opinion is contrary to a false opinion, whether the opinion of the true denial or the contrary opinion. [On taking 'good' and 'bad' in sentences and thought.] (23a27-23b2)

Aristotle here fusses with a notion of logical equivalence while considering different sentences and their meanings. He takes up this matter in virtually the same way in *Prior Analytics A46.* In any case, this shows that he distinguished a sentence, here taken as a formal object, from its meaning or content.

Aristotle specifically addresses the topic of a given sentence having more than one meaning in *On Interpretation 8.* To establish a point of contrast he first mentions some sentences in which each word has only one meaning (18a12–17). He then cites an instance of a sentence in which a given word is *artificially* designated as having more than one meaning. He writes:

> If one word [ἓν ὄνομα] has two meanings, which do not combine to make one, the affirmation itself is not one [οὐ μία κατάφασις]. If, for instance, you give the name 'garment' alike to a horse and a man, then it follows that "garment is white" would not be one but two affirmations, nor would "garment is not white" be one denial but two. (18a18 21)

Surely this treatment of the topic not only indicates his making a clear distinction between a sentence and the proposition it expresses, but it also strikingly rings of his experimenting with a notion of reinterpretation familiar to modern logicians. This is not to claim that Aristotle is a model-theoretic logician. Still, it is evident that in this case Aristotle has retained the word 'garment' but *reinterpreted* it twice. And while his example is elementary and serves as an illustration, he might

just as easily have reinterpreted an entire sentence. In this connection it is worth noticing that Aristotle regularly used the definite neuter article 'τό' (inflected as appropriate in the context of his exposition) as moderns use quotation marks to mention a word or an expression or even to mention an entire sentence.

The matter of a given sentence expressing more than one proposition occupied Aristotle's attention in *Sophistical Refutations*, where his discussion is considerably more developed than has customarily been acknowledged. Surely recognizing a given sentence as having more than one meaning is evidence of making a distinction between its syntax and its semantics. *Sophistical Refutations* is replete with such examples, especially, for instance, where Aristotle treated ambiguity and equivocation.[27]

We can now turn to Aristotle's model of the grammar of the formal language of the underlying logic depicted in *Prior Analytics*. His notation there is quite elementary: he employed only upper case Greek letters as schematic placeholders for terms in categorical sentences. And he never provided abbreviations for his logical constants. Nevertheless, he specifically treated sentence *patterns* and their *logical relationships* in a genuinely syntactic manner.

The grammar of Aristotle's formal language

In respect of the theory of predicating of substance outlined in *Categories*, as treated also in *Metaphysics* and *Topics*, and that underlies his notion of predication in *Prior Analytics*, Aristotle understood there to be four ways that an attribute or property — ἴδιον, πάθος, ποιόν (see *Top. 1.5:*102a18-30 on ἴδιον) – can be related (or belong) to a substance or subject - οὐσία or ὑποκείμενον (Table 2).

Table 2.

Kinds of substance attribution	
1.	Every individual of a given kind has a given property.
2.	No individual of a given kind has a given property.
3.	Some individual of a given kind has a given property.
4.	Some individual of a given kind does not have a given property.

These attributions involve ontic relationships that exist independent of a knower: they obtain or they do not obtain. Aristotle referred to such matters generally

[27]In *SR* Aristotle treated the syllogistic deductive process as well, but there his focus was on *semantic matters*. In particular, he treated the fallacies as though they formally violate what a deduction (*sullogismos*) is as this topic is treated in *Pr. An.* For example, in the case of ambiguity, while a given argument with an ambiguity has one grammatical pattern, which helps to make it appear to be a deduction, it really has two underlying logical patterns. And in the case of equivocation, while an argument with an equivocal expression has a given grammatical pattern that makes it appear to be a deduction, it really has, with the addition of a fourth term (in relation to a standard three term deduction), an underlying logical pattern different than a deduction. Nothing results necessarily in these cases.

as πράγματα (*pragmata*; singular *pragma*), or states of affairs, facts, and he used 'τὸ εἶναι' — "to be [the case]" — and 'τὸ μὴ εἶναι' — "not to be [the case]" — to qualify them (cf. on his using 'ἀληθής' and 'ψευδής' in this connection). From these facts about existence Aristotle conceived four ways that a human being could express – that is, predicate (κατηγορεῖν) — these substance/attribute relationships linguistically. In the process he invented four logical constants to capture these relationships - and he *explicitly named* each, although without an expression for 'logical constant', in *Prior Analytics A4*: 26b30-33 (cf. *A23*: 40b23-26). Thus, corresponding to the four ontic relationships above, there are four possible predications of a subject by a participant (3).

Table 3.

Aristotle's four logical constants		
Logical Constant	**Predication**	**Modern Abbreviation**
1. Τὸ παντὶ ὑπάρχειν (belongs to every)	A given property is predicated of [is said to belong to] every member of a given kind.	*a*
2. Τὸ μηδενὶ ὑπάρχειν (belongs to no)	A given property is predicated of [is said to belong to] no member of a given kind.	*e*
3. Τὸ τινὶ ὑπάρχειν (belongs to some)	A given property is predicate of [is said to belong to] some member of a given kind.	*i*
4. Τὸ μὴ τινὶ ὑπάρχειν (does not belong to some) Τὸ μὴ παντὶ ὑπάρχειν (belongs not to every)	A given property is not predicated of [is said not to belong to] some member of a given kind.	*o*

Correspondingly, there are four categorical sentence patterns that Aristotle used throughout his logical investigations in *Prior Analytics*. His most commonly used schematic representations of the four categorical sentences are represented in Table 4.

Concerning any categorical sentence AB, then, A can be taken, or predicated, of B in four ways. 'A' and 'B' here are schematic letters that hold places for terms, or non-logical constants. The four kinds of sentence involve the four kinds

Table 4.

Aristotle's model for each kind of categorical sentence		
Sentence pattern	**Kind of categorial sentence**	**Modern expression**
1. τὸ A παντὶ τῷ B ὑπάρχει (A belongs to every B)	Universal attributive	A*a*B
2. τὸ A μηδενὶ τῷ B ὑπάχει (A belongs to no B)	Universal privative	A*e*B
3. τὸ A τινὶ τῷ B ὑπάχει (A belongs to some B)	Partial attributive	A*i*B
4. τὸ A τινὶ τῷ B μὴ ὑπάχει (A does not belong to some B)	Partial privative	AoB

of predication, which themselves reflect the four ontic relationships.[28] Aristotle thought of a categorical sentence as having a special pattern that distinguishes it from other kinds of sentence, namely, from those of natural Greek. Moreover, he thought of each of the four kinds of categorical sentence as itself fitting a special pattern. This is most evident in his treating the syllogisms and non-syllogisms in *Prior Analytics A4–6*. Thus, we can extract Aristotle's syntax rule, according to his formal grammar, for forming a simple categorical sentence in a given object language pertaining to a given domain and express it as follows.

CSFR1 A categorical sentence in a given language consists in combining a non-logical constant with any one of the four logical constants with another non-logical constant in this order.

This rule identifies the *pattern* of a categorical sentence in Aristotle's formal language. Aristotle's expression for 'non-logical constant' is 'ὅρος', or 'term'. The term in the first position is called the predicate term (τὸ κατηγορούμενον or ὅρος κατηγορούμενον), the term in the second position is called the subject term (τὸ ὑποκείμενον or ὅρος ὑποκείμενος). In natural Greek it is customary, but not a strict practice (*On Int.* 20b1–12), to place the subject of a sentence before the predicate/verb. But in *Prior Analytics* we see that Aristotle quite deliberately placed the predicate term before the logical constant, which acts as a verb, and

[28]Aristotle takes the following expressions to amount to the same thing: 'A belongs to every B' and 'A holds of every B'; 'A follows all B'; and 'A is predicated of every B'. While Aristotle used 'ὑπάρχειν' (*huparchein*), 'ἀκολουθεῖν' and 'κατηγορεῖσθαι' respectively, his preference to use '*huparchein*' for the logical constants was not accidental but an important reflex of his theory of substance.

then place the subject term after the logical constant. In addition, the logical constants themselves are rather artificial constructions aimed to reproduce linguistically what he took to be conditions of being, as in *Categories* and *Metaphysics.* It is evident that Aristotle thought of a categorical sentence as formally constructed by concatenating, stringing or combining, a predicate term (or non-logical constant) with a logical constant with a subject term (or non-logical constant) *strictly* in this order.

We might also extract two additional categorical sentence formation rules that have a rather more semantic character, but which nevertheless bear on the logical pattern of a sentence.

CSFR2 The two non-logical constants in every categorical sentence are not identical.[29]

CSFR3 The two non-logical constants in every categorical sentence are homogeneous with respect to grammatical category, that is, both non-logical constants are substantives.

This seems to be Aristotle's practice, at least, for the most part. There is a passage in *Prior Analytics A36* that confirms this (CSFR3).

> For we state this without qualification about them all: that terms must always be put in accordance with the cases of the *nouns* [κατὰ τὰς κλήσεις τῶν ὀνομάτων] ... (48b40 41; see 48b39 49a2; cf. *Pr. An. A39–40*).

Conversion otherwise would seem unintelligible. According to modern standards, we might also formulate a fourth rule, which is surely implicit in Aristotle's thinking.

CSFR4 Nothing is a categorical sentence except in virtue of these rules.

[29]This is a restriction on the language that anticipates its use in scientific discourse. However, Aristotle does recognise identity in his formal logic *per se* (see, e.g. *Pr. An. A41* and *B15*). There is an interesting instance of Aristotle's treating this matter in *Po. An. A3* where he establishes the inadequacy of circular reasoning for demonstration (but not its invalidity). He writes, after remarking that he can accomplish this demonstration by using three or even two terms: "When if A is the case, of necessity B is, and if B then C, then if A is the case C will be the case. Thus, given that if A is the case it is necessary that B is the case and if B is the case that A is the case (this is what is to proceed in a circle [however long the loop]), let A be C. Hence, to say that if B is the case A is the case is to say that C is the case; and to say this is to say that if A is the case C is the case. But C is the same as A. Hence, it follows that those who declare that demonstrations may proceed in a circle say nothing more than that if A is the case A is the case. And it is easy to prove everything in this way" (72b37 73a6). Surely Aristotle considered such sentences as "Every horse is a horse", or, metalogically, "Every A is an A". However, since he was preeminently concerned with 'deriving something other' (which requirement appears in his definition of '*sullogismos*') for the proposes of extending knowledge in the sciences, such a restriction on the language makes perfect sense and does not do violence to his logical acumen.

There is no analogous, strong syntax rule for forming the negation, or contradictory, of a given affirmative sentence in *Prior Analytics* as there is at places in *On Interpretation.* However, in *Prior Analytics A46* he holds that denials require the use of a negative operator.

> Consequently, it is evident that 'is not-good' is not the denial [ἀπόφασις] of 'is good'. If, therefore, 'affirmation' or 'denial' [ἢ φάσις ἢ ἀπόφασις] is true about every single <predicate>, then if 'is not-good' is not a denial, it is evident that it must be a sort of affirmation [κατάφασις]. But there is a denial of every affirmation [καταφάσεως δὲ πάσης ἀπόφασις ἔστιν], and, therefore, the denial of this affirmation is 'is not not-good'. (51b31–35).

For Aristotle a genuine denial, as distinguished from an affirmation, involves a negative operator, whether as an adverb attached to a verb (predicate), or as a pronominal adjective attached to a non-logical constant (or as part of the logical constant).[30]

We can represent Aristotle's thinking on sentence formation as prescribed in his formal language as follows (Figure 3):

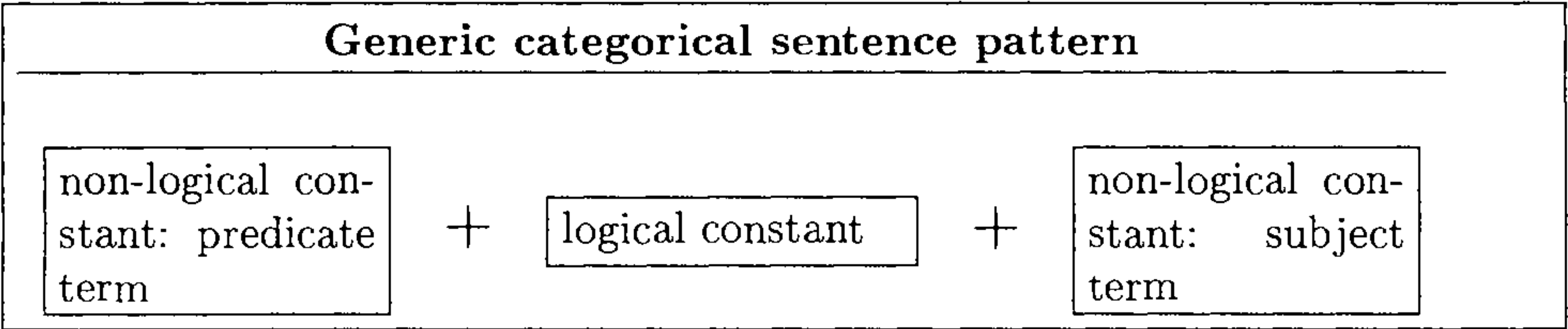


Figure 3.

Now, in *Prior Analytics*, as contrasted with his treatment of sentence formation in *On Interpretation*, Aristotle fixed the word order in a sentence. The order of the constituent parts of a categorical sentence does not change as it might in a natural language. The syntax of a categorical sentence is strict since *its use is anticipated in the syllogistic deduction process*, and this process requires precision.

In connection with treating language formally, Aristotle frequently writes of 'taking' (λαμβάνειν) or 'not taking' A of B in one of four ways. He writes, for example, about predicating in general that it is necessary to take something of something: ἀνάγκη λαβεῖν τι κατά τινος (*Pr. An. A23*: 40b31). He often uses 'AB', or similar expressions, to indicate any categorical sentence (*Pr. An. A25*: 42b6). This way of addressing predication tends to treat a sentence as uninterpreted, although, of course for Aristotle, not completely. A categorical sentence may be

[30] See below this section on Aristotle's distinguishing an opposite sentence from an affirmation and a denial. In short, while contraries are opposites (as are contradictories), each contrary sentence might be an affirmation, that is, designating attribution and not privation, as in "Every man is good" and "Every man is bad".

understood to express taking one term about another term as a formal matter. This is especially the case in *Prior Analytics A23*, which is an especially proof-theoretic chapter (§5.1). Moreover, he often writes in the same manner about taking sentences of one or another pattern, for example, as starting points of argumentation. In addition, he often uses the word 'πρόβλημα' (*problêma*) to indicate, not a particular sentence with a particular meaning, but to refer to each of the four *kinds* of categorical sentence. Consider in *Prior Analytics A4* where he uses '*problêma*' to indicate a sentence pattern:

> All the *problêmata* are proved through this figure [καὶ ὅτι πάντα προβλήματα δέικνυται διὰ τούτου τοῦ σχήματος]. (26b30–31; cf. *A27*: 43a16–19, *A28*: 44a36–37, & *A29*: 45a36–38)

'*Problêmata*' here does not refer to *problems* in a given domain as he uses this word in, for example, *Problems* and elsewhere, but to the four kinds of categorical sentence: it is used purely in reference to a formal object. We see that Aristotle's formal language, at least in respect of sentences, while not a purely uninterpreted object (as in string theory), is nevertheless sufficiently formal to exemplify the defining structures or patterns of categorical sentences.[31] This indicates that Aristotle took his logical (formal or artificial) language represented in *Prior Analytics* to be a syntactic object for the purpose of defining an underlying logic.

Aristotle also considered the relationship of negation in a somewhat syntactic manner, notwithstanding that his semantics is just below the surface. He writes in *On Interpretation* that whatever can be affirmed can also be denied, whatever denied can be affirmed, and that each attributive sentence and each privative sentence has its own opposite (17a30 33). This is all formal. In *On Interpretation 7* he recognized two kinds of sentence that are opposites (τὰ ἀντικέιμενα): contradictories (αἱ ἀντιφάσεις) and contraries (τὰ ἐναντία). This corresponds exactly with what he writes in *Metaphysics 5.10* on contrariety *in things*. In *On Interpretation 7* he defines contradictories in a loosely syntactic manner as follows:

> Now, I call an affirmation [κατάφασιν] *contradictorily opposed* [ἀντιφατικῶς ἀντικεῖσθαι] to a denial [ἀπόφασιν] when what the one denotes [σημάινουσαν] universally and the other not universally. (17b16 18)

And then he provides some examples

> "Every man is white" [to] "Not every man is white" [πᾶς ἄνθρωπος λευκός — οὐ πᾶς ἄνθρωπος λευκός] and "No man is white" [to] "Some

[31] In addition, Aristotle uses three sets of **schematic letters** to mark places for terms, one set for each figure: for the first *ABΓ* (ABC), the second *MNΞ* (MNX), and the third *ΠΡΣ* (PRS). He names terms by their schematic positions — first (or major), middle, last (or minor) — and he calls the first and last ἄκρα (extremes). His use of the terms 'σχῆμα' (figure or arrangement) and 'διάστημα' (interval) are further indications. In addition, in his practice of substituting actual terms for schematic letters when determining that a premiss-pair pattern is inconcludent, he sets out such terms according to the schematic order for each figure's schematic letters: first figure — PMS; second figure — MPS; third figure — PSM.

> man is white" [οὐδεὶς ἄνθρωπος λευκός - ἔστι τις ἄνθρωπος λευκός]. (17b18–20)

This definition is used throughout *Prior Analytics*. Aristotle models these sentences as shown in Table 5.

Table 5.

Contradictories	
Aristotle's text	**Modern notation**
1. A belongs to every B — A does not belong to some B.	A*a*B – A*o*B
2. A belongs to no B — A belongs to some B.	A*e*B - A*i*B

The following two syntactic relationships hold between these different sentences:

1. Whenever a sentence fitting the pattern A*a*B is taken, then a sentence fitting the pattern A*o*B cannot be taken; and whenever a sentence fitting the pattern A*o*B is taken, then a sentence fitting the pattern A*a*B cannot be taken.

2. Whenever a sentence fitting the pattern A*e*B is taken, then a sentence fitting the pattern A*i*B cannot be taken; and whenever a sentence fitting the pattern A*i*B is taken, then a sentence fitting the pattern A*e*B cannot be taken.

Here Aristotle leaves the schematic letters uninterpreted — or unsubstituted — and asserts the formal, logical relationships that exist between sentences fitting such patterns. In fact, from his text on contradictories (cited above), we can extract Aristotle's rule for their formation and express it as follows.

Contradictory formation rule

The contradictory of a given sentence, whether attributive or privative, is formed by retaining the predicate and subject terms (non-logical constants) as given and replacing the logical constants as follows:

1. In the case of a universal attributive sentence, the universal attributive logical constant is replaced by the partial privative logical constant.

2. In the case of a universal privative sentence, the universal privative logical constant is replaced by the partial attributive logical constant.

3. In the case of a partial attributive sentence, the partial attributive logical constant is replaced by the universal privative logical constant.

4. In the case of a partial privative sentence, the partial privative logical constant is replaced by the universal attributive logical constant.

In *On Interpretation 7* Aristotle defines contraries in a loosely syntactic manner in the following way:

> I call a universal affirmation and a universal denial *contrarily opposed* [ἐναντίως (ἀντικεῖσθαι) δέ τὴν τοῦ καθόλου κατάφασιν καὶ τὴν τοῦ καθόλου ἀπόφασιν] (17b20–21)

His example is the following:

> "Every man is just" [to] "No man is just" [πᾶς ἄνθρωπος δίκαιος — οὐδεὶς ἄνθρωπος δίκαιος]. (17b21–22)

Of course, in both contradictory sentences and contrary sentences, the subject terms and the predicate terms in the one are the same in the other. Aristotle indicated this a little earlier in *On Interpretation 7*:

> Now if someone states universally of a universal that something belongs or does not belong [ὅτι ὑπάρχει ἢ υή], there will be contraries. (17b3–5)

Perhaps he states this more emphatically in *On Interpretation 14*: "for contraries are among things that differ most in respect of the same thing [περὶ τὸ αὐτό]" (23b22–23); and again in *Categories 11*: "the nature of contraries is to belong to the same thing, either in species or in genus [περὶ ταὐτὸν ἢ εἴδει ἢ γένει]" (14a15 16). In *On Interpretation 6*: 17a33–37 (34–35) he writes on contradictories that: "I mean opposites [ἀντικεῖσθαι] that [affirm and deny] the same thing of the same thing [τὴν τοῦ αὐτοῦ κατὰ τοῦ αὐτοῦ] and not ambiguously [μὴ ὁμωνύμως]". This definition is also used throughout *Prior Analytics.* Aristotle models these sentences as in Table 6.

Table 6.

Contraries				
Aristotle's text			**Modern notation**	
A belongs to every B	—	A belongs to no B.	A*a*B	A*e*B

The following syntactic relationships hold between the different sentences:

1. Whenever a sentence fitting the pattern A*a*B is taken, then a sentence fitting the pattern A*e*B cannot be taken.

2. And whenever a sentence fitting the pattern A*e*B is taken, then a sentence fitting the pattern A*a*B cannot be taken.

We can extract a formation rule for contraries analogous to that for contradictories from Aristotle's text (cited above) and express it as follows.

Contrary formation rule:

> The contrary of a given sentence, whether attributive or privative, is formed by retaining the predicate and subject terms as given and replacing the universal logical constants as follows:

1. In the case of a universal attributive sentence by replacing the universal attributive logical constant with the universal privative logical constant.

2. In the case of a universal privative sentence by replacing the universal privative logical constant with the universal attributive logical constant.

What Aristotle writes in *On Interpretation* and in *Categories* corresponds exactly with what he does and with what he writes in *Prior Analytics B15* about the formal relationships among categorical sentences.

> I say that verbally [κατὰ τὴν λέξιν] there are four ⟨pairs of⟩ opposite sentences [προτάσεις], to wit: 'to every' [παντὶ] ⟨is opposed⟩ to 'to no' [οὐδενί]; and 'to every' [παντὶ] ⟨is opposed⟩ to 'not to every' [οὐ παντί]; and 'to some' [τινὶ] ⟨is opposed⟩ to 'to no' [ὀυδενί]; and 'to some' [τινὶ] ⟨is opposed⟩ to 'not to some' [οὐ τινί]. In truth, however, there are three, for 'to some' and 'not to some' are only opposites verbally. Of these, I call the universal sentences contraries ('to every' is contrary to 'to none', as, for example, 'every science is good' [πᾶσαν ἐπισήμην ἔιναι σπουδάιαν] is contrary to 'no science is good' [μηδέμιαν ἔιναι σπουδάιαν]) and the other pairs of sentences opposites [sc., contradictories]. (63b23 30)

Aristotle's concern for argumentational skill and logical syntax

The syntactic character of Aristotle's treatment of opposition is all the more assured when we place his logical investigations in the context of his concern with argumentation, as it pertains to both axiomatic discourse and disputational discourse. Aristotle was eminently occupied in *Sophistical Refutations* and *Topics* with equipping his students with argumentational skills that they could employ quickly and with facility and keenness. These two treatises surely served as student handbooks. Perhaps his introduction to *Topics* exemplifies this concern.

> The purpose of the present treatise is to discover a method [μέθοδος] by which we shall be able to reason deductively [συλλογίζεσθαι] from generally accepted opinions about any problem set before us and shall

> ourselves, when sustaining an argumentation, avoid saying anything self-contradictory [ὑπεναντίον].[32] (100a18–21; cf. *Top.1. 2*)

Aristotle did not have his students memorize certain texts — a set of specific, stock speeches — to acquire this skill, as was a common practice at the time. Rather, he expected them to become familiar with the structural — formal or syntactic — aspects of cogent and fallacious reasoning. In effect, Aristotle aimed to have his students become accomplished logicians. His formal interests are especially evident in his closing remarks in *Sophistical Refutations 34*. He writes, in connection with remarking that his logical investigations are entirely new:

> For the training given by the paid teachers of eristic argumentation resembled the pedagogy of Gorgias. For some of them required their students to learn by heart speeches that were either rhetorical or consisted of questions and answers, in which both sides thought that the rival argumentations were for the most part included. Hence the teaching that they gave to their students was rapid but unscientific [ἄτεχνος]; for they conceived that they could train their students by imparting to them not an art but the results of an art ... he has helped to supply his need but has not imparted an art [τέχνην] to him. ... [While there was much information available having to do with rhetoric] whereas regarding deductive reasoning [τοῦ συλλογίζεσθαι] we had absolutely no earlier work to quote but were for a long time laboring at tentative researches. (183b36–184b3)[33]

Aristotle also took up developing argumentational skills in *Prior Analytics*, especially at *A24-46*, the chapters that follow the formal representation of his deduction system. He was particularly concerned in these chapters with developing an individual's ability to establish (κατασκευάζειν) or to destroy (ἀνασκευάζειν) an argumentation.[34] This theme is wholly consonant with his treatment of argumentation in *Sophistical Refutations* and *Topics*. Indeed, the title of his works on formal logic, 'τὰ ἀναλυτικά' — a topic specially treated in *Prior Analytics A45* — signifies his concern with the formal aspects of argumentation. Analysis (ἀνάλυσις; ἀναλυεῖν) is a process of transforming one syllogism in any one figure into another syllogism of another figure if both syllogisms prove the same *problêma*

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[32] We use E. S. Forster's (1960) translation of *Topics* and below his (1955) translation of *Sophistical Refutations* with significant modifications. Cf. L.-A. Dorion's (1995) French translation of *Sophistical Refutations*.

[33] This passage continues and ends the treatise with the following remark that might have been addressed to modern critical readers. "If, therefore, on consideration, it appears to you that, in view of such original conditions, our system is adequate when compared with the other methods which have been built up in the course of tradition, then the only thing which would remain for all of you, or those who follow our instruction, is that you should pardon the lack of completeness of our system and be heartily grateful for our discoveries" (184b3–8).

[34] See *Pr. An. A26-28* and summary at *A30*: 46a3–10. For example, Aristotle writes (*A26*): "... a universal positive *problêma* is most difficult to establish [κατασκευάσαι] but easiest to refute [ἀνασκευάσαι]" (43a1-2).

(§6.2). Aristotle aimed to promote his students' facility with reasoning syllogistically to establish and to refute arguments by studying the *logical relationships* among sentence patterns and among patterns of elementary arguments. This is analogous to a modern logician's studying the formal relationships among the rules of propositional logic.

Aristotle's mathematical disposition toward the study of grammar

In *On Interpretation* Aristotle treated sentences in natural languages metalinguistically. His practice there is much the same, although without the complexity, as that of a modern grammarian whose natural language is, say, English, and who writes an English grammar. Aristotle used Greek to mention Greek as this grammarian would use English to mention English. However, Aristotle went considerably farther than a grammarian in his treating the syntactic aspects of a language because he thought of his linguistic investigations as laying an epistemological — or *formal* — foundation for various axiomatic sciences, the *apodeiktikai epistêmai*. As a logician formalizing a deduction system, Aristotle continued in *Prior Analytics* to develop a formal grammar where a grammarian of a natural language might leave off in *On Interpretation*.

Aristotle always took Greek, whether explicitly or implicitly, as the background language for discourse in any specialized domain. In this connection, we might say that Aristotle took Greek as his master language, although he never formulated the matter using just such an expression. Still, he recognized that each of the special axiomatic sciences was equipped, or ought to be equipped, with its own specialized terminology, or vocabulary, appropriate to its domain. Aristotle indicated his having a notion of a specialized vocabulary in *Metaphysics 4.2*. There he wrote about using terminology across sciences and thus indicated, albeit negatively, that each science has its own terminology.

> For a term belongs to different sciences, not merely because it is used in many ways, but when its definition can be referred neither to a single subject matter nor to a common ground. (1004a24 25)

That Aristotle had a clear notion of *universe of discourse*, although, again, without an equivalent expression in Greek, is evident from his treatment of the genus of a given science in *Posterior Analytics A7, 9–10, 28*.[35] This notion is poignantly expressed also in *Metaphysics 10.4*: "a single science covers a single genus and therefore deals with the complete differences in that genus" (1055a31–32).[36] Perhaps *Posterior Analytics A7* (cf. *A8–10*) expresses his notion of universe of discourse most plainly.

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[35]Cf. Aristotle's treatment of different discourses in the various branches of mathematics in *Po. An. A5*.

[36]Cf. what Aristotle writes in *Meta. 4.2*: 1003b12–15 when considering the subject of philosophy and being *qua* being: "so whatever is said of one subject matter belongs to one science. Accordingly, whatever is said in reference to a single nature is a single science; for such statements, too, in some way or other, refer to a single subject matter". Cf. *Pr. An. A30*: 46a15–22.

> Thus you cannot prove anything by crossing from another kind — for example, something geometrical by arithmetic. There are three things involved in demonstrations: one, what is being demonstrated, or the conclusion [τὸ συμπέρασμα] (this is what holds of some kind in itself); one, the axioms [τὰ ἀξιώματα] (axioms are the items from which the demonstrations proceed); third, the underlying kind [τὸ γένος τὸ ὑποκείμενον] whose attributes [τὰ πάθη] — that is, the items incidental to it in itself — the demonstrations make plain.
>
> Now the items from which the demonstrations proceed may be the same [τὰ αὐτὰ]; but where the kinds are different [τὸ γένος ἕτερον], as with arithmetic and geometry, you cannot attach arithmetical demonstrations to what is incidental to magnitudes – unless magnitudes are numbers. ... Arithmetical demonstrations always contain the kind with which the demonstrations are concerned, and so too do all other demonstrations. Hence the kind must be the same, either *simpliciter* or in some respect, if a demonstration is to cross [ὥστ' ἢ ἁπλῶς ἀνάγκη τὸ αὐτὸ ἔιναι γένος ἢ πῇ, ἐι μέλλει ἡ ἀπόδειξις μεταβαίνειν]. *That it is impossible otherwise is plain; for the extremes and the middle terms must come from the same kind* [ἐκ γὰρ τοῦ αὐτοῦ γένους ἀνάγκη τὰ ἄκρα κὰι τὰ μέσα ἔναι], since if they do not hold in themselves, they will be incidentals.[37]
> For this reason you cannot prove by geometry that there is a single science of contraries, nor even that two cubes make a cube. (Nor can you prove by any other science what pertains to a different science, except when they are so related to one another that the one falls under the other — as, for example, optics is related to geometry and harmonics to arithmetic.) Nor indeed anything that holds of lines not as lines and as depending on the principles proper to them — for example, whether straight lines are the most beautiful of lines, or whether they are contrarily related to curved lines; for these things hold of lines not in virtue of their proper kind but rather in virtue of something common. (75a38–75b20)

Not only has Aristotle indicated his notion of universe of discourse in relation to a genus, but he has also indicated that he worked with a notion of category mistake. This matter also is treated in *Sophistical Refutations*.

That different scientific domains are distinguished in one or another discourse is an important part of Aristotle's discussion of fallacious reasoning in *Sophistical Refutations*. There he treats a kind of fallacious reasoning that violates the boundaries of different domains. In *Sophistical Refutations 8*: 169b20–23 he remarks that a sophistical refutation, while it is usually a spurious deduction of the

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[37] In *Po. An. A6*: 74b27 33, for example, Aristotle made this point about the different genera in an interesting way by stating that when the middle term of a deduction is necessary the conclusion must be necessary and thus germane to one science and not to another. 'Necessary' here is used in its modal sense.

contradictory of a given sentence, might, nevertheless, be a genuine deduction (i.e., a refutation) but one that is *not germane* to the subject matter under discussion. The deductive reasoning while genuine "only seems to be, but is really not, germane to the subject at hand [ἀλλὰ καὶ τὸν ὄντα μὲν φαινόμενον δὲ οἰκεῖον τοῦ πράγματος]". In *Sophistical Refutations 9* Aristotle draws a distinction between the function of a scientist and that of a dialectician, or, that is, of a logician. In this connection he writes about demonstrations and refutations special to a given science.

> So we shall need to have scientific knowledge of everything; for some refutations will depend on the principles of geometry and their conclusions, others on those of medicine, and others on those of the other sciences. Moreover, spurious refutations [οἱ ψευδεῖς ἔλεγχοι] also are among things which are infinite [as are, perhaps, the sciences and their demonstrations (170a22–23)]; for every art has a spurious proof peculiar to it, geometry a geometrical proof and medicine a medical proof. By 'peculiar to an art' [τὸ κατὰ τὴν τέχνην] I mean 'in accordance with the principles of that art' [τὸ κατὰ τὰς ἐκείνης ἀρχάς]. (170a27–34)

Aristotle returns to this matter in force in *Sophistical Refutations 11*, where he treats the discipline of logic as the dialectical art of the deductive principles common to all intelligible, cogent discourse. He also distinguishes the sophist from the eristic in respect of their motives (171b29-34). In this connection he establishes a clear notion of universe of discourse.

> Then there are those spurious deductions that do not accord with the method of inquiry peculiar to the subject yet seem to accord with the art concerned. For false geometrical figures are not contentious (for the resultant fallacies accord with the subject-matter of the art), and the same is the case with any figure illustrating something which is true, for example, Hippocrates' figure or the squaring of the circle by means of lunules. ... [Bryson's method of squaring is sophistical because it does not accord with the subject-matter.] ... And so any merely apparent deduction on these topics is a contentious argumentation, and any deduction that merely appears to accord with the subject-matter [κατὰ τὸ πρᾶγμα], even though it be a genuine deduction, is a contentious argumentation [because it only appears to accord with the subject-matter]. (171b11–22)

A little later in this discussion Aristotle provides some examples to illustrate his meaning.

> For example, the squaring of the circle by means of lunules is not contentious, whereas Bryson's method is contentious. *It is not proper to transfer the former outside the sphere of geometry because it is based on principles that are special to geometry* [κὰι τὸν μὲν οὐκ ἔστι μετενεγκεῖν

> ἀλλ' ἢ πρὸς γεωμετρίαν μόνον, διὰ τὸ ἐκ τῶν ἰδίων εἶναι ἀρχῶν], whereas the latter can be used against many disputants, namely, all those who do not know what is possible and what impossible in any particular case; for it will always be applicable. And the same is true of the way in which Antiphon used to square the circle. Or, again, if someone were to deny that it is better to take a walk after dinner because of Zeno's argumentation, it would not be a medical argument; for it is of a general application. (172a2–9)

It is evident, then, that Aristotle recognized there to be any number of special sciences, each with its own domain and topical sub-language, each of which is a fragment of a whole, or master, language.

Aristotle's focus shifts from a general concern with grammar, as in *On Interpretation*, to a more specialized concern with language and grammar in *Prior Analytics* and *Posterior Analytics* and in *Metaphysics*. There language is treated (1) as *modified* from natural language according as its universe of discourse is delimited and specialized and (2) as more rigorously formalized for the purposes of precision and deduction. Aristotle's emphasis on the simple sentence in *On Interpretation* repays him well in *Prior Analytics* where he treats his deduction system with more rigor. His linguistic and argumentational analyses in *On Interpretation*, and in *Sophistical Refutations* and *Topics*, provided the foundation for his formulating the simple grammar of the artificial language in *Prior Analytics*.

Now, while Aristotle seemed always to have natural language in the background when he undertook his logical investigations, his thinking was surely disposed toward constructing an artificial language. And while he surely did not work with a fully uninterpreted calculus, he nevertheless had already moved toward developing a notion of a precise scientific language for extended deductive discourse in each of the special axiomatic sciences. And here he developed some stringent requirements for intelligible discourse. In particular, in *Metaphysics 11.5* he expressed a requirement that in scientific discourse *one word have one meaning*, and if it were to have more than one meaning this should be made patently clear. He treats this topic there in conjunction with treating the law of non-contradiction in the following way:

> Those, therefore, who are to communicate with one another by way of argumentation [λόγου] must have some common understanding. ... Each word must therefore be intelligible [γνώριμον ἕν] and indicate something definite, not many things, but only one [καὶ μὴ πολλά, μόνον δὲ ἕν]; and if it has more than one meaning it must be made plain in which of these the word is being used. He, therefore, who says that "this is and is not" denies what he affirms, with the consequence that he declares the word to signify what it does not signify; but this is absurd. Consequently, if "this is" signifies something, it is absurd to assert truly its contradictory.

> Accordingly, if a word signifies something, and this is truly asserted, this [connection] must be necessary; but what necessarily is cannot ever not be; and so opposite sentences concerning the same thing cannot be true together ... [again] opposite sentences concerning the same thing can never be true together. (1062a11–23)[38]

Aristotle's scientific languages eschew the ambiguity that abounds in natural languages. Surely his concern for a precise syntax is a reflex of his concern for scientific precision. Again, we have the testimony of *Sophistical Refutations* to make Aristotle's case; this point is especially evident there when Aristotle treats fallacious reasoning involving ambiguity and equivocation as well as making many questions into one.

In connection with his study of logic, then, what he writes in *Metaphysics* and in *Sophistical Refutations* can be taken as a move toward developing a *logically perfect language*, albeit restricted in its scope to a specific, delimited domain. Nevertheless, Aristotle's impulse and that of a modern logician are correspondingly the same when their respective focuses are on the deduction process. Now, of course, each of these scientific languages is a topical sub-language of a given master language, in this case Greek, and as such each is an object, if not a natural, language. We might think of this as natural Greek departmentalized; or, rather, that the languages of mathematics and biology, for example, are specialized topical sub-languages with a tailored Greek as their mode of expression. In any case, Aristotle's requirements for scientific discourse, in connection with his syllogistic logic, indicate that his treatment of language in *On Interpretation*, *Prior Analytics*, *Metaphysics*, *Sophistical Refutations*, and even *Categories* is (1) thoroughly metalinguistic and, thus, (2) especially occupied with syntax. Aristotle aimed to formalize scientific discourse, not only with a polished deduction system, but also with precisely formulated linguistic requirements concerning both its syntactic and semantic dimensions.

Aristotle's formal language is not strictly an uninterpreted calculus awaiting an interpretation as a modern logician understands this matter. Its vocabulary consists only in (1) four fully interpreted logical constants and (2) a number of schematic (upper case Greek) letters that function purely as metalinguistic placeholders for terms in categorical (or predicational) sentences. These schematic letters, however, are generally uninterpreted in a way familiar to modern logicians; but they are not variables.[39] There are no genuine variables, whether bound or free, ranging over individuals in a given domain in Aristotle's formal language. Indeed, there is no need for variables, since the system lacks quantification theory and works with patterns appropriate to a *term logic*. Nor, then, are there any non-

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[38] See *Cat. 1* on equivocal (ὁμώνυμος) and univocal (συνώνυμος) names; cf. *Cat. 5*: 3b7–9. Also cf. *Meta. 4.4*: 1006a28–1006b20 in the context of defining 'having a meaning'.

[39] J. Corcoran (1974: 100) has called these "metalinguistic variables"; cf. R. Smith (1984: 590, 595) who refers to them as "syntactic variables for terms". We believe that Aristotle takes his letters to be schematic letters in a way similar to W. O. Quine's meaning of "dummy" letters (1970:12; 1982: 33, 145–146, 160-162, 289, 300-301).

logical constants in his formal language. Non-logical constants pertain to a given universe of discourse along with its object language. Again, there is no need for any non-logical constants. His formal language does not anticipate quantification and the existence of variables in a given object language.

It may be that considerations of natural language underlay Aristotle's thinking when he constructed his artificial language. However that may be, where a natural language has sentences, and this holds of an interpreted language or an interpretation of a formal language, Aristotle's formal language does not have sentences *per se*, but only formalized sentence patterns. A sentence possesses a truth-value; a sentence pattern does not. Indeed, just as with a modern formal language, Aristotle's artificial language is not strictly a language, since it contains no true or false sentences. So, while his language is interpreted in respect of its logical constants, it is not a fully interpreted object in respect of (1) its not being bound to a particular universe of discourse and (2) its having schematic letters holding places for terms that anticipate a given universe of discourse. In respect of the first point, his language is formally applicable to every domain but is itself specific to none — it is topic neutral. And in respect of the second point, 'term' is a metalinguistic name for a formal part of a categorical sentence, that part which is filled by a non-logical constant, a name or substantive. Aristotle's definition of 'term' (ὅρος) in *Prior Analytics A1* is consonant with his practice.

> I call that a term into which a premiss may be broken up, that is, both that which is predicated and that of which it is predicated (whether or not 'is' or 'is not' is added or divides them). (24b16–18)

There are no terms in Aristotle's formal language, only schematic letters holding places for non-logical constants (terms); and, of course, a schematic letter is not itself a term. The word 'term' (ὅρος) exists in Aristotle's metalanguage. In addition, there are no logically true sentences in Aristotle's syntax language.[40] Thus, there are absolutely no truth-conditions for sentence patterns in Aristotle's formal language. In principle, this is exactly the case with modern logics, save for a logic involving identity and tautology. Aristotle had genuinely syntactic concerns, although, again, without the sophistication and rigor of a modern mathematical logician, but, nevertheless, with an intelligence sufficient for having accomplished many of the same results as a modern logician. Aristotle's formal language is entirely a metalogical (metalinguistic) device used to objectify and exemplify, to explicate, and to study his logic, and, moreover, it was conceived by him to be applicable equally to all the axiomatic or axiomatizable sciences.[41]

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[40]We might make one exception to this. While Aristotle's syllogistic system seems to eschew identity, he does cite some instances of categorical sentences with the same subject and predicate terms. Surely Aristotle would recognize that in these cases it is impossible for such sentences to be false, just as he recognized the impossibility of the compound sentence expressing the law of the excluded middle, and any sentence expressing an instance of this law, being false. But such sentences do not serve his scientific interests, and thus his logic is accordingly restricted. Cf. n29.

[41]Even a modern mathematical logician, when constructing a formal language, has some in-

2.2 Truth conditions for object language sentences

In relation to establishing truth conditions for sentences, it is customary for a modern logician to speak about 'giving an interpretation' of a formal language. In this respect, then, a modern logician would (1) specify a non-empty domain as a universe of discourse, (2) specify the meanings of all logical constants, (3) establish definitions of 'true' and 'false', and (4) establish conditions under which a given interpreted sentence is either true or false. However, in connection with truth conditions, Aristotle did not employ a modern system of interpretations and reinterpretations; he seemed not to work with model-theoretic notions. Thus, we do not find him saying that "a given sentence is true under a given interpretation". Aristotle, however, does use a method of substituting non-logical constants for schematic letters in categorical sentence patterns. And we do witness him establish meanings for his logical constants and truth conditions for sentences, categorical or otherwise. Above we treated their syntax and now we treat their semantics, principally focussing on categorical sentences.

Since the four categorical sentence patterns are not themselves sentences possessing a truth-value, we might wonder what conditions Aristotle required to produce a categorical sentence according to his definition of '*logos*' or '*apophansis*', that is, beyond his sentence formation rules. It is immediately apparent that he would specify a universe of discourse — that is, he would introduce genuine non-logical constants, or, what amounts to the same thing, he would apply his formal language to a given domain. The following passage from *Posterior Analytics A10* establishes that this is so.

> Every demonstrative science [πᾶσα ἀποδεικτικὴ ἐπιστήμη] is concerned with three things: [1] what it posits to exist [ὅσα τε εἶναι τίθεται] (these items constitute the kind [τὸ γένος] of which it studies the attributes [παθημάτων] which hold of it in itself); [2] the so-called common axioms [τὰ κοινὰ λεγόμενα ἀξιώματα], that is, the primitives from which its demonstrations proceed; and [3] thirdly, the attributes [τὰ πάθη] where it assumes what each of them means [τί σημαίνει ἕκαστον]. (76b11–16)

Aristotle also provided meanings for his logical constants. In addition, he defined 'truth' and 'falsity', which he understood to pertain to sentences, and he provided the conditions under which a given sentence is true or false. We can take his discussions, particularly in *Posterior Analytics*, on the genus of a science as evidence

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tended interpretation in mind as pertains to variables, logical constants, and non-logical constants. When a logician identifies a notation designating variables, logical constants, non-logical constants, he/she already has in mind sentence, derivation, and the meanings of the logical constants. A logician never escapes language and indeed invents a formal language always anticipating its interpretations. Moreover, the distinction between logical syntax and semantics is one that exists for the most part in thought only, and even in thought the distinction is not complete.

of the requirement that a non-empty domain be specified. In this section we examine his definitions of the logical constants, his definitions of 'true' and 'false', and the conditions under which a sentence is true or false. There also is a section on Aristotle's notion of existential import and a final note on his intensional notion of truth. We begin with a brief statement on the importance he attributed to meaning.

Aristotle on meaning in general

Aristotle gave special attention to the matter of meaning in various treatises. In connection with semantics in general, he defined 'having a meaning' in relation to intelligible discourse in *Metaphysics 4.4*, concerning the principle of non-contradiction, in the following, rather stipulative, way:

> Suppose 'man' has the meaning 'two-footed animal'. By 'having a meaning' [τὸ ἕν σημάινει] I mean this: if 'man' is 'two-footed animal', then if anything is a man, its 'being two-footed' will be what its 'being a man' is. (1006a31-34)

Aristotle recognized that the meanings assigned to words, and the words, or meaningful sounds themselves, are conventions. Accordingly, he expected those engaged in intelligible discourse to agree that one word have one meaning, or if many meanings that this be made clear: "let us suppose ... that a word has a meaning and one specific meaning [σημᾶινόν τι τὸ ὄνομα κὰι σημᾶινον ἕν]" (*Meta. 4.4*: 1006b12-13; cf. *Meta. 11.5*: 1062a11-23). Moreover, he made absolutely clear that we not confuse a word with its denotation.

> As to having a meaning, what we insist on is that the meaning is not the object referred to [itself] (since then 'musical' and 'white' and 'man' could have a single meaning or referent, and all would be one, and those terms would be synonymous). [What we mean is that] it will not be possible to be and not to be the same thing [τὸ αὐτὸ], except ambiguously; for example, if we call a 'man' what others were to call a 'non-man'. *The question is not whether the same thing can at the same time be and not be a man in name, but in fact* [τὸ δ' ἀπορούμενον οὐ τοῦτό ἐστιν, ἐι ἐνδέχεται τὸ αὐτὸ ἅμα ἔιναι καὶ μὴ ἔιναι ἄνθρωπον τὸ ὄνομα, ἀλλὰ τὸ πρᾶγμα]. (1006b15-22)

Aristotle took up this matter throughout most of *Metaphysics 4.4*, where he was careful to state the necessity for clear definition and meaning in connection with the law of non-contradiction. He wrote that "to signify its being means that its being is not something else" (1007a26-27); and "there must, accordingly, be some meaning in the sense of indicating a thing's being" (1007b16-17). Here again Aristotle expected that words be used carefully and precisely in order better to reflect in thought what exists independently of thought. Then, later in *Metaphysics 4.7* he wrote that

> basic to all these argumentations [viz., eristic argumentations] are definitions. And definition [ὁρισμὸς] arises out of the necessity of stating what we mean; for the sentence of which the word is a sign becomes a definition [ὁ γὰρ λόγος οὗ τὸ ὄνομα σημεῖον ὁρισμὸς ἔσται]. (1012a21-24)

Then, in connection with meaning in relation to truth and falsity, Aristotle wrote in *Metaphysics 4.8*:

> Against all such argumentations, however, it must be asked [as at *Meta. 4.4*: 1006a18–22] ... not that something is or is not, but that something has meaning [οὐχὶ εἶναί τι ἢ μὴ εἶναι ἀλλὰ σημαίνειν τι]; so that we must converse on the basis of definition [ἐξ ὁρισμοῦ] by grasping what falsity and truth mean. (1012b5-8)

We shall treat truth and falsity more fully below, but note here that Aristotle is quite clear about the importance of establishing meaning[42] and about the relationship between meaning and existence.[43]

Defining the logical constants

Aristotle provides some explicit definitions of his logical constants in *Prior Analytics* and elsewhere. On 'belonging to every' and 'belonging to no' he writes in *Prior Analytics A1*: "I call 'belonging to every' or 'to none' universal" (24a18). He adds:

> For one thing to be in another as a whole is the same as for one thing to be predicated of every one of another. We use the expression 'predicated of every' when none of the subject can be taken of which the other term cannot be said, and we use 'predicated of none' likewise. (24b26-30)

In *Posterior Analytics A4* he writes on universal predication in much the same way:

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[42] On meaning, see *Top. 1.5* & *7.2-5* and *Po. An. B13-14*.

[43] Cf. *SR* on confusing a word with an object it denotes. At the outset of *SR1*, where Aristotle introduces the subject matter of fallacious argumentation, just after defining 'refutation' (ἔλεγχος), he remarks that argumentations might appear to be cogent when they are not. He then writes, setting the tone for what follows, that this might be due to "several causes, of which the most fertile and widespread is the argumentation that depends on names. For, since it is impossible to argue by introducing the actual things under discussion, but we use names as symbols in the place of things, we think that what happens in the case of names happens also in the case of the things, just as people who are counting think in the case of their counters" (165a4-10).

> I say that something holds of every case if it does not hold of some cases and not of others, nor at some times and not at others. For example, if animal holds of every man, then if it is true to call this a man, it is true to call him an animal too; and if he is now the former, he is the latter too. (73a28-31; cf. *A4*: 73b25-74a3 on 'holding universally' ['τὸ καθόλου'])

In *On Interpretation 7* he defines 'predicating universally' indirectly, when he identifies some sentences as indeterminate. He writes:

> It is necessary when asserting [ἀποφαίνεσθαι] as either belonging or not belonging [ὡς ὑπάρχει τι ἢ μή] sometimes to something universal [καθόλου] sometimes to an individual [καθ' ἕκαστον]. Now, if someone states universally of a universal that something belongs or does not belong, there will be contrary sentences [εναντίαι ἀποφάνσεις]. I mean by stating universally of the universal, for example, "Every man is white" and "No man is white". (17b1–6)

In *Prior Analytics A1* when he treats 'belonging to some' and 'not belonging to some' Aristotle writes in a rather eclipsed manner: "I call 'belonging to some', 'belonging not to some', or 'belonging not to every' partial [ἐν μέρει]" (24a18-19). He seems to have taken their meanings as evident to his audience. In addition, Aristotle takes 'belonging to some' in two ways, implicitly in *On Interpretation 7* but rather more explicitly in *Prior Analytics.* (1) In its *determinate* (διωρίσμενος) meaning, 'some' means, as in 'A belongs to some B', that 'some Bs are A' and 'some Bs are not A' but not that 'possibly all Bs are A'. The determinateness of a sentence pertains to its having only one meaning. A participant knows that, of a given kind, some indeed have and some indeed do not have a given property. (2) In its *indeterminate* (ἀδιόριστος) meaning, 'some' means 'at least one, possibly all'. Here a participant does not know, in the case of 'A belongs to some B', whether some Bs are not A or every B is an A. He writes in *Prior Analytics A1* that

> I call belonging or not belonging without a universal or partial indeterminate [ἀδιόριστον] as, for example, "The science of contraries is the same" or "Pleasure is not a good". (24a19-22)

Again, an indeterminate sentence is ambiguous. Aristotle does not usually cite a partial sentence, that is, one specifically using a partial logical constant, to identify indeterminateness. Rather, he usually cites a *general* sentence, such as "Men are white" or "Pleasure is good". In such cases he remarks that while 'men' is used universally, the sentence is indeterminate: it could mean "Some men are white" and "Some men are not white" or "All men are white" (*On Int. 7*: 17a38-17b16). There he indicates the indeterminateness of "Man is white" and "Man is not white". He notes that

> the universal 'man' is not used universally in the sentence. For the word 'every' does not signify the universal but that it is taken universally [τὸ γὰρ πᾶς οὐ τὸ καθόλου σημαίνει ἀλλ' ὅτι καθόλου].[44] (17b11-12)

This matter is clearly stated in *Prior Analytics A27* where he counterposes a sentence being determinate to its being indeterminate. There Aristotle comments on developing argumentational skills and instructs his students to select things that follow the subject as a whole since "a syllogism is through universal premisses". He continues this thought and thereby clarifies the meaning of 'indeterminate':

> Now, if it is indeterminate [ἀδιορίστου], it is unclear [ἄδηλον] whether the premiss is universal [καθόλου], whereas if it is determinate this is evident [διωρισμένου δὲ φανερόν]. (43b14–15; cf. *Pr. An. A4*: 26b21–25)

He also reveals his understanding when he establishes the inconcludence of a few patterns of premiss pairs by "proving it from the indeterminate" . In *Prior Analytics A4*, in connection with showing that a pair of premisses — the major a universal attributive or privative, the minor a partial privative — does not result in a syllogism, Aristotle notes that this must be established from the indeterminate. He writes:

> Moreover, since "B does not belong to some C" is indeterminate, that is, it is true if B belongs to none as well as if it does not belong to every (because it does not belong to some), ... (26b14-16; cf. *Pr. An. A5*: 27b16-23)

In this way, then, Aristotle defined his logical constants.

Finally, in this connection, Aristotle distinguishes variously between kinds of declarative sentence (*apophansis*). There are, *first*, the affirmation (*kataphasis*) and the denial (*apophasis*). Using Aristotle's terminology developed in *Prior Analytics*, we can take a *kataphasis* to be a positive, or attributive (κατηγορικός), sentence and an *apophasis* as a negative, or privative (στερητικός), sentence that uses a negative operator. *Second*, a sentence can be singular (καθ' ἕκαστον), partial, or particular (ἐν μέρει; κατὰ μέρος), or universal (καθόλου). The first of these determinations usually captures the quality of a sentence, the second its quantity. *Third*, a sentence can be either determinate (διωρισμένον or διοριστός) or indeterminate (ἀδιόριστον), as we noted above. These distinctions are more sharply defined in *Prior Analytics* than they are in *On Interpretation*, but the two works are generally in accord on these sentential determinations.[45] We shall examine the kinds of sentence more fully when we treat their truth-values in relation to the matter of existential import.

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[44]In *On Int.* 17b12-16 Aristotle rules out such sentences as "Every man is every animal". Aristotle takes a universal to be a secondary substance and not an individual. Later logicians considered 'taking a term universally' to mean taking a term to be distributed.

[45]Interestingly, Aristotle used '*huparchein*' in both works, but formalizes its use in *Pr. An.* In any case, we take his practice in both treatises as a reflex of his theory of substance.

Defining truth and falsity

In *On Interpretation 9* Aristotle treats the notions of truth and falsity especially in relation to examining contrary and contradictory sentences. There he provides definitions of 'true' [ἀληθής] and 'false' [ψευδής] that accord exactly with Alfred Tarski's treatment of the topic in "The concept of truth in formalized languages".[46] Aristotle writes:

> For, if it is true to assert [εἰ γὰρ ἀληθὲς εἰπεῖν] that something is white or not white, then it is necessarily [ἀνάγκη ἐ)ιναι] white or not white. And if it is [καὶ ἐι ἔστι] white or not white, it was true to affirm or deny it [ἀληθὲς ἦν φάναι ἢ ἀποφναι]. And, if it is not [in fact] white [έι μὴ ὑπάρχει], then to say that it is will be false [ψεύδεται]; if to say that it is will be false [καὶ εἰ ψεύδεται], then it is not white [οὐχ ὑπάρχει]. And so, it is necessary that the affirmation or the denial be true [ὥστ' ἀνάγκη τὴν κατάφασιν ἢ τὴν ἀπόφασιν ἀληθῆ ἕναι]. (18a39-18b4)

The upshot of this discussion is to affirm that every declarative sentence is either true or false and that "the truth of sentences consists in *corresponding with states of affairs* [ὥστε, ἐπεὶ ὁμοίως οἱ λόγοι ἀληθεῖς ὥσπερ τὰ πράγματα]" (19a32-33). In *Categories 12* Aristotle writes in much the same vein, but he states the case somewhat more strongly when he addresses various meanings of 'prior'.

> The existence of a man is reciprocal in relation to the true sentence about him as it follows from there being [such] a man [τὸ γὰρ εἶναι ἄνθρωπον ἀντιστρέφει κατὰ τὴν τοῦ ἔιναι ἀκολούθησιν πρὸς τὸνἀληθῆ περὶ αὐτοῦ λόγον]. For if a man exists, then the sentence asserting [ὁ λόγος ᾧ λέγομεν] that a man exists will be true [ἀληθὴς]. And conversely, if the sentence asserting [ὁ λόγος ᾧ λέγομεν] that a man exists is true [ἀληθὴς], then the man exists. The true sentence [ὁ μὲν ἀληθὴς λόγος], however, is in no way the cause of the [given] state of affairs [ἀίτιος τοῦ ἔιναι τὸ πρᾶγμα]; and yet the state of affairs [τὸ πρᾶγμα] seems somehow to be the cause of the truth of the sentence [πως ἀίτιον τοῦ ἔιναι ἀληθῆ τὸν λόγον]. *For a sentence is called true or false as the state of affairs exists or does not exist* [τῷ γὰρ ἔιναι τὸ πρᾶγμα ἢ μὴ ἀληθὴς ὁ λόγος ἢ ψευδὴς λέγεται]. (14b14-22)

Aristotle is quite clear about distinguishing a sentence pattern from a sentence, and a sentence from its denotation, or state of affairs, or even from its sense or meaning. Again, we have ample evidence of this topic treated more fully throughout *Sophistical Refutations* and *Rhetoric*.

We can supplement what Aristotle writes on truth and falsity in these works with what he writes in *Metaphysics 4.7-8* in connection with his discussion of the laws of non-contradiction and excluded middle.

[46] Tarski states his semantic definition in relation to natural language as follows: "a true sentence is one which says that the state of affairs is so and so, and the state of affairs indeed is so and so" (in Corcoran 1990: 155; cf. 154-165).

> And the possibility of a middle between contradictories is excluded; for it is necessary either to assert or to deny one thing of another [ἀλλ' ἀνάγκη ἢ φάναι ἢ ἀποφάναι ἓν καθ' ἑνὸς ὁτιοῦν]. This is clear from the definition of truth and falsity [δῆλον δὲ πρῶτον μὲν ὁρισαμένοις τί τὸ ἀληθὲς καὶ ψεῦδος]; *for to deny what is or to affirm what is not is false, whereas to affirm what is and to deny what is not are true; so that any sentence that anything is or is not states either what is true or what is false* [τὸ μὲν γὰρ λέγειν τὸ ὂν μὴ εἶναι ἢ τὸ μὴ ὂν εἶναι ψεῦδος, τὸ δὲ τὸ ὂν εἶναι καὶ τὸ μὴ ὂν υὴ εἶναι ἀληθές, ὥστε καὶ ὁ λέγων εἶναι ἢ μὴ ἀληθεύσει ἢ ψεύσεται, ἀλλ' οὔτε τὸ ὂν λέγεται μὴ εἶναι ἢ εἶναι οὔτε τὸ μὴ ὄν]. Hence, either what is is affirmed or denied, or else what is not is affirmed or denied. There can be no middle ground. (*4.7*: 1011b23-29 & *11.6*: 1063b15-18; cf. *On Int. 14*: 23b29-30)

We shall return to this matter in connection with Aristotle's notion of existential import.

Aristotle, then, is quite clear about 'truth' and 'falsity' applying to sentences (*logoi*) and not to states of affairs (*pragmata*), which he characterized using 'εἶναι', or 'being [the case]', and 'μὴ εἶναι', or 'not being [the case]'. However, Aristotle sometimes uses the words '*alêthês*' and '*pseudos*' in relation to *pragmata* where we would prefer using 'is the case' and 'is not the case' and thus would avoid making a category mistake. Consider, for example, what he writes in *Metaphysics 5.12* where he defines 'possibility' [δυνατόν] and 'impossibility' [ἀδύνατον]. (Here we transliterate, rather than translate, and mark in bold face, the Greek for 'true' and 'false' to objectify Aristotle's meanings.)

> [In the case where 'impossibility' means the opposite of 'possibility', the impossible is] the contrary of what is necessarily ***alêthês*** [τὸ ἐναντίον ἐξ ἀνάγκης ἀληθές]: that the diagonal of a square is commensurable with its side is impossible, because that is something ***pseudos*** [ὅτι ψεῦδος τὸ τοιοῦτον], and its direct contrary, incommensurability, is not only ***alêthês*** [ἀληθὲς] but also necessary; that it is commensurable is, therefore, not only ***pseudos*** [ψεῦδος] but also necessarily ***pseudos*** [ἐξ ἀνάγκης ψεῦδος]. On the other hand, the contrary of this, the "possible", holds when it is not necessary for its contrary to be ***pseudos*** [ψεῦδος]: it is possible for a man to be seated, for it is not of necessity ***pseudos*** [ψεῦδος] that he is not seated. The possible, then, means: (1) what is not of necessity ***pseudos*** [τὸ μὴ ἐξ ἀνάγκης ψεῦδος σημαίνει]; (2) what is ***alêthês*** [τὸ ἀληθές]; (3) what may be ***alêthês*** [τὸ ἐνδεχόμενον ἀληθὲς εἶναι]. (1019b23-33)

It is not uncommon for Aristotle, and he suggests that it is a common practice, to use '*alêthês*' and '*pseudês*' to refer to both *sentences* (*logoi*) and *states of affairs* (*pragmata*). In fact, he explicitly makes this point in *Metaphysics 9.10* where he remarks that being and nonbeing are commonly assessed according to the 'true' or the 'false', that is, by using the words '*alêthês*' and '*pseudês*'. He writes:

> This use depends on things being combined or dissociated [τοῦτο δ' ἐπὶ τῶν πραγμάτων ἐστὶ τῷ συγκεῖσθαι ἢ διῃρῆσθαι]; *so that he who thinks that what is dissociated is dissociated, and what is combined is combined, holds the truth, whereas he whose thought is contrary to the state of affairs is in error* [ὥστε ἀληθεύει μὲν ὁ τὸ διῃρημένον οἰόμενος διῃρῆσθαι κὰι τὸ συγκέιμενον συγκεῖσθαι, ἔψευσται δὲ ὁ ἐναντίως ἔχων ἢ τὰ πράγματα]. When, therefore, is there or is there not what is called truth or falsity?
>
> We must inquire into what we mean by this. For it is not because we truly [ἀληθῶς] hold you to be white that you are white; but it is because you are white that we who assert this speak truly [ἀληθεύομεν]. (1051b2-9)

Here he uses '*alêthês*' and '*pseudês*' to mean true and false in relation to sentences. He also frequently uses, as he does here, the verbs 'ἀληθεύειν' and 'ψεύδεσθαι'. Continuing later in this same passage he writes:

> As to "being" [τὸ εἶναι] in the sense of the true [ὡς τὸ αληθὲς] and "not being" [τὸ μὴ εἶναι] in the sense of the false [ὡς τὸ ψεῦδος], there are two cases: in one case there is truth [ἀληθὲς] if the combination [ἐι σύγκειται] [of subject and attribute] exists, and falsity [ψεῦδος] if there is a dissociation [τὸ δ' ἐι μὴ σύγκειται]; in the other case, however, whatever is, is as it is, or it is not at all. Here truth is the knowledge of these things [τὸ δὲ ἀληθὲς τὸ νοεῖν ταῦτα]. (1051b33-1052a2)

Aristotle also makes much the same point succinctly in *Metaphysics 5.29*.

> The "false" [τὸ ψεῦδος] refers (1) to a state of affairs as not the case [ὡς πρᾶγμα ψεῦδος]: and this, on the one hand, because it is not put together or cannot be put together. ... States of affairs [πράγματα] then are said to be not the case [ψευδῆ] whether *because they themselves are not* or because the appearance derived from them is of something that is not.
>
> Next, (2) *a false account, in so far as it is false, is the account of things that are not. Hence, every account is false which is an account of something other than that of which it is true* [λόγος δὲ ψευδὴς ὁ τῶν μὴ ὄντων, ᾗ ψευδής, διὸ πᾶς λόγος ψευδὴς ἑτέρου ἢ οὗ ἐστὶν ἀληθής]; for example, an account of a circle is false of a triangle.[47] (1024b17-19, 24-28)

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[47]Aristotle later writes that "just as we declare states of affairs to be 'false' which occasion a 'false' appearance" (1025a5-6). He refers in this passage to when the diagonal of the square is said to be commensurate with the side as a 'sense in which a state of affairs is not' (1024b21). We translate '*adunaton*' in relation to ontic matters by 'impossible', in relation to sentences by 'absurd'.

We encounter an analogue of this ambiguity in English, where 'true' and 'false' have a range of meanings, including the genuine and the spurious. We might be inclined to accuse Aristotle of making a category mistake, and perhaps there are cases where he slips. However, it is obvious here that Aristotle does not commit a category mistake, but that he uses the same word with two meanings corresponding to the two contexts. He does not confuse a sentence (*logos* or *apophansis*) with a corresponding state of affairs (*pragma*), although we might wish that he had always used different expressions to distinguish their being or not being. Thus, in relation to states of affairs (*pragmata*), we could properly translate '*alêthês*' by 'is the case' and '*pseudês*' by 'is not the case'.

Again, Aristotle is quite clear about *pragmata* either being or not being, obtaining or not obtaining. This is something ontic. He affirms, in one sense, that combination and division are mental acts, in particular, acts of predicating one thing of another either attributively or privatively, which are participant relative. In *Metaphysics 6.4* he writes:

> Now, being [τὸ δὲ ὡς ἀληθὲς ὄν] in the sense of being true and nonbeing [καὶ μὴ ὄν ὡς ψεῦδος] in the sense of being false are concerned with union [σύνθεσιν] and division [διαίρεσιν] and, taken together, with the relation of contradictories [ἀντιφάσεως]. For there is truth when an affirmation corresponds to a combination in beings and when a denial corresponds to a dissociation among beings; whereas there is error [or nonbeing] when the opposite relations hold [τὸ μὲν γὰρ ἀληθὲς τὴν κατάφασιν ἐπὶ τῷ συγκειμένῳ ἔχει τὴν δ' ἀπόφασιν ἐπὶ τῷ διῃρήμενῳ, τὸ δὲ ψεῦδος τούτου τοῦ μερισμοῦ τὴν ἀντίφασιν]. (1027b18–23)

A little later in this same passage he remarks that "the false and the true are *not in things* [οὐ γὰρ ἐστι τὸ ψεῦδος καὶ τὸ ἀληθὲς ἐν ταῖς πράγμασιν], as if the good were true and the bad were forthwith false; *but they are in thought* [ἀλλ' ἐν διανοίᾳ]" (1027b25-27). We could not ask him to be more clear. He adds, nevertheless:

> However, *since unification and separation are in thought and not in things*, 'being' in this sense differs from 'being' in the chief sense. For to predicate or deny what something is, or that it is of some sort, or that it is so much, or the like *requires thinking* [ἡ διάνοια]. (1027b29-33)

And thoughts become 'materialized', or expressed, by means of sentences. Aristotle makes this point at *Metaphysics 4.7* as follows:

> Still, every concept and thought is expressed either as an affirmation or a negation [ἔτι πᾶν τὸ διάνοητὸν καὶ νοητὸν ἡ διανοία ἢ κατάφασιν ἢ ἀπόφασιν]; this is clear from the definition [ἐξ ὁρισμοῦ] of truth and falsity. When a sentence either asserts or denies, it expresses either truly or falsely [ὅταν ἀληθεύῃ ἢ ψεύδηται].[48] (1012a2-4).

[48]This passage continues: "Whenever a sentence either asserts or denies [φᾶσα ἢ ἀποφᾶσα], it expresses either truly [ἀληθεύει] or falsely [ὅταν δὲ ὡδί, ψεύδεται]" (1012a4-5).

The distinctions that modern logicians believe themselves to have invented were surely anticipated, if not invented, by Aristotle in ancient times. *Categories* helps to make this even more evident.

Aristotle's treatment of truth conditions in Categories

In *Categories*, as he sharpens the distinction between a sentence and its denotation, Aristotle defines truth and falsity by affirming that a sentence is not a substance (*ousia*). In *Metaphysics* he makes it clear that a substance, or a subject (*hupokeimenon*), maintains the same identity and yet admits of contrary qualities. In this connection he defines contraries, or contrariety, in *Metaphysics 10.4* as they pertain to substance.[49]

> Since things which differ may be more or less different, there is a greatest difference; this I call contrariety [ἐναντίωσιν]. We can show inductively that contrariety is the greatest difference. ... Hence the distance between extremes is a maximum, and this constitutes the relation of contrariety. ... From these considerations, then, it is clear that contrariety is perfect difference.[50] (1055a3-6, 9-10, 16-17)

A given individual, then, at one time might be warm or good and at another time cold or bad, but he/she cannot be both warm and cold or good and bad at the same time in the same respect. For Aristotle a substance has a capacity for contraries, but does not itself change identity. In *Categories 5* he discusses the mistake of thinking that a sentence (*logos*) or an opinion (δόξα [*doxa*]), which is expressed by a sentence, admits of contrariety. He writes that

> the same sentence [ὁ αὐτὸς λόγος] appears to be both true and false. For example, if the sentence "Someone sits" is true, but if he rises, the same sentence becomes false. And likewise with opinions [ἐπὶ τῆς δόξης]. For if someone believes truly the sentence "Someone sits", then upon the person rising he will believe falsely if he still holds the same opinion about him [περὶ αὐτοῦ δόξαν]. (4a23-28)

[49]In *Meta. 5.10* Aristotle also writes that: "'contrary' means [1] attributes whose genera are different and which cannot at the same time be present in the same thing; [2] things which differ most in the same genus; [3] attributes which differ most in the same subject; [4] things which diverge most from the same potentiality; and [5] things that differ widely either in themselves, in genus or in form" (1018a25-31).

[50]In *Meta. 10.4* Aristotle continues by writing that: "all this being so, it is evident that one thing can have only one direct contrary; a difference separates two things; therefore contrariety, being complete difference, is a relation between two things. ...hence, a complete difference between different things in the same genus is the greatest possible. We have shown also that such complete difference is contrariety; for a complete difference is one that separates the species of the same genus. ...for a single science covers a single genus and therefore deals with the complete differences in that genus" (1055a19-20, 22-23, 26-29, 31-33). Cf. *On Int. 14*: 23b22-23. See also above §2.1 on Aristotle's notion of universe of discourse.

Aristotle continues here to remark that whenever a substance admits of contrary qualities it is due to a change *within itself.* However, in the case of a sentence (*logos*) and an opinion (*doxa*) each

> *remains in itself unaltered in any and every respect*; but it is *because* of a change in the fact [τοῦ δὲ πράγματος κινουμένου] that the contrary applies to them. For the sentence "Someone sits" remains the same; but according to changing conditions [του δὲ πράγματος κινηθέντος] it becomes at one time true and at another time false. As with sentences, so too with opinions [ἐπὶ τῆς δόξης]. (4a34-4b2)

At 4b2-6 Aristotle asserts that it is the special property (ἴδιον) of substance to admit of changes (μεταβολή) within itself, but that opinions and sentences do not admit of such changes. He next forcefully states that anyone maintaining that a sentence admits of contrary qualifications is speaking nonsense.

> It is not because a sentence and an opinion [ὁ γὰρ λόγος καὶ ἡ δόξα] take on contrary qualities that they are said to take on contraries, but because of what has happened to something else. For it is because the fact is or is not that case [τῷ γὰρ τὸ πρᾶγμα ἔιναι ἢ μὴ ἔιναι] that a sentence [λόγος] is called true or false, and not that it can itself receive contrary qualities. For absolutely nothing [ἁπλῶς γὰρ οὐδὲν ὑπ' οὐδενὸς] can alter either a sentence or an opinion, and so, since they cannot receive contraries nothing changes in them. (4b6-13)

Only substances can admit of such changes (4b13-14). Here again Aristotle affirms the difference between a sentence and the state of affairs denoted by the sentence as he treats truth and falsity. In *Categories 10* he develops this distinction with even more precision.

> Nor is what underlies [τὸ ὑπὸ] an affirmation and a denial an affirmation and a denial. An affirmation [κατάφασις] is an affirmative sentence [λόγος καταφατικὸς], a denial [ἀπόφασις] is a denying sentence [λόγος ἀποφατικὸς]. But what underlies [ὑπὸ] either an affirmation or a denial is not a sentence [οὐδέν ἐστι λόγος]. Still, these things are said to be opposed to each other as affirmation and denial; there is the same manner of opposition. For just as an affirmation is opposed to a denial — for example, "Someone sits" and "Someone does not sit" — so are opposed the things that underlie each sentence [τὸ ὑφ' ἑκάτερον πρᾶγμα] — the sitting and the not sitting. (12b5-16)

As Aristotle had distinguished a word and its object, here he distinguishes a sentence from what it expresses. He clearly grasps the difference between a sentence and its denotation. Later in this same discussion in *Categories 10* he turns to a position he treats in *On Interpretation 7*:

> It is evident that affirmations and denials are opposed in none of those ways we have already treated. For only in relation to [contradictory] sentences is it always necessary for one to be true the other to be false. With contraries it is not always necessary for one to be true the other false, nor with relatives, nor with possession, nor with privation. For example, health and sickness are contraries, but neither the one nor the other is either true or false; likewise with the relatives the double and the half. Nor are privation and possession such as sight and blindness. Generally, *nothing that is said without combination is either true or false* [ὅλως δὲ τῶν κατὰ μηδεμίαν συμπλοκὴν λεγομένων οὐδὲν οὔτε ἀληθὲς οὔτε ψεῦδός ἐστιν]. All the opposites just treated are said without combination [ἄνευ συμπλοκῆς]. (13a37-13b12)

Thus, Aristotle distinguishes a sentence from its denotation, and he establishes that the truth or falsity of a given sentence, which are ontic determinations, depends upon correspondence with the states of affairs or facts, which also are ontic matters, denoted by the sentence as being the case (τὸ ἔιναι) or not being the case (τὸ μὴ ἔιναι).[51]

Truth-values of contradictory and contrary sentences

In *Metaphysics 5.10* Aristotle defines 'opposite' (*antikeimenon*) as having a variety of meanings: contradiction, contrariety, correlation, privation, possession. What he writes there about *being* corresponds exactly with what he writes in *On Interpretation 7* having to do with contradictory and contrary *sentences*. In *Metaphysics 10.4* he states of substance that "the primary form of contrariety [πρώτη δὲ ἐναντιώσις] is that between a positive state and a privation [ἕξις καὶ στερησίς ἐστιν]" (1055a33-34).[52] He continues there to distinguish contradiction from contrariety as follows:

> Opposition [ἀντίχεται] may take the form of contradiction [ἀντίφασις] or of privation [στέρησις] or of contrariety [ἐναντιότης] or of relation [πρός τι]. The first of these is contradiction, and contradiction admits of no intermediate, whereas contraries do; it is clear that contradictories and contraries are not the same. (1055a38–1055b3)

Aristotle defines 'contrary' and 'contradictory' in *Prior Analytics B15*: 63b23-30 in a way that exactly comports with what he writes in *On Interpretation*, but in *Prior Analytics* in relation to the logical constants.

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[51] Below this section we take up whether meaning is an extensional or an intensional determination according to Aristotle.

[52] In *Meta. 10.5* Aristotle writes on contraries that "it is rather an extreme which has something between it and its opposite. In that case it must be an opposite either as a denial or as a privation. It cannot be the denial or privation, for why would it be of the greater rather than of the less? Hence, it must be the privative denial of both" (1056a14-18).

> I say that verbally there are four <pairs of> opposite sentences [προτάσεις], to wit: [1] 'to every' and 'to no', [2] 'to every' and 'not to every', [3] 'to some' and 'to no', and [4] 'to some' and 'not to some'. In truth, however, there are three, for 'to some' and 'not to some' are only opposites verbally. Of these, I call the universal sentences contraries [ἐναντίας μὲν τὰς καθόλου] ('to every' is contrary to 'to none', as, for example, "Every science is good" is contrary to "No science is good") and the other pairs of sentences opposites [τὰς δ' ἄλλας ἀντικειμένας]. (63b23-30)

Only opposites cannot belong to the same thing at the same time. According to the principle of opposition, both opposites obtaining at the same time is impossible (*Meta. 10.5*: 1055b37-1056a3). When he turns in *On Interpretation 7* to treat sentences, he states that "the denial ... must deny the same thing the affirmation affirms of the same thing" (17b39-40).[53] And there also he makes this point about contraries: "but what constitutes sentences as contrary is having two contrary meanings, not having two contrary subjects" (23b6-7). And, "it is impossible for opposite sentences [τὰς ἀντικειμένας φάσεις] to be true about the same thing" (21b17-18). In fact, when one of a pair of contradictories is true the other is necessarily false. This is not the case with contraries where both might be false, but not both true. In *On Interpretation 7* he writes:

> But I call the universal affirmation [τὴν τοῦ καθόλου κατάφασιν] and the universal denial [καὶ τὴν τοῦ καθόλου ἀπόφασιν] contrarily opposite ... Hence, these cannot be true together, but it is possible that their opposites [i.e., sub-contraries] can be true of the same thing. (17b20-26)

Among the relationships Aristotle understands to exist between categorical sentences are two pairs of contradictories and one pair of contraries, all of which he employs in his deduction system. Now, rather than grasping some syntactic relationships among categorical sentences as *taking* a given sentence and then as a result being able or not able to *take* another sentence, Aristotle treats their semantic relationships as follows.

For contradictories:

1. If a sentence fitting the pattern A*a*B is true, then a sentence fitting the pattern A*o*B is necessarily false; if a sentence fitting the pattern A*a*B is false, then a sentence fitting the pattern A*o*B is necessarily true.
   If a sentence fitting the pattern A*o*B is true, then a sentence fitting the pattern A*a*B is necessarily false; if a sentence fitting the pattern A*o*B is false, then a sentence fitting the pattern A*a*B is necessarily true.

[53] This passage continues: "whether of something partial or universal, taken as universal or as not universal" (17b40-18a1).

2. If a sentence fitting the pattern A*e*B is true, then a sentence fitting the pattern A*i*B is necessarily false; if a sentence fitting the pattern A*e*B is false, then a sentence fitting the pattern A*i*B is necessarily true.
   If a sentence fitting the pattern A*i*B is true, then a sentence fitting the pattern A*e*B is necessarily false; if a sentence fitting the pattern A*i*B is false, then a sentence fitting the pattern A*e*B is necessarily true.

For contraries:

1. If a sentence fitting the pattern A*a*B is true, then a sentence fitting the pattern A*e*B is necessarily false; but if a sentence fitting the pattern A*a*B is false, then a sentence fitting the pattern A*e*B is not necessarily true but might be false.

2. If a sentence fitting the pattern A*e*B is true, then a sentence fitting the pattern A*a*B is necessarily false; but if a sentence fitting the pattern A*e*B is false, then a sentence fitting the pattern A*a*B is not necessarily true but might be false.

Aristotle states all this in so many words, although not as rigorously, and it is obviously an important part of his treatment of sentences in *Prior Analytics*. This position is explicitly treated in *On Interpretation* and it is used throughout his analyses in *Prior Analytics*, notably there in relation to conversion (*A2*: 25a17-19) and to his treating *reductio* proofs (e.g., those of Baroco in *A5*: 27a30-27b3 and of Bocardo in *A6*: 28b17-20).

Aristotle on existential import

Aristotle noticed some difficulty concerning the semantics of some sentences, particularly indeterminate sentences and those having to do with future events (*On. Int.* 9).[54] Still, his considerations of these matters in *On Interpretation* (*6*, *7-8*, *9-11*, *12*, *13*) seem well resolved. He reaffirms that every sentence is either true or false, although determining this in one or another case may be difficult or sometimes impossible. Modern logicians, however, seem more puzzled by considerations of existence, and they fear that Aristotle's logic leads to peculiar violations of the

[54]There has been much discussion about 'the sea-battle episode' in *On Int. 9*, where Aristotle seems to question the truth that every sentence is either true or false. In truth Aristotle refutes a sophistic argumentation – one that aims to establish an ontology of predestination as following from the truth of the law of excluded middle as applied to both sentences and states of affairs – by reaffirming that "it does not make any difference whether any people made the contradictory statements or not. For clearly this is how the actual things are even if someone did not affirm it and another deny it. For it is not because of the affirming or denying that it will be or will not be the case, nor is it a question of ten thousand years beforehand rather than any other time" (18b36-19a1). Here again Aristotle distinguishes between *what is* the case and *knowing what* is the case. Thus, he always preserves the principle that a given sentence is either true or false.

square of opposition, particularly in relation to contradictories.[55] However, this matter was not especially troubling to Aristotle. In *Categories 10* he addresses this matter in the following way.

> It is evident that affirmations and denials are opposed in none of those ways we have already treated. For only in relation to [contradictory] sentences is it always necessary for one to be true the other to be false. With contraries it is not always necessary for one to be true the other false, nor with relatives, nor with possession, nor with privation. For example, health and sickness are contraries, but neither the one nor the other is either true or false; likewise with the relatives the double and the half. Nor are privation and possession such as sight and blindness. Generally, *nothing that is said without combination is either true or false* [ὅλως δὲ τῶν κατὰ μηδεμίαν συμπλοκὴν λεγουμένων οὐδὲν οὔτε ἀληθὲς οὔτε ψεῦδός ἐστιν]. All the opposites just treated are said without combination [ἄνευ συμπλοκῆς].
>
> However, it might seem that some such thing follows in the case of contraries said with combination [κατὰ συμπλοκὴν] – [as in] the sentence "Socrates is ill" is contrary to "Socrates is well". Yet, even in these cases it is not always necessary that one sentence be true and the other be false. For, if Socrates *exists*, one is true and the other is false; but if Socrates *does not exist*, both [sentences] are false. For neither will the sentence "Socrates is ill" nor the sentence "Socrates is well" be true if Socrates himself does not exist.
> As for affirmations and denials [ἐπὶ δὲ γὲ πῆς καταφάσεως καὶ τῆς ἀποφάσεως], if [the subject] does not exist, then neither sentence is true. But if [the subject] exists, even then one or the other will not always be true. The sentence "Socrates has sight" is the opposite of the sentence "Socrates is blind" [in the sense in which 'opposite' is applied] to privation and possession. For, if he [viz., Socrates] exists, it is not necessary that the one sentence be true and the other false (since until the time when it is natural for him to have sight both sentences are false). While if Socrates does not exist then both sentences are false: both "He has sight" and "He is blind".
> However, concerning affirmation and negation [i.e., contradictories] the one will always be false the other true *whether or not [the subject] exists*. For take the sentence "Socrates is ill" and the sentence "Socrates is not ill", if he exists it is evident that the one or the other must be true or false. It is the same if he does not exist. If Socrates exists, the sentence [expressing] that he is sick is false, but the sentence [expressing] that he is not sick is true. Thus, it is characteristic [ἴδιον] of

[55] On the topic of existential import and related matters see: A. Church 1964; R. M. Eaton 1959: 157-234; Kneale and Kneale 1962: 45-67; Cohen and Nagel (Corcoran 1993): 41-68; W. T. Parry & E. Hacker 1991: 179-185; W. T. Parry 1966; I. Copi 1986: 177-193; and J. Lukasiewicz 1958: 59-67.

> these only — sentences opposed as affirmation and denial [viz., contradictories] — that the one is always true and the other always false in all cases will hold of those opposites only which are in the same sense opposed as affirmative and negative sentences. (13a37-13b35)

We cite here again a passage from *Categories 12* where he writes about truth and falsity.

> The existence of a man is reciprocal in relation to the true sentence about him as it follows from there being [such] a man [τὸ γὰρ ἔιναι ἄνθρωπον ἀντιστρέφει κατὰ τὴν τοῦ ἔιναι ἀκολούθησιν πρὸς τὸν ἀληθῆ περὶ αὐτοῦ λόγον]. For if a man exists, then the sentence [ὁ λόγος] asserting that a man exists will be true. And conversely, if the sentence asserting that a man exists is true, then the man exists. The true sentence, however, is in no way the cause of the [given] state of affairs [τὸ πρᾶγμα]; and yet the state of affairs [τὸ πρᾶγμα] seems somehow to be the cause of the truth of the sentence. *For a sentence is called true or false as the state of affairs exists or does not exist.* (14b14-22)

Aristotle affirms that the truth or falsity of a given sentence depends, first and foremost, upon whether what it expresses corresponds to a given state of affairs. In this connection, then, it depends upon whether the objects denoted by the subject exist or do not exist, whether what is asserted is the case or is not the case. The cause, or ground, of the truth of a sentence is the state of affairs it denotes. This notion is underwritten by his notion of substance. "Were there no individuals existing of whom it could thus be affirmed, it could not be affirmed of the species; and were there no primary substance, nothing else could so much as exist" (*Cat. 5*: 2a38-2b1). Aristotle has remarked in what way existence is the cause of a sentence being true (*Meta. 4.7*: 1011b23-29).

With what he writes in *Categories 10* and *12* and elsewhere on truth and falsity, we can make sense of the semantics of the various sentences that Aristotle treats concerning their existential import. In general, in the case of existence, a sentence is true or false as, respectively, the state of affairs denoted by the sentence is the case or is not the case; in these cases there are no empty classes. In the case of non-existence, no affirmation is true because it affirms something to be the case that is not the case, and every privative, that is, every sentence with a negative operator, is true because it truly expresses what is not the case.

Below we set out Aristotle's semantics according as he considers sentences to be: (1) singular (καθ' ἕκαστον), universal (καθόλου), or partial (ἐν μέρει; κατὰ μέρος); (2) attributive (κατηγορικός; ἕξις) or privative (στερητικός) — affirmative [positive] or negative: an affirmation (κατάφασις) or a denial (ἀπόφασις); (3) determinate (διοριστός) or indeterminate (ἀδιόριστος); (4) having a subject that exists or having a subject that does not exist. Aristotle understands an opposite (ἀντικείμενον) of a given affirmative sentence to be either (1) a contrary (ἐναντίον), which may or may not involve a negative operator, or (2) a contradictory (ἀντίφασις), *which always involves a negative operator.* In our treatment of

sentences below we always take a sentence to be a simple sentence according to Aristotle's stipulation in his formal language.

The singular sentence

A singular sentence predicates, attributively or privatively, one thing of a particular *this*, a primary substance (πρώτη οὐσία), and this particular is not predicable of anything else (*On Int.* *7*: 17a38-17b1; *Cat.* *5*: 2a11-14; *Pr.* *An.* *A27*: 43a25-29, 39-40). Every singular sentence is determinate. The opposite of a given attributive singular sentence is either its contrary or its contradictory. A contrary of an attributive singular sentence does not have a negative operator and is always another attributive singular sentence (sc. an affirmation). Since, for Aristotle, every denial involves a negative operator, there are no privative contraries in the case of singular sentences. Using a negative operator ('μή' or 'οὐ') in the case of an attributive singular sentence, either adverbially as attached to a verb, or logical constant, or as prefixing an entire sentence, always results in the contradictory of the given singular sentence. *Prior Analytics* treats singular sentences only incidentally. Moreover, *Prior Analytics* does not prefix a given sentence with '*ou*' to produce its negation. Table 7 represents Aristotle's thinking on the semantics of singular sentences.[56]

The universal sentence

A universal sentence predicates, attributively or privatively, one thing of every or of no member of a given kind, a secondary substance (δεύτερα οὐσία). Every universal sentence is determinate. The opposite of a given attributive universal sentence is either its contrary or its contradictory. A contrary of an attributive universal sentence might or might not involve a negative operator, as in both *On Interpretation* and *Prior Analytics.* The negative operator in these cases appears as a pronominal adjective modifying the subject (or as part of the logical constant, which nevertheless modifies the subject). The contradictory of a given universal attributive sentence involves a negative operator in both *On Interpretation* and *Prior Analytics.* The negative operator in these cases might appear as prefixing an entire sentence, as in *On Interpretation* but not in *Prior Analytics*, or adverbially as attached to a verb as part of the logical constant. Table 8 represents Aristotle's thinking on the semantics of universal sentences.

The partial sentence

A partial sentence predicates, attributively or privatively, one thing of some or of not every member of a given kind. Here there are instances of both determinate and indeterminate sentences. General sentences, for example, "Man is white" or "Pleasure is good", lack a universal quantifier and thus can be interpreted as denoting both some and all; their meaning is not determinate but ambiguous.

[56]In Tables 7–9 the '1' pertains to *On Interpretation* and the '2' to *Prior Analytics*; 'T' = true and 'F' = false.

Table 7.

Semantics of Singluar Sentences		
Given affirmation	**Opposites**	
	Contrary	**Contradictory**
1. Socrates ails. 2. Ailing belongs to Socrates.	1. Socrates fares. 2. Faring belongs to Socrates	1. Socrates does not ail. Not Socrates ails. 2. Ailing does not belong to Socrates.
Corresponding truth values		
Existent subject		
When an affirmation is T ...	its contrary is F.	its contradictory is F.
When an affirmation is F ...	its contrary may be T or F.	its contradictory is T.
Non-existent subject		
No affirmation is T. Every affirmation is F.	No affirmation is T. Every affirmation is F.	But every denial is T.
When an affirmation is F...	its contrary is F.	its contradictory is T.

The opposite of a given determinate partial attributive sentence is either its sub-contrary (as modern logicians name it) or its contradictory. A sub-contrary of a given partial attributive sentence might, as in both *On Interpretation* and *Prior Analytics*, or it might not involve a negative operator. The negative operator in the case of a sub-contrary appears adverbially as attached to a verb, or as part of a logical constant. The contradictory of a given partial attributive sentence involves a negative operator in both *On Interpretation* and *Prior Analytics*. The negative operator in the case of a contradictory may appear as a pronominal adjective as part of the logical constant, or as merely modifying the subject, or as prefixing an entire sentence as in *On Interpretation*. Table 9 represents Aristotle's thinking on the semantics of partial sentences.[57]

Denying a given privative sentence does not seem to have been treated by Aristotle in either *On Interpretation* or *Prior Analytics*, although there are suggestions in *On Interpretation*. Because he lacks a notion of double negation, or, at least, it seems, a strong notion of double negation, Aristotle does not treat denying a privative sentence save for reverting to its already given affirmation. This is particularly true in *Prior Analytics*, where he does not negate an entire sentence with '*ou*' as in *On Interpretation*. Rather, there he begins with an affirmation, then

[57]While Aristotle does not have an expression for 'sub-contrary', he does recognise their existence: *On Int. 10* at 20a16–23. In Table 9 we interpolate to complete Aristotle's thinking.

Table 8.

Semantics of Universal Sentences		
Given affirmation	**Opposites**	
	Contrary	**Contradictory**
1. Every man is good. 2. Good belongs to every man.	1. Every man is bad. No man is good. 2. Bad belongs to every man. Good belongs to no man.	1. Not – every man is good. 2. Good does not belong to some man. Good does not belong to every man.
Corresponding truth values		
Existent subject		
When an affirmation is T ...	its contrary is F.	its contradictory is F.
When an affirmation is F ...	its contrary may be T or F.	its contradictory is T.
Non-existent subject		
No affirmation is T. Every affirmation is F.	No affirmation is T. Every affirmation is F. But every denial is T.	But every denial is T.
When an affirmation is F...	its contrary may be T or F.	its contradictory is T.

provides its contradictory, and then looks back at the original affirmation to obtain the contradictory of the negation: he does not, then, negate the negation to obtain its *own* contradictory. As he remarked at different places, each affirmation has its own, one negation, each negation its own, one affirmation — and he takes them as pairs.

Finally, in respect of existential import, according to his definitions of true and false, Aristotle holds that "to deny what is or to affirm what is not is false, whereas to affirm what is and to deny what is not are true; so that any sentence that anything is or is not states either truly or falsely" (*Meta. 4.7*: 1011b26-28). This notion is again poignantly expressed in *Metaphysics 5.7*:

> 'Being' [τὸ ἔιναι] means [σημάινει] the 'true' when something is the case and 'not being' [τὸ μή ἔιναι] <means> the 'not true' but the 'false' when something is not the case; likewise for affirmation and denial [ἐπὶ καταφάσεως καὶ ἀποφάσεως]; for example, that "Socrates is musical" means that this is the case <but we know that this sentence is false>, or that "Socrates is non-white" means that this is the case

Table 9.

Semantics of Partial sentences		
Given affirmation	**Opposites**	
	[Sub-Contraries]	**Contradictory**
1. Some man is good.	1. Some man is bad. Some man is not good.	1. Not – some man is good. No man is good
2. Good belongs to some man.	2. Good does not belong to some man. Bad belongs to some man.	2. Good belongs to no man.
Corresponding truth values		
Existent subject		
When an affirmation is T ...	its sub-contrary may be T or F.	its contradictory is F.
When an affirmation is F ...	its sub-contrary is T.	its contradictory is T.
Non-existent subject		
No affirmation is T. Every affirmation is F.	No affirmation is T. Every affirmation is F. Every denial is T.	But every denial is T.
When an affirmation is F...	Every affirmation is F. Every denial is T.	its contradictory is T.

<but, again, we know that this state of affairs is not the case>. But that "the diagonal <of the square> is not commensurate <with the side>" means that this is not the case <and this, of course, is not the case, and so the sentence is true>. (1017a31-35)

Thus, for Aristotle, every sentence *affirming* something of a non-existent subject is false because it affirms, incorrectly, something to be the case that is not the case. In addition, every sentence *denying* — that is, using a negative operator – something of a non-existent subject is true because it affirms, correctly, something not to be the case that indeed is not the case. The correspondence, or non-correspondence as the case may be, of thought to being is foundational in Aristotle's thinking about truth and falsity. Now, when we turn back to *On Interpretation* 6, just after where he writes that an affirmation states something of something, a denial denies something of something, we can better understand Aristotle's meaning.

> Now it is possible to state of what does belong that it does not belong, of what does not belong that it does belong, of what does belong that it does belong, and of what does not belong that it does not belong [ἐπεὶ δὲ ἔστι καὶ τὸ ὑπάρχον ἀποφαίνεσθαι ὡς μὴ ὑπάρχον καὶ τὸ μὴ ὑπάρχον ὡς ὑπάρχον καὶ τὸ ὑπάρχον ὡς ὑπάρχον καὶ τὸ μὴ ὑπάρχον ὡς μὴ ὑπάρχον]. (17a26-29)

When someone asserts of what does not belong, or is not the case, that it does not belong, or is not the case, he/she speaks truly.

Truth-value: an extensional or intensional determination?

Considering the meanings Aristotle assigns to his logical constants might lead one to believe that the truth-value of a categorical sentence is determined extensionally. Recall, for example, that he defines 'belonging to every' by writing that "for one thing to be in another as a whole ...". This suggests his taking the application of the universal attributive logical constant extensionally with respect to non-empty domains. However, Aristotle continues this statement by writing that "...is the same as for one thing to be predicated of every one of another", or "to hold in every case". His writing here seems shy of an extensional determination.

Now, while interpreting the relationship of terms in a categorical sentence as that between classes, or even of sets, has been fruitful, this interpretation, nevertheless, does not reproduce Aristotle's own understanding. It has become a common practice to define the truth of a categorical sentence in this way as follows (using traditional expressions; 'A' and 'B' are placeholders).

> "Every A is a B" is true *iff* the extension of A is entirely included in the extension of B.
> "No A is B" is true *iff* the extension of A is disjoint with the extension of B.
> "Some A is B" is true *iff* the extension of A intersects with the extension of B.
> "Some A is not B" is true *iff* the extension of A is not entirely included in the extension of B.

Perhaps in some cases Aristotle did envisage terms to relate extensionally as classes. Nevertheless, this interpretation does not take into account Aristotle's notions of *attribution* and *privation*, which pertain to things (*pragmata*), and his notions of *predicating of* and *not predicating of*, which pertain to sentences (*apophanseis*). Underlying his theory of predication in *Categories*, *On Interpretation*, and in *Prior Analytics* is his theory of substance in *Categories* and *Metaphysics*. Briefly, for Aristotle attribution is ontic and independent of a participant, while predication is intentional or linguistic and participant dependent. Predicating is an activity of a human being *reflecting in thought* what exists or does not exist outside of thought. For Aristotle, insofar as he considers matters of logic, *thought*

follows being. And his notion of substance (*ousia*) is precisely that in which properties (*pathêmata*) *inhere or do not inhere.* A substance, whether primary or secondary, *has* or *does not have* one or another property. This explains Aristotle's use of '*huparchein*' in the logical constants.

In order better to grasp term relationships and to see Aristotle as not determining truth extensionally, we might consider his objection to platonic forms. For Plato, an individual person's being two-footed, for example, is just his *participation* in the transcendent form 'two-footedness'. Aristotle, rejecting the reality of such transcendent forms and their putative explanatory value, rather thought of an individual person's being two-footed as his *having* this property or attribute – as this characteristic inhering in, or as immanent in, a subject. A sentence expressing this relationship would be attributive. To express the notion of a horse, on the other hand, as *not having* rationality would be privative. Aristotle treats attribution, in so far as a human expresses attribution in thought by means of sentences, rather fully in *Categories* as well as at places in *Metaphysics*.

Since his theory of substance underlies his theory of predication, we ought rather to say that truth for Aristotle is determined intensionally (or, perhaps, 'possessionally'), remembering his correspondence notion of knowledge. Thus, (using his sentential expressions) we have the following (see n17):

> "A belongs to every B" is true *iff* every individual B has property A.
> "A belongs to no B" is true *iff* no individual B has property A.
> "A belongs to some B" is true *iff* some individual B has property A.
> "A does not belong to some B" is true *iff* some individual B does not have property A.

Aristotle uses '*hupokeimenon*' to refer to the subject of a categorical sentence, and he uses '*pathê*' or '*idion*' to refer to a property attributable or not attributable to a subject. '*Pathê*' contains a notion of 'affect', and surely not a notion of 'class', but of something 'happening to' an individual *this*. Indeed, again, this is just his meaning of the categories (*Cat. 4*). If we add, as Aristotle sometimes did, that property A is essential and not accidental to subject B, then we understand him to mean that having property A, or being A, is just what it means to be B. We again cite a passage from *Metaphysics* tersely to illustrate his thinking.

> Suppose 'man' has the meaning 'two-footed animal'. By 'having a meaning' I mean this: if 'man' is 'two-footed animal', then if anything is a man, its 'being two-footed' will be what its 'being a man' is. (1006a31-34)

Thus, we can take the following expressions of the universal attributive logical constant (and the corresponding sentence patterns) to be generally equivalent in meaning for Aristotle.

> "A belongs to every B." "A holds of every B." "Having property A belongs to every B." "Being A is a property of every B."

2.3 The deduction system of Aristotle's underlying logic

In *Prior Analytics* Aristotle turned his attention away from object language discourses and toward objectifying the formal deduction apparatus used to establish scientific theorems. He was especially concerned to determine "how every syllogism is generated" (25b26-31). He refers here to the elements of syllogistic reasoning, which consist in elementary two-premiss valid arguments — the syllogisms — that, when chained together, make up longer syllogistic (deductive) discourses. Aristotle's project was to identify all the panvalid patterns of such elementary arguments. In this way he explicitly treated deduction rules and their logical relationships in *Prior Analytics.* These rules include both the one-premiss conversion rules and the two-premiss syllogism rules. He accomplished this by exhaustively treating *each and every* possible argument pattern relating to both the one-premiss and two-premiss rules: (1) those patterns of arguments having a premiss-set of just one categorical sentence with only two different terms - the conversions; and (2) those patterns of arguments having a premiss-set of just two categorical sentences with only three different terms — the syllogisms. Aristotle limited his study of multi-premiss arguments to their two-premiss patterns because two categorical sentences, taken together, have 'the fewest number of terms and premisses through which something different than what was initially taken follows necessarily'.[58] The syllogisms, then, are the building blocks of longer deductive discourses.

We have thus far examined the grammar of categorical sentences as Aristotle treated this matter with the artifice of his formal language. We have also treated their semantics. We turn now to the syntactic matter of generating, or transforming, sentences from given categorical sentences according to stipulated rules. This defines the deduction system of Aristotle's underlying logic. Briefly, the deduction system as presented in *Prior Analytics A1-2, 4-6* consists in the following (Table 10:

Table 10.

Aristotle's deduction system	
1.	Four kinds of categorical sentence.
2.	Two pairs of contradictories.
3.	One pair of contraries.
4.	Three one-premiss deduction rules: the conversion rules.
5.	14 two-premiss deduction rules; the syllogism rules.
6.	Two kinds of deduction: direct, or probative deduction, and indirect, or *reductio*, deduction.

Below we extract the rules Aristotle used for forming one-premiss arguments and the three corresponding conversion rules and his rules for forming two-premiss arguments and the corresponding syllogism rules.

[58]The text here is a gloss. See *Pr. An. B2*: 53b18-20 and *Po. An. A3*: 73a7-11 *B11*: *B24-27.* Also see the definition of '*sullogismos*' in *Pr. An. A1*: 24b18-20.

The one-premiss conversion rules

In *Prior Analytics A2* Aristotle treats converting (τὸ ἀντιστρέφειν; conversion: ἀντιστροφή) the predicate and subject terms of categorical sentences to extract certain deduction rules. To do this he treats each of the four kinds of categorical sentence metalogically; that is, he treats at one time, say, all universal privative sentences, by treating the one sentence pattern that they all fit. Aristotle models an object language conversion as an elementary one-premiss argument pattern — where a given sentence is in the role of premiss and its conversion is in the role of conclusion. In this way, without considering object language arguments but only their patterns, he determines which sentences logically convert with respect to terms and which do not by establishing that the argument pattern that they fit is panvalid. Aristotle determines that of the four kinds of categorical sentence three logically convert and thus their panvalid argument patterns can serve as deduction rules. Interestingly, he first states each of the three conversion rules in a *sentence* (25a5-13), as we would expect a rule to be expressed. Moreover, he treats the conversions in *Prior Analytics A2* exactly as he does the syllogism rules in *Prior Analytics A4-6* (§3.2 and n68). Aristotle assumes his reader's familiarity with converting, since he does not define conversion *per se*: he takes it as obvious that conversion involves *changing the places* of the subject and predicate terms in a given categorical sentence while leaving its logical constant unchanged (save for *per accidens* conversion). Aristotle considers these formal transformations to be *deduction rules* because something different than what is initially taken is established to follow necessarily in each instance. Their being universal in this respect underlies their rule nature.

Aristotle does not explicitly state any general syntax rules for forming premiss-sets or for forming premiss-conclusion arguments. Still, in this connection he tends to be more explicit in the case of the syllogisms than in the case of the conversions. In any case, we can easily extract from his practice of treating conversion in *Prior Analytics* the following conversion premiss formation rules.

CPFR1 A conversion premiss-set consists in one and only one of any of the four categorical sentences.

This rule could be generalized for any one-premiss argument.

CPFR2 The two non-logical constants in the categorical sentence in a conversion premiss-set are not identical.

CPFR3 Nothing is a conversion premiss-set except in virtue of these rules.

The following are one-premiss argument formation rules implicit in Aristotle's treatment of conversion.

CAFR1 A one-premiss conversion argument consists in one and only one categorical sentence in the role of premiss and one and only one categorical sentence in the role of conclusion.

CAFR2 The logical constant in the conclusion sentence is the same as the logical constant in the premiss sentence.[59]

CAFR3 The non-logical constant in the predicate position in the conclusion sentence is the same as the non-logical constant in the subject position in the premiss sentence, and the non-logical constant in the subject position in the conclusion sentence is the same as the non-logical constant in the predicate position in the premiss sentence.

CAFR4 Nothing is a one-premiss conversion argument except in virtue of these rules.

We can now turn to the three conversion rules Aristotle established in *Prior Analytics A2*.

CR1-CR3 below are Aristotle's statements of the three deduction rules involving conversion. Our formulations of his statements using a modern notation exactly reproduce Aristotle's meaning, both in his manner of expression in *Prior Analytics* and in his using them there. In the boxes below, the texts of Aristotle's models appear on the left, our modern representation of the panvalid argument patterns on the right.[60] We treat Aristotle's logical methodology for establishing the conversion rules below (§3.1).

CR1 "It is necessary for a universal privative premiss of belonging to convert with respect to its terms" (25a5–6).

"First, then, let premiss AB be universally privative" (25a14):	***e* simple conversion**
"Now, if A belongs to none of the Bs, then neither will B belong to any of the As." (25a15–16)	1. A*e*B ∴ B*e*A

This can be expressed syntactically as: whenever A*e*B is taken then B*e*A can be taken. Thus, a sentence fitting the pattern B*e*A logically follows from a given sentence fitting the pattern A*e*B. Aristotle treats the *e* conversion first.

CR2 "And the attributive (τὴν κατηγορικὴν) premiss necessarily converts [with respect to its terms], though not universally but in part" (25a7–9).

[59]This rule, of course, applies to simple conversion, which likely was the beginning point for ancient study of categorical sentence transformations. Here we witness Aristotle's recording the results of his study. Strictly, an *a* sentence does not convert, although he does not say as much. Although he recognized that an *o* sentence is implied by its e counterpart, he did not provide an *e* conversion *per accidens*. Aristotle noted that an *o* sentence does not convert, and he provided a counterexample to establish this. Interestingly, since an *e* sentence converts simply, both an *o* sentence and its converse are deducible; this, of course, does not establish that the one is a logical consequence of the other.

[60]We here have gathered from his text relating to the *e* conversion something of his thinking about premiss formation and argument formation that our statements of rules below capture. This manner of writing does not characterize the texts for the *a* and *i* conversions, which he treats rather summarily; rather it is appropriately taken as given.

On the universal attributive: "And if A belongs to every B, then B will [necessarily] belong to some A" (25a17–18)	***a* conversion *per accidens*** 1. A*a*B ∴ B*i*A

This can be expressed syntactically as: whenever A*a*B is taken then B*i*A can be taken. Thus, a sentence fitting the pattern B*i*A logically follows from a given sentence fitting the pattern A*a*B.

CR3 "The affirmative (τὴν καταφατικὴν) must convert partially [with respect to its terms]" (25a10–11).

On the partial attributive: "And similarly if the premiss is partial: if A belongs to some of the Bs, then necessarily B belongs to some of the As" (25a20–21)	***i* simple conversion** 1. A*i*B ∴ B*i*A

This can be expressed syntactically as: whenever A*i*B is taken then B*i*A can be taken. Thus, a sentence fitting the pattern B*i*A logically follows from a given sentence fitting the pattern A*i*B. Aristotle remarks that the partial privative sentence does not convert: "...but the privative premiss need not [convert]" (25a12-13).

We might wish that Aristotle had stated each rule more rigorously. Nevertheless, it is evident that he construes these conversions to be rules and that he treats them syntactically. And, although he is quick to provide an example to help illustrate his thinking in each case,[61] his doing so no more subverts his syntactic analysis and configuration than does providing a counterargument subvert this for a modern logician. Moreover, as we show below in relation to the syllogisms (§3.2), it is evident from his discourse in *A2* that Aristotle thinks of these transformations as metalogical patterns of arguments whose premiss-sets are single sentences.

The two-premiss syllogism rules

Aristotle noted that two premisses with three different terms is the fewest number by which someone could deduce a sentence that is neither (1) a repetition nor (2) a conversion. Accordingly, to fulfill his principal concern in *Prior Analytics*, he demonstrated which of these *elementary* two-premiss argument patterns have only valid argument instances and which *elementary* patterns have only invalid argument instances. The results of his study, particularly in *Prior Analytics A4-7*, serve as elements, or principles — in particular, as deduction rules — in his deduction system. Aristotle thought of syllogistic reasoning as progressively linking

[61] We cite here Aristotle's examples in the case of each rule. CR1: "For instance, if no pleasure is a good, neither will any good be a pleasure" (25a6-7). CR2: "For instance, if every pleasure is a good, then some good will be a pleasure" (25a9-10). CR3: "for if some pleasure is a good then some good will be a pleasure" (25a11-12).

the conclusions of two premiss arguments — to wit, the syllogisms — until a final conclusion (theorem) is reached. Here again we extract his syntactic two-premiss deduction rules before taking up his logical methodology (§3.2).

In order systematically to extract all possible panvalid patterns of two-premiss categorical arguments, Aristotle considered in *Prior Analytics A4-6* every possible arrangement of any two of the four kinds of categorical sentence with three different terms. Working with a notion of 'form' of argument that is genuinely syntactic, he systematically treated patterns of two *protaseis* (sentences), or premiss-pair patterns, and their corresponding argument patterns, and he treated neither premisses nor arguments *per se*. To do this he treated each premiss-pair pattern metalogically; that is, he treated at one time, say, all such patterns of two universal attributive sentences in a given figure, by treating the one premiss-pair pattern that they all fit in that figure. Aristotle modeled an object language syllogism as an elementary two-premiss argument pattern — where two given sentences are in the role of premises and one sentence is in the role of conclusion. In this way, without considering object language arguments but only their patterns, he determined which argument patterns are panvalid and which are not panvalid. Thus, any argument fitting a panvalid pattern is valid; its validity might be recognized by virtue of its fitting such a pattern. Aristotle determined that 14 such premise-pair patterns are concludent and thus that at least one corresponding argument pattern is panvalid and can thereby serve as a deduction rule. Aristotle first states each of the syllogism rules in a *sentence*, as we would expect a rule to be expressed. He then treats each of the argument patterns schematically, that is, he models each as a two-premiss argument pattern to determine its panvalidity. Arguments are introduced (1) to establish that certain premise-pair patterns are inconcludent and (2) to serve as instances of panvalid argument patterns or of paninvalid argument patterns.

We take Aristotle at his word when he states, on numerous occasions in both *Prior Analytics* and *Posterior Analytics*, that "every demonstration [πᾶσα ἀπόδειξις] and every deduction [πᾶς συλλογισμὸς] will be through only three terms ... it will also be from two premisses [or intervals] and no more, for three terms are two premises" (*Pr. An. A25*: 42a30-33; cf. *Pr. An. A25*: 41b36-37 and *Po. An. A19*: 81b10 & *A25*: 86b7-8 among many other passages). From this statement and his practice throughout, we can extract his rules for syllogistic[62] premise-pair formation as follows.

SPFR1 A syllogism premiss-set consists in two and only two of the four kinds of categorical sentence.

Aristotle provides additional text in *Prior Analytics A23* that confirms our taking him to have such a rigid rule. We shall refer to this text again when we treat

[62] We use 'syllogistic' in this section, and sometimes elsewhere, to refer to the syntax relations among categorical sentences and not strictly to refer to consistent sets of categorical sentences or valid categorical arguments. Cf. his definition of '*sullogismos*' in *Pr. An. A1*: 24b18-20; see below §5.1 n93.

Aristotle's notion of deducibility (§5.1).

> Now, if someone should have to deduce A of B, either as belonging or as not belonging, then it is necessary for him to take something about something. If, then, A should be taken about B, then the initial thing will have been taken. But if A should be taken about C, and C about nothing nor anything else about it, nor some other thing about A, then there will be no syllogism, for nothing results of necessity through a single thing having been taken about one other. Consequently, another premiss must be taken in addition. If, then, A is taken about something else, or something else about it or about C, then nothing prevents there being a syllogism, but it will not be in relation to B through the premisses taken. Nor when C is taken to belong to something else, that to another thing, and this to something else, but it is not connected to B: there will not be a syllogism in relation to B in this way either. (40b30-41a2)

Aristotle makes it abundantly clear that taking three terms in two sentences is possible in only three ways. He established this implicitly in *Prior Analytics A4-6*, but he makes this explicitly part of his argumentation in *A23* where he effectively treats deducibility and the completeness of his logic. He writes there that "every demonstration and every deduction must necessarily come about through the three figures" (41b1-3; cf. 40b19-22 & *A28*: 44b6-8, 19-20). His fuller statement at *A23* follows:

> For, in general, we said that there cannot ever be any syllogism of one thing about another without some middle term having been taken which is related in some way to each according to the kinds of predications. For a syllogism, without qualification, is from premises; a syllogism in relation to this term is from premisses in relation to this term; and a syllogism of this term in relation to that is through premisses of this term in relation to that. And it is impossible to take a premiss in relation to B without either predicating or rejecting anything of it, or again to get a syllogism of A in relation to B without taking any common term, but <only> predicating or rejecting certain things separately of each of them. As a result, something must be taken as a middle term for both which will connect the predications, since the syllogism will be of the term in relation to that. If, then, it is necessary to take some common term in relation to both, and if this is possible in three ways (for it is possible to do so by predicating A of C and C of B, or by predicating C of both A and B, or by predicating both A and B of C), and these ways are the figures stated, then it is evident that every syllogism must come about through some one of these figures. (41a2-18)

Thus, we can extract three additional syllogistic premiss formation rules.

SPFR2 The two categorical sentences in a syllogistic premiss-set consist in three different non-logical constants (terms).

SPFR3 None of the three non-logical constants in a syllogistic premiss-set appears twice in the same categorical sentence. Thus, the one categorical sentence in a syllogistic premiss-set has a non-logical constant in common with the other sentence in the premiss-set.

SPFR4 Any of the four logical constants may appear in each of the two categorical sentences in a syllogistic premiss-set.

Aristotle nowhere states SPFR4 in rule fashion, but it is evident from his treatment of the premise-pair patterns throughout *Prior Analytics A4-6* that he consciously works with such a rule of premiss formation.

The salient feature of his exposition in this connection is the crucial role he attributes to the middle term. He makes this quite emphatic in *Prior Analytics A28*:

> It is also clear that one must take things which are the same, not things which are different or contrary, as the terms selected for the investigation. This is because, in the first place, the examination is for the sake of the middle term, and one must take as middle something the same, not something different. (44b38-45a1; cf. *A29*: 45b36-46a2)

Again, there is a rule of the middle term for taking pairs of sentences as premisses to form syllogistic arguments: "there cannot ever be any syllogism of one thing about another without some middle term having been taken which is related in some way to each according to the kinds of predications" (*A23*: 41a2-4). This, of course, is a theme highly resonant in *Posterior Analytics A & B*, and this might best be captured as follows from *Posterior Analytics B4*: "A deduction proves something of something through the middle term [ὁ μὲν γὰρ συλλογισμὸς τὶ κατὰ τινὸς δέικνυσι διὰ τοῦ μέσου]" (91a14-15). Aristotle recognized three possible positions for the term shared by each of the categorical sentences in a syllogistic premise-set. He called this the middle term, and he named three figures, "first", "second", and "third". Accordingly, he had three rules for constructing a syllogistic premise-set for each of the three figures (σχήματα; singular σχῆμα). While he names the first figure at the end of *Prior Analytics A4* (26b33) Aristotle defines it at the beginning. And thus we have his rule for forming a first figure syllogistic premise-set.

SPFR5 I call the middle [term] which both is itself in another and has another in it — this is also the middle in position — and call both that which is itself in another and that which has another in it extremes [or extreme terms]. (25b35–37)

Aristotle's rule for forming a second figure syllogistic premiss-set is the following:

SPFR6 When the same thing belongs to all of one term and to none of the other, or to all of each or none of each, I call such a figure the second. In it, I call that term the middle which is predicated of both and call those of which this is predicated extremes; the major extreme is the one lying next to the middle, while the minor extreme is the one farther from the middle. The middle is placed outside the extremes and is first in position. (26b34–39)

Aristotle's rule for forming a third figure syllogistic premiss-set is the following:

SPFR7 If one term belongs to all and another to none of the same thing, or if they both belong to all or none of it, I call such a figure the third. By the middle in it I mean that term of which they are both predicated, and by extremes the things predicated; by major extreme I mean the one farther from the middle and by minor the one closer. The middle is placed outside the extremes and is last in position. (28a10–15)

Thus, there are only three syntactic arrangements of middle (or common) terms, called the three figures: "[1] by predicating A of C and C of B, or [2] by predicating C of both A and B, or [3] by predicating both A and B of C" (41a15-16). In the context of his logical investigations in *Prior Analytics A4-6*, we can state a rule implicit in his treatment of the syllogisms.

SPFR8 Nothing is a syllogistic premiss-set except in virtue of these rules.

Thus far we have represented Aristotle's rules for forming premise-sets of syllogistic arguments. He also has syntax rules for forming syllogistic premise-conclusion (P-c) arguments, that is, in particular, rules concerning the relationships of terms (1) to each other in the conclusion of a syllogistic argument in relation to (2) those in the premise-set in connection with each figure. His rules involve taking sentences to form P-c arguments consisting of a set of two sentences — call them *protaseis* (premises) — and a single sentence — call this the συμπέρασμα (*sumperasma*), or conclusion. These rules anticipate the rules of syllogistic inference.

SAFR1 A two-premiss syllogistic argument consists in two and only two categorical sentences in the role of premisses and one and only one categorical sentence in the role of conclusion.

SAFR2 Any of the four logical constants can appear in each of the three categorical sentences composing a syllogistic argument.

SAFR3 Each syllogistic argument consists in three and only three different non-logical constants (terms); no non-logical constant appears twice in a categorical sentence in a syllogistic argument.

SAFR4 Of the two different non-logical constants in the conclusion sentence, the one appears once in one of the two premiss sentences, the other once in the other premiss sentence.

SAFR5 In the first figure the predicate term of the conclusion is the term predicated of the middle term in the premiss-set; the subject term of the conclusion is the term in the premiss-set of which the middle term is predicated.

SAFR6 In the second figure the predicate term of the conclusion is the subject term of the first or major premiss; the subject term in the conclusion is the subject term of the second or minor premiss.

SAFR7 In the third figure the predicate term of the conclusion is the predicate term of the first or major premiss; the subject term in the conclusion is the predicate term of the second or minor premiss.

SAFR8 Nothing is a two-premiss syllogistic argument except in virtue of these rules.

In each figure, the predicate term of the conclusion both is the major term and its presence identifies the major premiss of a syllogistic argument; the subject term of the conclusion both is the minor term and its presence identifies the minor premiss of a syllogistic argument. These rules compass syllogistic P-c argument formation for Aristotle in *Prior Analytics*. Thus, we can set out the patterns of the three figures as follows, using our abbreviations of Aristotle's logical constants for convenience (Table 11.[63]

Aristotle understood that the order of a given set of two categorical sentences taken as premisses does not affect their implying a given categorical sentence taken as a conclusion. Premiss order is not important for logical consequence. On occasion he reversed the order in which he presented the two sentence patterns in the

[63] *Explanation of Table 11.* In *Pr. An. A4-6* Aristotle established the syntax of the syllogisms or, more generally, of elementary syllogistic arguments. Throughout *A4-6* he worked with schematic letters for three terms in various premiss-pair patterns according to three figures. He used, respectively, ABΓ, MNΞ, ΠΡΣ (ABC, MNX, PRS). In each case, whether he stated first the major or the minor premiss pattern, he always understood the predicate term (P) of the conclusion pattern (PS or PxS) to be the first, or major, term — and to identify the major premiss – in the premiss-pair pattern and the subject term (S) of the conclusion pattern to be the last, or minor, term and to identify the minor premiss. We use 'x', 'y', and 'z'as placeholders for any of the four logical constants. The term repeated in the premisses is the middle term (M). This syntax is strict. Aristotle always considered the conclusion of an argument to fit the sentence pattern PS (or PxS) and not its converse. (He did not, however, specifically show how a given premiss-pair pattern does not result in a syllogism when the conclusion is converted, say, in the case of Barbara, which we know to have no valid instances. But see *Pr. An. B22*.) In this connection, when he treated Camestres and Disamis, he specifically converted the derived sentence pattern to preserve this syntax. We have set out this syntax with our modern notation, which exactly reproduces Aristotle's meaning.

Table 11.

Synopsis of Aristotle's syllogistic argument patterns								
First figure: **PMS:** ($AB\Gamma$) PxM, MyS|PzS			**Second figure:** **MPS:** (MNΞ) MxP, MyS|PzS			**Third figure:** **PSM:** (ΠΡΣ) **PxM, SyM|PzS**		
AB	1.	PxM	MN	1.	MxP	ΠΣ	1.	PxM
$B\Gamma$	2.	MyS	MΞ	2.	MyS	ΡΣ	2.	SyM
$A\Gamma$	?	PzS	NΞ	?	PzS	ΠΡ	?	PzS

premisses when he considered them in *Prior Analytics A4-6*.[64] This shows that he understood this to be so. Nevertheless, in order to treat his premise-pair patterns *systematically*, he treated them as ordered pairs in the framework of his strict syllogistic syntax. Here the premiss order matters significantly (1) for systematically treating all possible combinations (sc. patterns) of two categorical sentences in the role of premisses and (2) for relating terms in the conclusion to those in the premises. In general, he first treated the universal sentences as premises, then combinations of universal and partial sentences, and finally combinations of partial sentences.

Now, of the 192 possible combinations of syllogistic premises, Aristotle identified 14 that result in syllogisms when terms are substituted for the placeholders.[65] As he did with the one-premiss conversion rules, he did with the two-premiss syllogistic rules. He first provided a sentence stating the rule before he represented it schematically, and then he provided a metalogical proof of its panvalidity. Below we provide Aristotle's texts in *Prior Analytics* that model his 14 two-premiss syllogism rules, our modern notation to the right.[66]

SR1 Whenever, then, three terms are so related to each other that the last is in the middle as a whole and the middle is either in or not in the first as a whole, it is necessary for there to be a complete syllogism of the extremes. (25b32-35) 1aa

Barbara (25b37–39)	1. AaB
	2. BaC
For if A is predicated of every B and B of every C, it is necessary for A to be predicated of every C.	∴ AaC

[64]This is the case, for example, when Aristotle treats third figure patterns. See *Pr. An A6*: 28a26-36, which includes Felapton, and *A6*: 28b5-31, which treats Disamis, Datisis, and Bocardo. For Aristotle, as for modern logicians, premiss order is independent of implication.

[65]On there not being a fourth figure see below §3.5.

[66]The '1aa', for example, after Aristotle's statement of a rule, refers to the figure and the patterns of each premiss sentence. In addition, rather than compose two statements for each of two of Aristotle's rule but which are expressed in one sentence, we have cited the passage twice, as, for example, with the statement covering both Barbara and Celarent (SR1 & SR2).

SR2 Whenever, then, three terms are so related to each other that the last is in the middle as a whole and the middle is either in or not in the first as a whole, it is necessary for there to be a complete syllogism of the extremes. (25b32–35) 1*ea*

Celarent (25b40–26a2) Similarly, if A is predicated of no B and B of every C, it is necessary that A will belong to no C.	1. A*e*B 2. B*a*C ∴ A*e*C

SR3 If one of the terms is universal and the other is partial in relation to the remaining term, then when the universal is put in relation to the major extreme (whether this is positive or privative) and the partial is put in relation to the minor extreme (which is positive), then there will necessarily be a complete syllogism. (26a17-20) 1*ai*

Darii (26a23–25) For let A belong to every B and B to some C. ...it is necessary for A to belong to some C.	1. A*a*B 2. B*i*C ∴ A*i*C

SR4 If one of the terms is universal and the other is partial in relation to the remaining term, then when the universal is put in relation to the major extreme (whether this is positive or privative) and the partial is put in relation to the minor extreme (which is positive), then there will necessarily be a complete syllogism. (26a17-20) 1*ei*

Ferio (26a25–27) And if A belongs to no B and B to some C, then it is necessary for A not to belong to some C.	1. A*e*B 2. B*i*C ∴ A*o*C

SR5 When the terms are universal, there will be a syllogism when the middle belongs to all of one term and none of the other, no matter which one the privative is in relation to. (27a3-5) 2*ea*

Cesare (27a5–9) For let M be predicated of no N but to every X. ...so that N belongs to no X.	1. M*e*N 2. M*a*X ∴ N*e*X

SR6 When the terms are universal, there will be a syllogism when the middle belongs to all of one term and none of the other, no matter which one the privative is in relation to. (27a3-5) 2*ae*

Camestres (27a9–14) Next, if M belongs to every N but to no X, then neither will N belong to any X.	1. M*a*N 2. M*e*X ∴ N*e*X

SR7 If the middle is universal only in relation to one term, then when it is universal in relation to the major extreme (whether positively or privatively) but partially with respect to the minor and oppositely to the universal ... then it is necessary for the privative partial syllogism to come about. (27a26-32) 2*ei*

Festino (27a32–36) For if M belongs to no N and to some X, it is necessary for N not to belong to some X.	1. M*e*N 2. M*i*X ∴ N*o*X

SR8 If the middle is universal only in relation to one term, then when it is universal in relation to the major extreme (whether positively or privatively) but partially with respect to the minor and oppositely to the universal ... then it is necessary for the privative partial syllogism to come about. (27a26–32) 2*ao*

Baroco (27a36–27b1) Next, if M belongs to every N but does not belong to some X, it is necessary for N not to belong to some X.	1. M*a*N 2. M*o*X ∴ N*o*X

SR9 For when both terms are positive [and universal], then there will be a syllogism that one extreme belongs to some of the other extreme. (28a37–39) 3*aa*

Darapti (27a17–26) When both P and R belong to every S, it results of necessity that P will belong to some R.	1. P*a*S 2. R*a*S ∴ P*i*R

SR10 And when one term is privative and the other affirmative [both universal], then if the major term should be privative and the other term affirmative, there will be a syllogism that one extreme does not belong to some of the other. (28b1–3) 3*ea*

Felapton (28a26–30) And if R belongs to every S but P to none, then there will be a deduction that P of necessity does not belong to some R.	1. P*e*S 2. R*a*S ∴ P*o*R

SR11 If one term is universal in relation to the middle and the other term is partial then when both terms are positive it is necessary for a syllogism to come about, no matter which of the terms is universal. (28b5–7) 3*ia*

Disamis (28b7–11) For if R belongs to every S and P to some, then it is necessary for P to belong to some R.	1. P*i*S 2. R*a*S ∴ P*i*R

SR12 If one term is universal in relation to the middle and the other term is partial then when both terms are positive it is necessary for a syllogism to come about, no matter which of the terms is universal. (28b5–7) 3*ai*

Datisis (28b11–15) Next, if R belongs to some S and P to every S, then it is necessary for P to belong to some R.	1. P*a*S 2. R*i*S ∴ P*i*R

SR13 But if one term is positive, the other privative, and the positive term is universal, then when the minor term is positive, there will be a syllogism. (28b15–17) 3*oa*

Bocardo (28b17–20) For if R belongs to every S and P does not belong to some, then it is necessary for P not to belong to some R.	1. P*o*S 2. R*a*S ∴ P*o*R

SR14 If the privative term is universal, then when the major term is privative and the minor positive there will be a syllogism. (28b31–33) 3*ei*

Ferison (28b33–35) For if P belongs to no S and R belongs to some S, then P will not belong to some R.	1. P*e*S 2. R*i*S ∴ P*o*R

Having now set out Aristotle's deduction system, we turn to examine the logical, or metalogical, methodology by which he established these deduction rules.

3 ARISTOTLE'S LOGICAL METHODOLOGY FOR ESTABLISHING DEDUCTION RULES

In this section we examine the methods by which Aristotle established the rules of his syllogistic deduction system. We also consider his methods for establishing that

certain elementary argument patterns cannot serve as rules. To fulfill a principal purpose in *Prior Analytics*, Aristotle demonstrated (1) which premiss patterns of one sentence are concludent and which inconcludent when the terms of the given sentence are converted in the conclusion. He also demonstrated (2) which premiss patterns of two sentences are concludent and which inconcludent — that is, which patterns when 'interpreted' result in a syllogism and which patterns do not result in a syllogism. The results of his study, particularly in *Prior Analytics A2*, and *4-7*, serve as rules in his deduction system.

To separate the elementary two-premiss argument patterns with only valid instances from the elementary two-premiss argument patterns with only invalid instances, Aristotle used two decision procedures in his metalogic: (1) the method of *completion*, which he so named, and (2) the method of *contrasted instances*,[67] which itself has three modes. There are no elementary argument patterns with both valid and invalid instances: none is neutrovalid. In the case of the two-premiss patterns, Aristotle's method is deductive, but not axiomatic, and enumerative. In the case of the one-premiss patterns, his method is deductive but not a process of completion *per se*.

Aristotle treated each conversion at *Prior Analytics A2* in exactly the same fashion as he treated the syllogisms at *A4-6*, namely, *not* as deductions *per se* but as elementary argument patterns having only valid instances. He performs metalogical deductions of the conversions at *A2* just as he does for the syllogisms at *A5-6*.[68] The points of similarity between his treatments of the one- and two-premiss argument patterns show that when he examined the formal properties of his logic, he treated the elementary panvalid argument patterns for syllogisms and conversions as deduction rules. In this respect they are equally species of a given genus. Moreover, Aristotle recognized there to exist valid arguments that are instances of these patterns. Again, in respect of a given genus, these instances are the same, namely, valid arguments. However, he never considered an instance of a conversion to be an instance of a syllogism, and vice versa, just as a modern logician would not consider an instance of a simplification to be an instance of an addition in propositional logic. Below we first treat Aristotle's logical methodology for establishing the conversion rules (§3.1) and then his methodology for establishing the syllogism rules (§3.2).

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[67] We follow W. D. Ross (1949: 302) in using the expression 'contrasted instances' to name Aristotle's method of establishing inconcludence.

[68] Aristotle uses the same expressions in treating conversions and syllogisms in *Pr. An.*: 'πρότασις' (premiss); 'ἀνάγκη' (it is *necessary* that .. [25a6, 10, 21]) and οὐκ ἀνάγκη (it is *not necessary* that ... [(25a23]); 'ἀναγκαῖον' (necessarily [25a8]) and οὐκ ἀναγκαῖον (not necessarily [25a12]). Some corresponding examples from *Pr. An. A4* to indicate this are: ἀνάγκη (25b38); ἀναγκαῖον (26a4); for the negation Aristotle writes "οὐκ ἔσται συλλογισμός" (26a7–8). He uses schematic letters ('A' and 'B') in exactly the same fashion. He equally treats the conversions as rules of deduction for syllogistic reasoning as he does the syllogisms at *A5-6*. Of the four possible conversions Aristotle treats each exactly, in principle, as he does when establishing and reducing the two-premiss syllogism rules.

3.1 Establishing the conversion rules

Aristotle treats converting the predicate and subject terms of each of the four kinds of categorical sentence in *Prior Analytics A2*. After expressing each conversion rule in a sentence (25a5-13), he then treats each schematically. He models each as a one-premiss argument pattern and provides a deduction in the metalanguage of *Prior Analytics* (25a14-26) to establish each conversion pattern to be a panvalid argument pattern. This establishes its suitability as a rule. Then, in *A4-6* he employs these conversions as rules in his metalogical deductions to establish the panvalid patterns relating to the syllogisms.

Since there are only three conversions we treat each of them below. Aristotle first treats establishing the panvalidity of the *e* conversion:[69]

> Now, if [ἐι] A belongs to none of the Bs, then neither will B belong to any of the As. For if it [B] does belong to some [A], for instance to C, it will not be true that A belongs to none of the Bs, since C is one of the Bs. [But it was taken to belong to none.] (25a15-17)

According to this text, then, we can represent Aristotle's rather 'intuitionist' deduction in the following familiar manner (Table 12).[70]

Aristotle next provides a proof of the panvalidity of the *a* conversion *per accidens*:

> And if [ἐι] A belongs to every B, then B will belong to some A. For if it [B] belongs to none [A], neither will A belong to any B; but it [A] was assumed to belong to every one [B]. (25a17-19)

Again, we can represent what he writes here in the following manner (Table 13). On the proof of the panvalidity of the *i* conversion rule (Table 14):

> If [ἐι] A belongs to some of the Bs, then necessarily B belongs to some of the As. (For if it [B] belongs to none [A], then neither will A belong to any of the Bs [but it ⟨A⟩ was assumed to belong to some ⟨B⟩].) (25a20-22)

Finally, in a fashion less analogous to his methods of establishing inconcludence of premise-pair patterns, Aristotle provides a counterexample to show that an *o* sentence does not convert (25a22-26; cf. 25a12-13). The result does not follow necessarily (οὐκ ἀνάγκη at 25a23 and οὐκ ἀναγκαῖον at 25a12). His proof of the paninvalidity of the *o* conversion follows:

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[69] Some logicians take this to be an instance of *ekthesis* early in *Pr. An.*; a later, more fully presented instance appears in *Pr. An. A6*: 28a23-26.

[70] We have aimed to represent something of Aristotle's own thinking, although we have simplified it here. Strictly, 'C=A' might not represent Aristotle's thinking. Surely he would recognize that some Cs might not be As, but the Cs Aristotle picks out are those of A to which B belongs, as a Venn diagram might easily show. Cf. Smith, 1989, xxiii xxv on 'setting out' and Smith, 1982 on *ekthesis*.

Table 12.

Establishing the *e* conversion rule		
Aristotle's text	**Modern notation**	
1. A belongs to none of the Bs	1. A*e*B	
? neither will B belong to any of the As	? B*e*A	
2. it [B] does belong to some [A]	2. B*i*A	assume
3. for instance [B belongs] to [every] C [to which A belongs]	3. C=A	basis of assumption
4. C is one of the Bs	4. B*i*C	basis of assumption
5. it will not be true that A belongs to none of the Bs [or, that is, A will belong to some B]	5. -(AeB) [A*i*B]	3,4 logic
6. [But it [A] was taken to belong to none [B].]	6. A*e*B & -(A*e*B) [A*i*B]	1,5 conj & contra
7. neither will B belong to any of the As	7. B*e*A	2–6 *reductio*

Table 13.

Establishing the *a* conversion *per accidens* rule		
Aristotle's text	**Modern notation**	
1. A belongs to every B	1. A*a*B	
? B will belong to some A	? B*i*A	
2. it [B] belongs to none [A]	2. B*e*A	assume
3. neither will A belong to any B	3. A*e*B	2 *e*-conversion
4. it was assumed [A] to belong to every one [B]	4. A*e*B & A*a*B	3,1 conj & contra
5. [B will belong to some A]	5. B*i*A	2–4 *reductio*

Table 14.

Establishing the *i* conversion rule		
Aristotle's text	**Modern notation**	
1. A belongs to some of the Bs	1. A*i*B	
? B belongs to some of the As	? B*i*A	
2. it [B] belongs to none [A]	2. B*e*A	assume
3. neither will A belong to any of the Bs	3. A*e*B	2 e-conversion
4. [it <A> was assumed] to belong to some <B>]	4. A*e*B & A*i*B	3 conj & contra
5. [B belongs to some of the As]	5. B*i*A	2–4 *reductio*

> But if [ἐι] A does not belong to some B, it is not necessary for B also not to belong to some A (for example if B is animal and A man: for man does not belong to every animal, but animal belongs to every man). (25a22-26)

"Man does not belong to some animal", which is obviously true, does not convert to "Animal does not belong to some man", which is obviously false. We can set this out in the following way (Table 15).

Table 15.

Establishing that converting an *o* sentence does not produce a necessary result		
Aristotle's text	**Aristotle's counterexample**	
1. A does not belong to some B	1. Man does not belong to some animal	T
? B also does not belong to some A	? [Animal does not belong to every man.]	F
	[But animal belongs to every man.]	T.

This proof uses an instance of the method of fact, which itself uses the principle that no false sentence is implied by a true sentence. It is evident that Aristotle's notion of logical consequence is the same in each case and comports with that of modern logicians (§5.3). His treatment of the conversion of categorical sentences

was aimed to establish rules. The three one-premiss conversion rules can be represented in the following schematic way with a modern notation that preserves exactly what Aristotle accomplished in *Prior Analytics A2* (Table 16):

Table 16.

Aristotle's one-premiss conversion rules		
a **conversion** *per accidens*	*e* **simple conversion**	*i* **simple conversion**
1. A*a*B	1. A*e*B	1. A*i*B
∴ B*i*A	∴ B*e*A	∴ B*i*A

3.2 Establishing the syllogism rules: deciding concludence of premiss-pair patterns

Each syllogism fitting one of the four first figure panvalid patterns is τέλειος (*teleios*), perfect or complete. This means that the necessity of its result following from the things initially taken, its being valid, is obvious, or evident through itself, to a participant (§5.1): nothing additional need be taken for this evidency (*A1*: 24b18-24, *A4*: 26b28-33). While Aristotle did not prove[71] the panvalidity of the *teleioi sullogismoi* patterns, he did think of the syllogisms fitting these patterns as proving certain kinds of conclusion, that is, in particular, as each proving a sentence in one of the four sentence patterns, or *problêmata*: A*a*B, A*e*B, A*i*B, and A*o*B.[72] And, moreover, while evidency of following necessarily is epistemic, Aristotle understood a *teleios sullogismos* to be grounded in a corresponding ontic reality that causes or makes this evidency possible, analogous to the truth-value of a sentence, and thus also its panvalid pattern.

Each syllogism fitting a second and third figure pattern is ἀτελής (*atelês*), imperfect or incomplete. The necessity of their conclusions is not obviously evident to someone but δυνατός, or potentially evident. Consequently, an epistemic process is required to make the validity of an *atelês sullogismos* evident, namely, a deduction. This distinction exactly characterizes the significant difference between Aristotle's treatment of second and third figure patterns from those of the first figure. Now, just as an *atelês sullogismos* needs a deduction to establish its validity, so does its

[71] W. D. Ross (1949: 22-28, 29), and other traditionalists such as J. N. Keynes (1906: 301) and R. M. Eaton (1959: 86, 120), maintain that the first figure syllogisms are not primitive but derived from the *dictum de omni et nullo*, which is variously conceived to be the "principle" or the "axiom" of Aristotle's system. On the other hand, J. Lukasiewicz (1958: 45, 46-47), J. Corcoran (1974: 109), and J. Lear (1980: 2-3), for example, consider them to be given without proof as self-evidently valid. Corcoran agrees with Lukasiewicz (1958: 46-47) to consider the passage cited in *Pr. An.* (24b26-30; cf. *Cat.* 1b10-12) that states the *dictum de omni et nullo* to be definitional; cf. J. W. Miller (1938: 26-27).

[72] He indicates this by writing, for example, of the first figure syllogisms that "all the problems *are proved* by means of this figure [πάντα τὰ προβλήματα δείκνυται διὰ τούτου τοῦ σχήματος]" (26b31), or by noting that the same results had been proved earlier (e.g., at 26b20-21).

pattern need a metalogical deduction to establish its panvalidity.[73] In each case of a second figure and a third figure panvalid pattern, Aristotle showed *by means of a metalogical deduction* that a given premise-pair pattern is concludent, that is, moreover, that such a premiss pattern results in an argument pattern such that an arbitrary argument fitting this pattern is a syllogism. Aristotle used the method of completion, τελειοῦσθαι or τελείωσις (*teleiousthai* or *teleiôsis*), which here is a deduction process carried out in the metalanguage of *Prior Analytics.*[74] Along with the three conversion rules, this process *explicitly* employs the four panvalid patterns of the first figure as deduction rules to establish which second and third figure argument patterns are panvalid and could themselves, then, serve as rules. Aristotle's interest here was to establish which elementary argument patterns have only valid argument instances. Every argument with semantically precise terms fitting one of these patterns is valid. In this way he identified 14 panvalid patterns in three figures.[75] Aristotle treated *each* pattern individually and not axiomatically; his metasystematic treatment of the panvalid patterns involves induction and is not strictly deductive.[76] What follows relates to the process of establishing, then, not the validity of object language arguments (sc. syllogisms), but the panvalidity of their patterns corresponding to concludent premise-pair patterns in the second and third figures.

The text concerning Camestres in the second figure illustrates that Aristotle used a deduction in his metalanguage to establish that a given argument pattern is in fact a panvalid pattern and *not* to establish that Camestres is itself a deduction or itself derived from another 'syllogism-axiom'.

> If [εἰ] M belongs to every N but to no X, then neither will N belong to any X. For if M belongs to no X, neither does X belong to any M; but M belonged to every N; therefore [ἄρα], X will belong to no N (for *the first figure has again been generated* [γεγένηται γὰρ πάλιν τὸ πρῶτον σχῆμα]). And since the privative converts, neither will N belong to any X. (27a9–14)

We can express exactly what Aristotle writes here in the manner of a deduction with which we are familiar. Notice that Aristotle converts the conclusion to maintain strict syllogistic syntax (Table 17).

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[73]See *Pr. An. A1*: 24b24-26; *A5*: 27a1-3, 27a15-18, 28a4-7; *A6*: 28a15-17, 29a14-16.

[74]The process of completion is, of course, also carried out in one or another object language.

[75] Aristotle recognized 14 syllogisms in three figures, where traditionalist logicians, or logicians referring to traditional logic, consider there to be 24 syllogisms in four figures. A good recent reference for this information is William T. Parry & Edward A. Hacker 1991; an excellent, older source is H. W. B. Joseph 1906.

[76]The following analogy helps to explain Aristotle's procedure. As a geologist might use a hammer to break open a given rock to determine whether it is or is not a geode, and, upon making the determination, place the object in one of two piles, so Aristotle used a metalogical deduction to determine whether a given elementary argument pattern belongs in the set of panvalid patterns and he used the method of contrasted instances to determine whether in the set of paninvalid patterns.

Table 17.

Establishing the panvalidity of a second figure pattern: Camestres		
Aristotle's text	**Modern notation**	
1. M belongs to every N	1. M*a*N	
2. but [M] to no X	2. M*e*X	
? neither will N belong to any X	? N*e*X	
3. M belongs to no X	3. M*e*X	2 repetition
4. neither does X belong to any M	4. X*e*M	3 *e*-conversion
5. M belonged to every N	5. M*a*N	1 repetition
6. X will belong to no N	6. X*e*N	4,5 Clearent
7. neither will N belong to any X	7. N*e*X	6 *e*-conversion

The panvalidity of each second and each third figure pattern is determined in just this manner, whether by direct (probative) or indirect (*reductio*) deduction using the conversion and *teleioi sullogismoi* rules. Table 18 indicates what first figure pattern Aristotle used in the deduction process to establish the panvalidity of patterns in the three figures.

Table 19 expresses in schematic notation exactly what Aristotle writes concerning each proof of the panvalidity of the patterns in *Prior Analytics A5-6* ('X' = contradiction).

It is important to understand that in *Prior Analytics A5-6* Aristotle used his syllogistic deduction system as part of his metalinguistic discourse to establish certain features of the system itself. By this means he demonstrated[77] that a given second or third figure argument pattern is panvalid since its conclusion pattern follows logically from a given premise-pair pattern, which is thus understood to be concludent. Proving that a given argument pattern is panvalid establishes knowledge that the pattern can serve as a rule, since its extension is universal: an arbitrary argument fitting this pattern is valid.[78] Establishing this is strictly a metalogical process. Moreover, he demonstrated this by using the four patterns of the *teleioi sullogismoi*; this is indicated by his saying that "the first figure is again generated" (27a12-13; cf. 27a8-9, 36). Aristotle intentionally used the first figure patterns as deduction rules in *A5-6*, and he explicitly mentioned their use in

[77]Aristotle's "showings" are demonstrations as he remarks at 27b3 (on Baroco), 28a22-23 (on Darapti), 28a28-29 (on Felapton), and at 28b13-14 (on Datisi). His using the verb 'δείκνυσθαι' at 27a8-9 (on Cesare), 27a14-15 (on Camestres and on Baroco), 28a29-30 (on Felapton), 28b14-15 (on Datisi), and at 28b20-21 (on Bocardo) confirms his intention.

[78]Perhaps we see Aristotle here apply to the study of logic his requirement, expressed in *Po. An. A4*, that something is proved universally of a kind when it is proved of an arbitrary instance of that kind. He writes: "something holds universally when it is proved of an arbitrary and primitive case. ...Thus, if an arbitrary primitive case is proved to have two right angles (or whatever else), then it holds universally of this primitive item, and the demonstration applies to it universally in itself" (73b32-33, 73b39-74a2).

Table 18.

Summary of Aristotle's texts on completing each panvalid pattern		
Pattern considered	**Manner of proof: syllogism completed ...**	**Syllogism used in the completion process**
Barbara	by means of (διὰ [*dia*]) the things initially taken or through itself (δι αὑτοῦ)	(Barbara)
Celarent	*dia* the things initally taken or through itself	(Celarent)
Darii	*dia* the things initally taken or through itself	(Darii)
Ferio	*dia* the things initally taken or through itself	(Ferio)
Cesare	probatively *dia* ...	Celarent
Camestres	probatively *dia* ...	Celarent
Festino	probatively *dia...*	Ferio
Baroco	by *reducto ad impossibile dia...*	Barbara
Darapti	probatively *dia...*	Darii
Felapton	probatively *dia...*	Ferio
Disamis	probatively *dia...*	Darii
Datisi	probatively *dia...*	Darii
Bocardo	by *reductio ad impossibile dia...*	Barbara
Ferison	probatively *dia...*	Ferio

this respect in his proofs concerning Cesare (27a12-13), Festino (27a36), Darapti (28a22), and Ferison (28b34-35). Aristotle's summary of *A4-6* at *A7* highlights this point.

> All the incomplete syllogisms are completed by means of [τελειοῦνται διὰ] the first figure. For they all come to a conclusion [περάινονται, i.e., are deduced] either probatively or through an absurdity, and in both ways *the first figure is generated* [γίνεται]. (29a30 36; cf. *A5*: 28a4–7 and *A6*: 29a14 16)

As he expressly stated he would, Aristotle determined in *A4-6* "how every syllogism is generated". We complete Aristotle's model of his logic by schematically setting out his 14 syllogism rules using our abbreviations of Aristotle's long form to express the logical constants. Table 20 summarizes all the panvalid patterns

Table 19.

Aristotle's metalogical deductions for each second and third figure panvalid pattern		
Cesare (27a5-9) 1. M*e*N 2. M*a*X ? N*e*X 3. N*e*M 1 *e*-con 4. M*a*X 2 rep 5. N*e*X 3,4 **Celar**	Camestres (27a9-14) 1. M*a*N 2. M*e*X ? N*e*X 3. X*e*M 2 *e*-con 4. M*a*N 1 rep 5 X*e*N 3,4 **Celar** 6. N*e*X 5 *e*-con	Festino (27a32–36) 1. M*e*N 2. M*i*X ? N*o*X 3. N*e*m 1 *e*-con 4. M*i*X 2 rep 5. N*o*X 3,4 **Ferio**
		Baroco (27a36–27b3) 1. M*a*N 2. M*o*X 3. N*o*X 3. N*a*X assume 4. M*a*N 1 rep 5. N*a*X 3 rep 6. M*a*X 4,5 **Barb** 7. M*a*X & M*o*X 6,2 conj; X 8. N*o*X 3-7 *reduct*
Darapti (28a19-22) 1. P*a*S 2. R*a*S ? P*i*R 3. P*a*S 1 rep 4. S*i*R 2 *a*-con 5. P*i*R 3,4 **Darii**	Felapton (28a26-30) 1. P*e*S 2. R*a*S ? P*o*R 3. P*e*S 1 rep 4. S*i*R 2 *a*-con 5. P*o*R 3,4 **Ferio**	Disamis (28b7–11) 1. P*i*S 2. R*a*S ? P*i*R 3. S*i*P 1 *i*-con 4. R*a*S 2 rep 5. S*i*P 3 rep 6. R*i*P 4,5 **Darii** 7. P*i*R 6 *i*-con
Datisi (28b11–13) 1. P*a*S 2. R*i*S ? P*i*R 3. S*i*R 2 *i*-con 4. P*a*S 1 rep 5. S*i*R 3 rep 6. P*i*R 4,5 **Darii**	Bocarado (28b17-20) 1. P*o*S 2. R*a*S ? P*o*R 3. P*a*R assume 4. R*a*S 2 rep 5. P*a*S 3,4 **Barb** 6. P*a*S P*o*S 5,1 conj;X 7. P*o*R 3-6 *reduct*	Ferison (28b33–35) 1. P*e*S 2. R*i*S ? P*o*R 3. P*e*S 1 rep 4. S*i*R 2 *i*-con 5. P*o*R 3,4 **Ferio**

that might serve as rules in Aristotle's deduction system.

Table 20.

Summary of Aristotle's two-premiss syllogism deduction rules using modern notation						
First Figure	Barbara 1. A*a*B 2. B*a*C ∴ A*a*C	Celarent 1. A*e*B 2. B*a*C ∴ A*e*C	Darii 1. A*a*B 2. B*i*C ∴ A*i*C	Ferio 1. A*e*B 2. B*i*C ∴ A*o*C		
Second Figure	Cesare 1. M*e*N 2. M*a*X ∴ N*e*X	Camestres 1. M*a*N 2. M*e*X ∴ N*e*X	Festino 1. M*e*N 2. M*i*X ∴ N*o*X	Baroco 1. M*a*N 2. M*o*X ∴ N*o*X		
Third Figure	Darapti 1. P*a*S 2. R*a*S ∴ P*i*R	Felapton 1. P*e*S 2. R*a*S ∴ P*o*R	Disamis 1. P*i*S 2. R*a*S ∴ P*i*R	Datisi 1. P*a*S 2. R*i*S ∴ P*i*R	Bocardo 1. P*o*S 2. R*a*S ∴ P*o*R	Ferison 1. P*e*S 2. R*i*S ∴ P*o*R

When we treat Aristotle's notion of deducibility (§5.1) we treat the process of completion more fully than here. Nevertheless, we here briefly explain his meaning of '*teleios*'. It is common to translate '*teleios*' by 'complete', or even 'perfect', as in 'complete deduction'. However, taking a syllogism not to be a deduction but a valid argument, we see that Aristotle would have taken a *teleios sullogismos* to be an argument whose validity is obviously evident to a participant. Thus, perhaps, a better translation of '*teleios sullogismos*', albeit a bit cumbersome but more faithful to Aristotle's meaning, would be 'valid argument whose validity is obviously apparent'; an *ateles sullogismos* would then be translated by 'valid argument whose validity is not obviously apparent'. Following this interpretation, then, '*teleiousthai*', which has been translated by 'to be completed', as in 'all the *ateleis sullogismoi* are completed by means of the first figure', would mean 'the validity of those valid arguments whose validity is not apparent is made evident by means of the first figure'. Aristotle used only the verbs 'τελειοῦσθαι' (*teleiousthai*) and 'ἐπιτελεῖσθαι' (*epiteleisthai*) in connection with using the patterns of the first figure. Thus, in the deduction process, the validity of a valid argument becomes evident when during the process a *teleios sullogismos* appears, or 'is generated'. Such an appearance in a chain of reasoning signals to a participant the cogency of the chain of inferences that links the conclusion sentence to the premiss sentences as a logical consequence.

3.3 Establishing inconcludence of premiss-pair patterns

Aristotle used the method of contrasted instances in *Prior Analytics A4-6* to show that a given premiss-pair pattern is inconcludent, that it does not result in a panvalid pattern — that no substitution of terms produces a syllogism, or

valid argument. Consequently, at one stroke this method establishes the paninvalidity of each of the four corresponding elementary argument patterns relating to each premiss-pair pattern. Aristotle devised three processes for establishing the inconcludence of a given premiss-pair pattern. Moreover, his method is significantly different than the method of counterargument. In this way he was able to determine each case in which no syllogism is possible. His purpose was to eliminate every elementary two-premiss argument pattern that could not serve as a deduction rule. Below we treat each of the three related processes, establish the conditions for concludence and inconcludence, and extract some semantic principles underlying Aristotle's methods (§3.4). We also address two possible objections to interpreting Aristotle as treating premiss-pair patterns (§3.5.).

The method of contrasted instances

Aristotle introduced his most commonly used method, the method of contrasted instances, for deciding inconcludence at *Prior Analytics A4* and used it throughout *A4-6.* He writes, in relation to the premiss-pair pattern *1ae*:

> However, if [ἐι]the first extreme follows [i.e., belongs to] all the middle and the middle belongs to none of the last, there will not be a syllogism of the extremes, for *nothing necessarily results* in virtue of these things being so. (26a2-5)

This sentence states a set of formal relationships of three terms in two universal sentences in the role of premisses for *not* generating a syllogism in the first figure. This passage states the conditions concerning the pattern P*a*M, M*e*S | for which no categorical sentence is a logical consequence of two other categorical sentences fitting this pattern. Thus, Aristotle eliminates four elementary argument patterns in the standard syntax as possible syllogisms by establishing their premiss pattern to be inconcludent. He continues:

> For *it is possible* [ἐνδέχεται] *for the first extreme to belong to all as well as to none of the last.* Consequently, neither a partial nor a universal conclusion results necessarily [γίνεται ἀναγκᾶιον]; and, since nothing is necessary because of these, there will not be a syllogism. Terms for belonging to every are animal, man, horse; for belonging to none, animal, man, stone. (26a5-9)

We can express what he writes here as follows (truth-values to the right) (Table 21):

For Aristotle this demonstrates[79] that "nothing necessarily results", that no valid argument (syllogism) is possible from sentences fitting this premiss-pair pat-

[79]He considers himself to demonstrate the inconcludence of certain premiss-pair patterns as is indicated by his writing, for example, that some results must be proved (δεικτέον) in another way (see *Pr. An. A5*: 27b20-21, 28; *A6*: 29a6).

Table 21.

Establishing inconcludence by the method of contrasted instances			
Pattern: PaM, MeS — PaS			
1	Animal [A] belongs to every man [M].	A*a*M	T
2	Man belongs to no horse [H].	M*e*H	T
?	Animal belongs to every horse.	A*a*H	T
Pattern:P*a*M, M*e*S|P*e*S			
1.	Animal [A] belongs to every man [M].	A*a*M	T
2.	Man belongs to no stone [S].	M*e*S	T
?	Animal belongs to no stone.	A*e*S	T

tern since, as he shows, the results "*could be otherwise*". Aristotle clearly uses neither the method of counterargument nor the method of counterinterpretation,[80] each of which requires finding an instance of an argument having true premisses and a false conclusion in the same form as a given argument. Rather, by substituting two sets of three terms for the schematic letters, he constructs two arguments each of whose premisses are known to be true sentences fitting the same premiss-pair pattern and whose conclusions also are known to be true sentences, but in the one argument it is an *a* sentence, in the other an *e* sentence. It is not possible to do this with a concludent pair, since every similar substitution that produces true sentences as premisses will result in at least one false sentence, either the *a* or the *e* sentence, in the conclusion. Thus, any two sentences of three terms fitting this premiss-pair pattern are shown never to result together in a valid argument. This premiss-pair pattern is inconcludent. No syllogism is possible in this case. It is evident, moreover, that Aristotle treats *at one time* in this way four argument patterns in the standard syntax for each premiss-pair pattern; he does not show that each of the four patterns is paninvalid by using counterarguments. With 26a5-9 Aristotle establishes a practice that he uses throughout *A4-6* to demonstrate inconcludence. This method of deciding inconcludence works for almost every premiss-pair pattern, noticeably failing in some instances when the minor premiss is a partial, and usually a privative, sentence.

This method of deciding inconcludence, while different than, is nevertheless easily adapted to the method of counterargument, but adapted at the metalogical level. Both methods achieve the same results. We can apply Aristotle's two sets of three terms to the two argument patterns but switch the terms for *belonging to none* to *belonging to every* and vice versa, and then make the substitutions in the argument patterns accordingly. Thus (Table 22):

[80]A counterargument is an argument in the same form as a given argument (whose invalidity is to be established) but has premisses that are true and a conclusion that is false. A counterinterpretation is an argument in the same form as a given argument (whose invalidity is to be established) but a model of the premiss-set is not a model of the conclusion.

Table 22.

Method of counterargument for establishing invalidity			
Pattern: P*a*M, M*e*S|P*a*S			
1.	Animal [A] belongs to every man [M].	A*a*M	T
2.	Man *e* stone [S].	M*e*S	T
?	Animal belongs to every stone.	A*a*S	F
?	Animal belongs to some stone	A*i*S	F
Pattern: P*a*M, M*e*S|P*e*S			
1.	Animal [A] belongs to every man [M].	A*a*M	T
2.	Man belongs to no horse [H].	M*e*H	T
?	Animal belongs to no horse.	A*e*H	F
?	Animal does not belong to some horse.	A*o*H	F

In these cases each sentence of the premiss-set is true and the respective conclusion sentences are false. Here, then, are counterarguments for the arguments provided by Aristotle, which may serve as modern counterparts to Aristotle's ancient method. The method of counterargument could also establish that a given pattern is paninvalid, insofar as no syllogistic pattern is neutrovalid, on the principle that two arguments in the same pattern are either both valid or both invalid.

It is apparent that Aristotle did *not* use the method of counterargument in *A4-6.* Moreover, Aristotle did not establish arguments to be invalid but argument patterns to be paninvalid, and, more specifically, he established the inconcludence of certain premiss patterns of two categorical sentences with places for three different terms in his search for syllogistic rules. He knew this procedure to establish the paninvalidity at one time of four elementary argument patterns.

Conditions of concludence and inconcludence

Aristotle did not invent a name for his method of deciding inconcludence, nor did he invent expressions that denote features or principles of his method. Nevertheless, it is apparent that he consciously worked with a notion of contrariety that pertains to two categorical sentences each of which is the conclusion of a different categorical argument whose premiss sentences fit the same premiss-pair pattern. Here we describe two conditions[81] that underlie Aristotle's two decision procedures, the one pertaining to concludent patterns the other to inconcludent patterns.

In the context of *Prior Analytics A4-6* "to be otherwise" or "it is possible to be otherwise" involves a notion of *contrariety* according to which it is logically

[81] Concerning these two conditions, the expression 'condition I' is an abbreviation for the four points under it; likewise for 'condition II'. This simplifies treating the semantic principles underlying Aristotle's thinking treated in §3.4.

Condition I: pertaining to concludent patterns resulting in a syllogism

	Of two sentences:
1.	Each sentence is the *conclusion* of a categorical argument, each argument has the same premiss-pair pattern into which are substutited different sets of three terms that produce *true* sentences.
2.	Each sentnece has a *different* set of predicate and subject terms
3.	Each sentence is a universal categorical sentence: the one at an *a* sentence, the other an *e* sentence
4.	It is *logically impossible* for both sentences to be true.

Condition II: pertaining to inconcludent patterns not resulting in a syllogism

	Of two sentences:
1.	Each sentence is the *conclusion* of a categorical argument, each argument has the same premiss-pair pattern into which are substutited different sets of three terms that produce *true* sentences.
2.	Each sentence has a *different* set of predicate and subject terms
3.	Each sentence is a universal categorical sentence: the one an *a* sentence, the other an *e* sentence
4.	It is *logically possible* for both sentences to be true.

possible for substitution instances for a given premiss-pair pattern to produce conclusion sentences that satisfy condition II. Were a given premiss-pair pattern concludent, it would be logically impossible for an arbitrary substitution instance to produce conclusion sentences not satisfying condition I.

The modified method of contrasted instances

Aristotle uses a modified method of contrasted instances for deciding inconcludence in *Prior Analytics A4* for treating only two premiss-pair patterns, P*a*M, M*o*S | and P*e*M, M*o*S | (26a39-b14), both of which are then immediately treated by his third (or second most used) method, that of deducing inconcludence from the indeterminate. We cite 26a39-b14 in its entirety to examine his modified method.

> Nor will there be a syllogism whenever [ὅταν] the term in relation to the major extreme is universally either attributive or privative, and the term in relation to the minor is partially privative .... (26a39-26b3)

This sentence refers to two sets of necessary relationships of three terms in two premiss sentences, the one universal, whether attributive or privative, the other partially privative, for *not* generating a syllogism in the first figure. The sentence in the minor premiss in each case is an *o* sentence. This passage states the conditions under which substituting sets of three terms into the premiss-pair patterns P*a*M,

M*o*S | and P*e*M, M*oS* |, covering eight argument patterns in the standard syntax, never results in a syllogism, and, accordingly, it asserts the paninvalidity of these eight argument patterns. Here, however, Aristotle noticeably uses a variant of his most commonly used method for deciding inconcludence. The passage continues:

> ... as, for instance, if A belongs to every B and B does not belong to some C, or if it does not belong to every C. For whatever part <of the last extreme> it may be that the middle does not belong to, the first extreme could follow [ἀκολουθήσει] all as well as none of this part. For let the terms animal, man, and white be assumed, and next let swan and snow also be selected from among those white things of which man is not predicated. Then, animal is predicated of all of one but of none of the other, so that there will not be a syllogism. (26b3-10)

We can express what he writes here concerning P*a*M, M*o*S | as we did above for P*a*M, M*e*S | (Table 23).

Table 23.

<table>
<tr><th colspan="4">Establishing inconcludence by
the modified method of contrasted instances</th></tr>
<tr><td colspan="4">Pattern: PaM, MoS|PaS</td></tr>
<tr><td>1.</td><td>Animal [A] belongs to every man [M]</td><td>AaM</td><td>T</td></tr>
<tr><td>2.</td><td>Man does not belong to some white [selecting swan] [W/S]</td><td>MoW/S</td><td>T</td></tr>
<tr><td>?</td><td>Animal belongs to every swan</td><td>AaW/S</td><td>T</td></tr>
<tr><td colspan="4">Pattern: PaM, MoS|PeS</td></tr>
<tr><td>1.</td><td>Animal [A] belongs to every man [M].</td><td>AaM</td><td>T</td></tr>
<tr><td>2.</td><td>Man does not belong to some white [selecting snow] [W/S].</td><td>BoW/S</td><td>T</td></tr>
<tr><td>?</td><td>Animal belongs to no snow</td><td>AeW/S</td><td>T</td></tr>
</table>

The difference here consists in his "selecting from among white things" to which "man" does not belong. Aristotle here takes his substances to exist, that they are members of non-empty classes. Moreover, he takes 'some' in its determinate sense. Still, condition II is satisfied for these conclusion sentences as in the case for P*a*M, M*e*S | and the others. Aristotle next turns to P*e*M, M*o*S | in the same fashion. He writes:

> Next, let A belong to no B and B not belong to some C, and let the terms be inanimate, man, white. Then, let swan and snow be selected from among those white things of which man is not predicated (for inanimate is predicated of all of one and of none of the other). (26b10-14)

Table 24.

Establishing inconcludence by the modified method of contrasted instances			
Pattern: P*a*M, M*o*S|P*a*S			
1.	Inanimate [I] belongs to no man [M]	I*e*M	T
2.	Man does not belong to some white [selecting snow] [W/S]	M*o*W/S	T
?	Inanimate belongs to every snow	I*a*W/S	T
Pattern: P*e*M, M*o*S|P*e*S			
1.	Inanimate [I] belongs to no man [M].	I*e*M	T
2.	Man does not belong to some white [selecting swan] [W/S].	M*o*W/S	T
?	Inanimate belongs to no swan	I*e*W/S	T

We can set this out as follows (Table 24).

This is similar to his most commonly used method of deciding inconcludence, and we can easily grasp Aristotle's application. The conclusion sentences again satisfy condition II.

The method of deducing inconcludence from the indeterminate

Finally, Aristotle uses a method of establishing that a given premiss-pair pattern is inconcludent by deciding its inconcludence from another pattern already determined to be inconcludent. This method is introduced at 26b14-21 in relation to the premiss-pair patterns P*a*M, M*o*S | and P*e*M, M*o*S |. Two matters are noteworthy: (1) Aristotle distinguishes an indeterminate (ἀδιόριστος) sentence from a (determinate) partial (μέρος) sentence; and (2) he alludes to his having shown the pattern P*a*M, M*e*S | (at 26a2-9) not to generate a syllogism. His method may be designated, as he himself virtually does, as the *method of deducing inconcludence from the indeterminate.* We cite the passage from *A4*, where he first introduces this method, but we examine the method more fully below in connection with another passage from *Prior Analytics A5.*

> Moreover, since "B does not belong to some C" is indeterminate, that is, it is true if B belongs to none as well as if it does not belong to every (because it does not belong to some), and since a syllogism does not come about when terms are taken such that B belongs to none (this was said earlier [at 26a2-9]), then it is evident that there will not be a syllogism on account of the terms being in this relationship either (for there would also be one in the case of these terms). It may <also> be proved similarly if the universal is put as privative. (26b14-21)

Since (1) B*o*C is indeterminate, B*o*C is true if B*e*C is true, and (2) P*a*M, M*e*S | has already been shown to result in nothing necessarily, then P*a*M, M*o*S | cannot result in anything necessarily. P*a*M, M*o*S | is a weaker form of P*a*M, M*e*S |, which had been shown to be inconcludent. When B*o*C is partial and indeterminate, that B does not belong to some Cs and possibly to no Cs, then, since nothing has been specified about those Cs that are B, we are lead back to the aspect just treated. In other words, it is possible either (1) that a substitution instance could result in sentences fitting A*a*C and A*e*C both being true or (2) that a substitution instance produces true premiss sentences and a false conclusion sentence. The sets of substitution terms in the premiss-pair pattern P*a*M, M*o*S | generate no necessary result. The proof is the same for the pattern P*e*M, M*o*S |.

This method of deciding inconcludence is perhaps better portrayed in the following passage from *A5* where Aristotle treats premiss-pair patterns of the second figure. Again, we cite the passage in its entirety.

> But whenever[ὅταν] the premisses are the same in form (that is, both are privative or both attributive), then in no way will there be a syllogism. (27b10-12)

This sentence refers to four sets of necessary relationships of three terms in two premiss sentences, the one universal, the other partial, whether minor or major, and both sentences having the same quality, for not generating a syllogism in the second figure. This passage states conditions of inconcludence covering the four premiss-pair patterns M*e*P, M*o*S –, M*a*P, M*i*S—, M*o*P, M*e*S—, and M*i*P, M*a*S—. Accordingly, it establishes the paninvalidity of sixteen argument patterns in the standard syntax. The passage continues:

> For let the premisses first be privative, and let the universal be put in relation to the major extreme (that is, let M belong to no N and not to some X). It is then possible [ἐνδέχεται] for N to belong to every X as well as to none. Terms for not belonging are black, snow, animal. (27b12-16)

Here Aristotle shows that the premiss-pair pattern M*e*P, M*o*S— is inconcludent. The usual practice would be to produce two arguments whose premiss sentences fit the same premiss-pair pattern and whose conclusion sentences satisfy condition II. He is able to get terms for *belonging to none*, but he cannot get terms for *belonging to every* to satisfy condition II. We can illustrate what he writes here as follows (Table 25):

This is all familiar. Aristotle then continues to treat the argument pattern M*e*P, M*o*S | P*a*S using the method of deducing from the indeterminate. He writes:

> We cannot get [οὐκ ἔστι λαβεῖν] terms for belonging if M belongs to some X and does not belong to some (for if N belongs to every X and M to no N, then M will belong to no X: but it was assumed to belong

Table 25.

Establishing inconcludence by the method of proving from the indeterminate		
Pattern: P*a*M, M*o*S|P*a*S		
1.	M*e*N	
2.	M*o*X	
?	M*o*X	
Pattern: M*e*P, M*o*S|P*e*S		
1. Black [B] belongs to no snow [S].	B*e*S	T
2. Black does not belong to some animal white [selecting swan] [W/S].	B*o*A	T
? Snow belongs to no animal	S*e*A	T

> to some). It is not possible to get [οὐκ ἐγχωρεῖ λαβεῖν] terms in this way, then, but *it must be proved from the indeterminate* [ἐκ δὲ τοῦ ἀδιόριστου δεκτέον]. For since "M does not belong to some X" is also true even if M belongs to no X and there was not a syllogism when it belonged to none [27a20–25], then it is evident that there will not be one in the present case either. (27b16–23)

Aristotle does not use his modified method of deciding inconcludence in this case because he could not get terms for *belonging to every* for the argument pattern M*e*P, M*o*S | P*a*S when the minor premiss is an *o* sentence. Every set of term substitutions exhibiting true premiss sentences results in producing a false conclusion sentence — a situation that could, in fact, establish the paninvalidity of this argument pattern by the method of fact. *However, he aims to establish the inconcludence of premiss-pair patterns and thereby the paninvalidity of their corresponding argument patterns.* Accordingly, he takes the *o* sentence to be indeterminate and refers back to his determining, by the method of contrasted instances, that the premiss-pair pattern M*e*P, M*e*S | does not generate a syllogism. Since (1) the minor premiss sentence in M*e*P, M*o*S | is a weaker privative of the minor premiss sentence in M*e*P, M*e*S |, and (2) the truth-value of a partial follows its corresponding universal (when true), then, since (3) M*e*P, M*e*S | is a stronger premiss-pair pattern than M*e*P, M*o*S | and (4) M*e*P, M*e*S | has been shown to be an inconcludent pattern, (5) what is true of M*e*P, M*e*S | is also true of M*e*P, M*o*S |; thus, M*e*P, M*o*S | is an inconcludent premiss-pair pattern. Aristotle's reasoning can be expressed as follows:

1.	M*e*P, M*e*S|	Premiss-pair pattern known to be inconcludent
2.	M*e*P, M*o*S|	*e* premiss weakening: Premiss-pair pattern to be established as inconcludent

Again, he takes his terms here to relate to non-empty classes. This method, of course, depends for its success upon a prior use of the method of contrasted instances for deciding inconcludence. Tables 26-28 below catalogue each instance of Aristotle's deciding inconcludence.[82]

3.4 Semantic principles underlying Aristotle's method of deciding invalidity

Aristotle considered the following expressions to be synonymous: "the results could be otherwise" or "it is [logically] possible for the results to be otherwise"; "nothing results necessarily"; "nothing follows necessarily"; "the results are not [logically] necessary"; and, in the case of the syllogisms, "there is not a syllogism of the extremes". These expressions, of course, relate to invalidity and paninvalidity and have their counterparts for validity and panvalidity. And, moreover, in relation to our topic, they all involve a notion of contrariety special to categorical sentences in the role of conclusions in categorical arguments.

Aristotle recognized and worked with syntactic and semantic principles by which he established that a given premiss-pair pattern is concludent and another inconcludent. This pertains to satisfying condition I and condition II respectively. Adapting Aristotle's method of establishing invalidity to the method of counterargument indicates the correctness of his method. However, our doing this does not provide insight into the principles underlying his method, which principally aims at premiss-pair patterns and consequently their corresponding argument patterns. These principles were unexpressed by Aristotle as others have been expressed by modern logicians respecting the method of counterargument. All the following principles pertain to premiss-pair patterns and to their corresponding categorical argument patterns and arguments. We believe these principles represent Aristotle's own thinking about deciding invalidity. Moreover, they provide an interesting insight into an ancient logic that might shed light on modern logics.

[82]In Tables 26-28, 'PP' refers to the various premiss-pair patterns Aristotle treats, and 'SL' refers to his providing schematic letters in treating a given pattern. Aristotle sets out terms according to the schematic order for each figure's schematic letters: first figure-PMS, second-MPS, third-PSM.

Table 26. Catalogue of premiss-pair patterns not generating a syllogis

PP	Treatment	Covering Statement	SL	Method of Establishing Inconc		
				Contrasted instances		Modified c
				Terms for belonging *PMS*	Terms for not belonging *PMS*	Terms for belon
ae	26a5-9	26a2-4	no	animal-man-horse	animal-man-stone	
ee	26a11-13	26a9-11	no	science-line-medicine	science-line-unit	
ia	26a33-36	26a30-33	yes	good-condition-wisdom	good-condition-ignorance	
oa	26a33-36	26a30-33	yes	good-condition-wisdom	good-condition-ignorance	
ie	26a36-39	26a30-33	yes	white-horse-swan	white-horse-raven	
oe	26a36-39	26a30-33	yes	white-horse-swan	white-horse-raven	
ao	26b3-10 26b14-20	26a39-26b3	yes			animal-man-wh /swan (26b3-
eo	26b10-14 26b20-21	26a39-26b3	yes			inanimate-man- /snow (26b10
ii	26b24-25	26b21-24	no	animal-white-horse	animal-white-stone	
oo	26b24-25	26b21-24	no	animal-white-horse	animal-white-stone	
io	26b24-25	26b21-24	no	animal-white-horse	animal-white-stone	
oi	26b24-25	26b21-24	no	animal-white-horse	animal-white-stone	

Table 27. Catalogue of premiss-pair patterns not generating a syllogism

PP	Treatment	Covering Statement	SL	Method of Establishing Inconclud		
				Contrasted instances		Modified cont
				Terms for belonging *MPS*	Terms for not belonging *MPS*	Terms for belong
aa	27a18–20	27a23–25	yes	substance-animal-man	substance-animal-number	
ee	27a20–23	27a23 25	yes	line-animal-man	line-animal-stone	
oa	27b4–6	27b9 ·10	yes	animal-man-raven	animal-white-raven	
ie	27b6–8	27b9 10	yes	animal-substance-unit	animal-substance-science	
eo	27b12–23	27b10 12 (27b34–36)	yes		black-snow-animal	
ai	27b23–28	27b10-12 (27b34–36)	yes		white-swan-stone	
oe	27b28–32	27b10-12 (27b34-36)	yes	white-animal-raven	white-stone-raven	
ia	27b32–34	27b10–12 (27b34 36)	no	white-animal-swan	white-animal-snow	
ii	27b38–39	27b36 ·38	no	white-animal-man	white-animal-inanimate	
oo	27b38–39	27b36—38	no	white-animal-man	white-animal-inanimate	
io	27b38–39	27b36 38	no	wite-animal-man	white-animal-inanimate	
oi	27b38-39	27b36-38	no	white-animal-man	white-animal-inanimate	

Table 28. Catalogue of premiss-pair patterns not generating a syllogis

PP	Treatment	Covering Statement	SL	Method of Establishing Inconclu		
				Contrasted instances		Modified con
				Terms for belonging *PSM*	Terms for not belonging *PSM*	Terms for belon
ae	28a30–33	(28b1–3) 28b3–4	yes	animal-horse-man	animal-inanimate-man	
ee	28a33 36	(28a37–39) 28a39 ·28b1	no	animal-horse-inanimate	man-horse-inanimaate	
ao	28b22–31	28b22 23	yes	animate-man-animal		
ie	28b36-38	28b36	no	animal-man-wild	animal-science-wild	
oe	28b39–29a2	28b38–39	no	animal-science-wild	animal-man-wild	
eo	29a2–6	28b38–39	yes		raven-snow-white	
ii	29a9-10	29a6–9	no	animal-man-white	animal-inanimate-white	
oo	29a9–10	29a6–9	no	animal-man-white	animal-inanimate-white	
io	29a9–10	29a6–9	no	animal-man-white	animal-inanimate-white	
oi	29a9-10	29a6–9	no	animal-man-white	animal-inanimate-white	

1. A given premiss-pair pattern is *concludent* if and only if every set of term substitutions satisfies condition I.

2. A given premiss-pair pattern is *inconcludent* if and only if no set of term substitutions satisfies condition I (or every set of term substitutions satisfies condition II).

3. Two arguments having sentences that fit the same premiss-pair pattern and sentences whose conclusions satisfy condition I cannot both be valid (but, of course, both may be invalid as with Darii).

4. Two arguments having sentences that fit the same premiss-pair pattern and sentences whose conclusions satisfy condition II are both invalid.

5. No argument having all true premisses and a false conclusion is valid.

6. A given argument pattern is *panvalid* if and only if it is logically impossible for an arbitrary argument fitting the given pattern to have true premisses and a false conclusion.

7. A given argument pattern is *paninvalid* if and only if it is logically impossible for an arbitrary argument fitting the given pattern to be valid.

8. A given argument is *valid* if it fits a panvalid argument pattern. The sentence in the conclusion follows necessarily (it cannot be otherwise) from the sentences comprising the premiss-set.

9. A given argument is *invalid* if it fits a paninvalid argument pattern. No sentence follows necessarily (the results can be otherwise) from other sentences in a premiss-set.

Of course, in respect of numbers 8 and 9, a pattern does not make a given argument valid or invalid. Rather an argument is valid just in case all the information contained in the conclusion sentence is already contained in the premiss sentences, invalid if more information is in the conclusion than in the premisss. Again, just as Aristotle believes that truth follows being, so does he believe that validity, or following necessarily, follows being: there is an ontic underpinning for a valid argument's validity just as there is for a true sentence's truth.

Some epistemic principles relating to the semantic principles listed above include the following:

1. It is sufficient for *knowledge of the concludence* of a given premiss-pair pattern to produce two arguments whose conclusion sentences satisfy condition I.

2. It is sufficient for *knowledge of the inconcludence* of a given premiss-pair pattern to produce two arguments whose conclusion sentences satisfy condition II.

In some cases, when the minor premiss is an indeterminate sentence, it is sufficient to demonstrate that the given pattern is a weaker form of a premiss-pair pattern already established to be inconcludent.

3. It is sufficient for *knowledge of the panvalidity* of a given argument pattern to produce either a direct or *reductio* deduction of its conclusion sentence pattern.

4. It is sufficient for *knowledge of the paninvalidity* of a given argument pattern to show that its premiss-pair pattern is inconcludent, or in some cases (§3.5) to show that a given argument pattern fits an argument pattern rejected in the case of a concludent premiss-pair pattern.

5. It is sufficient for *knowledge of the validity* of a given argument to show that it fits a panvalid argument pattern.

6. It is sufficient for *knowledge of the invalidity* of a given argument to show that it fits a paninvalid argument pattern, or to show that the sentences of its premiss-set fit an inconcludent premiss-pair pattern.

3.5 Determining the paninvalid argument patterns of concludent premiss patterns

Two problems seem to arise were it true that Aristotle exclusively treated patterns of premisses and argument patterns in *Prior Analytics A4-6* rather than directly treating arguments. Consider the following two arguments, A1 and A2, both of which are invalid, however, both of which also have a premiss pattern established to be concludent (i.e., relating to Barbara).

Table 29.

A1 Pattern: P*a*M, M*a*S|P*e*S			
1.	Animal [A] belongs to every mammal [M].	A*a*M	T
2.	Mammal belongs to every human [H].	M*q*H	T
?	Animal belongs to no human	A*e*H	F
A2 Pattern: P*a*M, M*a*S|S*a*P			
1.	Animal [A] belongs to every mammal [M].	A*a*M	T
2.	Mammal belongs to every human [H].	M*a*H	T
?	Human belongs to every animal.	H*a*A	F

These two arguments are obviously invalid on the principle that no argument is valid having true premisses and a false conclusion. We cite two other arguments that perhaps better illustrate the problems because their invalidity may not be immediately evident. Note that each premiss-pair pattern is concludent: Darii results from that in A3, Baroco in A4.

Table 30.

A3 Pattern: P*a*M, M*i*S|P*a*S			
1.	Surface [S] belongs to every table [T].	S*a*T	T
2.	Table belongs to some furniture [F].	T*i*F	T
?	Surface belongs to every furniture.	S*a*F	F
A4 Pattern: M*a*P, M*o*S|S*o*P			
1.	Container [C] belongs to every bottle [B]	C*a*B	T
2.	Container belongs not to every plastic [P].	C*o*P	T
?	Plastic belongs not to every bottle.	P*o*B	F

These arguments represent the two basic concerns that a modern logician might have. (1) Since not every argument relating to a concludent pattern is valid, how are the paninvalid argument patterns of a given *concludent* premiss-pair pattern identified when the given syllogistic syntax is standard? (2) Similarly, how are the paninvalid argument patterns of a given *concludent* premiss-pair pattern determined when the syllogistic syntax is converted in the conclusion? These questions pertain to every paninvalid pattern associated with a concludent premiss-pair pattern in any of the three figures, whether the pattern's conclusion is a universal or a partial sentence. Arguments A1 and A3 correspond to the first concern and arguments A2 and A4 to the second concern. All paninvalid argument patterns of the kind treated in *Prior Analytics* fall into one or other of these two classes. These two concerns are treated in turn immediately below.

Paninvalid patterns relating to a concludent pattern in the standard syntax

Looking back at Aristotle's treatment of Barbara (P*a*M, M*a*S | P*a*S) in *Prior Analytics A4* we notice that (1) he did not demonstrate its panvalidity but posited this pattern as obviously panvalid and (2) he did not demonstrate the paninvalidity of the argument pattern P*a*M, M*a*S | P*e*S, which has the same premiss-pair pattern as Barbara. Nor, for that matter, did he show that P*a*M, M*a*S | P*iS* is panvalid and that P*a*M, M*a*S | P*o*S is paninvalid. All these argument patterns have the same concludent premiss-pair pattern. Since Aristotle does not specially take up these concerns, we interpolate from the text to illuminate his thinking. We first consider the premiss-pair pattern represented in argument A1 from which Barbara results and then consider the premiss-pair pattern represented in argument A3 from which Darii results.

Take the following two arguments in the first figure, the one substituting 'animal', 'mammal', 'human' for *belonging to every*, the other 'animal', 'reptile', 'snake' for *belonging to none.*

Since "Animal belongs to every man" is recognized to be a necessary result, Aristotle understood that 'no other syllogistic result is logically possible'. Thus, *PaM*, M*a*S | is a concludent premiss-pair pattern. Having posited that an *a* sen-

Table 31.

Pattern: P*a*M, M*a*S|P*a*S			
1.	Animal [A] belongs to every mammal [M].	A*a*M	T
2.	Mammal belongs to every human [H].	M*a*H	T
?	Animal belongs to every human.	A*a*H	F
Pattern: P*a*M, M*a*S|P*e*S			
1.	Animal [A] belongs to every reptile [R].	A*a*R	T
2.	Reptile belongs to every snake [S].	R*a*S	T
?	Animal belongs to no snake.	A*e*S	F

tence is a necessary result of sentences in the given pattern P*a*M, M*a*S | (*A4*: 25b37-40), Aristotle would have immediately recognized that an *e* sentence could never follow logically: this satisfies Condition I — granting the one sentence excludes the other. It is logically impossible to find terms for P*a*M, M*a*S | P*e*S where both the premisses and the conclusion are true sentences. He might have added that it is logically impossible to derive something *not belonging universally* from something *belonging universally*. In the case of Celarent (P*e*M, M*a*S | P*e*S) it would be impossible to have a true *a* sentence as a conclusion when its corresponding *e* sentence is true.[83] Recognizing that for Aristotle it is trivially true that a partial sentence follows logically from a universal sentence of the same quality, we see that he would have determined that P*a*M, M*a*S | P*i*S is panvalid and that, likewise, P*a*M, M*a*S | P*o*S is paninvalid. Aristotle recognized that what happens here is similar to what happens in applying the method of deducing inconcludence from the indeterminate. The reasoning is the same for any syllogistic pattern in each figure whose conclusion pattern is universal.

Next consider the case when the result in the first figure is partial. The following two arguments illustrate that the premiss-pair pattern P*a*M, M*i*S | is concludent; substitute 'surface', 'table', 'furniture' for *belonging to every*, 'container', 'bottle', 'plastic' for *belonging to none*.

It is logically impossible not to satisfy Condition I for this premiss-pair pattern even though the necessary result is a partial sentence. Again, take the second figure pair pattern M*e*P, M*i*S | from which Aristotle gets Festino, M*e*P, M*i*S | P*o*S, and substitute terms for *belonging to every* and *belonging to none* as below. It is logically impossible to satisfy Condition II for this pattern.
However, in a concludent pattern where the necessary conclusion is partial, contrary to a concludent pattern whose necessary result is universal, it is possible to exhibit two arguments each with true premisses and a false conclusion. For M*e*P, M*i*S | P*a*S take the instance cited immediately above and for M*e*P, M*i*S | P*e*S take the following instance (Table 34):

[83] Applying the method of fact in this instance, a method surely known to Aristotle (see *B2*), we can demonstrate for ourselves that P*a*M, M*a*S | P*e*S is a paninvalid argument pattern, as also is the pattern with a weakened conclusion (P*a*M, M*a*S | P*o*S).

Table 32.

Pattern: P*a*M, M*i*S|P*a*S			
1.	Surface [S] belongs to every table [T].	S*a*T	T
2.	Table belongs to some furniture [F].	T*i*F	T
?	Surface belongs to every furniture.	S*a*F	F
Pattern: P*a*M, M*i*S|P*e*S			
1.	Container [C] belongs to every bottle [B]	C*a*B	T
2.	Container belongs to some plastic [P].	B*i*P	T
?	Container belongs to no plastic.	C*e*P	F

Table 33.

Pattern: M*e*P, M*i*S|P*a*S			
1.	Bi-pedal [B] belongs to no turtle [T].	B*e*T	T
2.	Bi-pedal beongs to some animal [A].	B*i*A	T
?	Turtle belongs to every animal.	T*a*A	F
Pattern: M*e*P, M*i*S|P*e*S			
1.	Intelligence [I] belongs to no building [B]	I*e*B	T
2.	Intelligence belongs to some animal [A].	I*i*A	T
?	Building belongs to no animal.	C*e*P	F

This signals that both a universal privative and a universal affirmative sentence cannot result from the concludent pattern M*e*P, M*i*S |. Here again we recognize Aristotle's familiarity with the method of fact (*Pr. An. B2-4*). Moreover, it is logically impossible to generate a true attributive sentence from a true privative sentence and a true universal from a true partial sentence. Having eliminated *a* and *e* sentences as possible results, Aristotle would have turned to *i* and *o* sentences to determine which is necessary. We know that Aristotle established the panvalidity of Festino by a metalogical deduction using Ferio (27a32-36). Recognizing that nothing affirmative can result from something privative, he would eliminate an *i* result. This leaves M*e*P, M*i*S | P*o*S as the only possible logically necessary result.[84]

Paninvalid patterns from a concludent pattern with conclusion conversion

This concern relates to altering the standard syllogistic syntax by converting the conclusion pattern — from P*x*S to S*x*P — while retaining the syntax of the premiss-pair pattern. Thus we have:

[84] Perhaps Aristotle reasoned in this way. This seems likely from his using the method of deducing inconcludence from the indeterminate. Perhaps, he deduced the panvalidity of M*e*P, M*i*S | P*o*S from the panvalidity of M*e*P, M*a*S | P*e*S (Cesare), a strong pattern, which was itself likely established by fiddling with conversions and premiss transposition of Celarent (P*e*M, M*a*S | P*e*S).

Table 34.

Pattern: M*e*P, M*i*S|P*a*S			
1.	Bi-pedal [B] belongs to no turtle [T].	B*e*T	T
2.	Bi-pedal beongs to some animal [A].	B*i*A	T
?	Turtle belongs to every animal.	T*a*A	F
Pattern: M*e*P, M*i*S|P*e*S			
1.	Good [G] belongs to no danger [D]	G*e*D	T
2.	Good belongs to some house [H].	G*i*H	T
?	Danger belongs to no house.	D*e*H	F

1. P*x*M	1. M*x*P	1. P*x*M
2. M*y*S	2. M*y*S	2. S*y*M
? S*z*P	? S*z*P	? S*z*P

This *appears* to double the possible results of premiss-pair patterns. If Aristotle specifically treated premiss-pair patterns and not arbitrary arguments — and thus did not use the method of counterargument — how would he have established the paninvalidity of an argument pattern whose premiss pattern has been determined to be concludent but whose conclusion is the converse of the standard syntax? Moreover, do any panvalid argument patterns emerge from inconcludent premiss-pair patterns when the conclusion is converted?

First, we may easily dispense with all concludent patterns whose necessary results are either *e* or *i* sentences because, involving simple conversion, panvalidity is preserved. This, of course, also applies to inconcludent patterns with *e* or *i* conclusions; paninvalidity in such cases is preserved. But what happens in the cases of an *a* or an *o* conclusion, say, in an argument fitting the premiss pattern of Barbara but with a converted conclusion, P*a*M, M*a*S | **S*a*P**, or one fitting the premiss pattern of Bocardo, P*o*M, S*a*M | **S*o*P**? In each of these two cases, representative of *a* and *o* sentences, panvalidity is ruled out. Neither an *a* sentence nor an *o* sentence admits of simple conversion: the *a* converts *per accidens*, the *o* does not convert. Thus, if either an *a* or an *o* sentence is a necessary result of a given concludent premiss-pair pattern, then, since neither kind of sentence converts simply, no argument fitting a concludent pattern whose conclusion is a conversion of the standard syntax is valid. These argument patterns are accordingly determined necessarily to be paninvalid. We need only refer to *Prior Analytics A2* on the conversion rules to grasp Aristotle's reasoning.[85]

Finally, it is not possible, by altering the standard syllogistic syntax by converting the conclusion pattern, to generate a panvalid argument pattern, that is, that a given inconcludent pattern become concludent. A given premiss-pair pattern is

[85] In *Pr. An. B22* Aristotle treats the matter of conversions but in a confusing way. R. Smith (1989: 216-219) is not happy with Aristotle's treatment of the matter and notes that in some ways he repeats what he accomplishes at *B5-7*.

either concludent or inconcludent. Notice that Aristotle encounters this matter with Camestres in the second and with Disamis in the third figure where in each instance he converts the derivation to re-establish the standard syntax. This indicates that for second and third figure argument patterns a conclusion conversion amounts to transposing the premisses and converting the major to the minor and the minor to the major term to re-establish the standard syllogistic syntax. In truth, doing this for second and third figure argument patterns amounts to treating every possible arrangement of two categorical sentences taken as premisses in those two figures. Consider, for example, the relationship between Cesare and Camestres and that between Disamis and Dimaris.

The same reasoning applies to the first figure in relation to the purported existence of a fourth, or indirect first, figure. Examining why this is so might help to reveal why Aristotle considers there to be only three figures. Each so-called fourth figure argument pattern with at least one convertible sentence pattern as a premiss is analyzable (*Pr. An. A45*) into a second or third figure pattern: into the second figure when the convertible sentence pattern is the minor premiss; into the third figure when the convertible sentence pattern is the major premiss. This accounts for 12 of the 16 possible premiss-pair patterns in the first figure. The patterns among these combinations are: Dimaris [Disamis (2)]; Fresison [Festino (2) or Ferison (3) or even Ferio (1)]; Camenes [Camestres (2) and with a weakened conclusion Camenop (Camestrop [2])]; and Fesapo [Felapton (3)] (see Parry 1991: 282-287). This leaves M*a*P, S*a*M |, M*a*P, S*o*M |, M*o*P, S*a*M |, M*o*P, S*o*M |. We treated M*a*P, S*a*M | above (this section) and saw that this pattern amounts to a simple conversion of the conclusion of Barbara, which is not logically possible. However, converting the conclusion of Barbara *per accidens*, which is logically possible, and transposing premisses produces Bramantip (similar to Darapti) — Barbarix is its counterpart in the first figure. Finally, it is clear to Aristotle that M*a*P, S*o*M |, M*o*P, S*a*M |, and M*o*P, S*o*M | amount to a premiss transposition required by converting the conclusions of first figure patterns. The most that might be obtained, then, is an *o* sentence. However, since an *o* sentence is not obtainable from the counterpart of these premiss-pair patterns in the first figure, these conclusion conversions, with their concomitant premiss transpositions, would also not be logically possible. Thus, seeing that Aristotle considers premiss-pair patterns, as discussed above, we can grasp his considering there to be only three figures.[86]

4 REFINING THE SET OF SYLLOGISM RULES

4.1 Establishing independence among deduction rules

As we remarked at the outset of this study (§1.2), the three different interpretations of Aristotle's logic are in significant agreement about the place of reduction in his system. In fact, they tend to consider the processes of completion

[86] L. Rose's discussion (1968: 57-79) of the fourth figure is very instructive.

(*teleiôsis*, *teleiousthai*), reduction (ἀναγωγή [*anagôgê*], ἀνάγειν [*anagein*]), and analysis (ἀνάλυσις [*analusis*], ἀναλυεῖν [*analuein*]) to be virtually identical.[87] It is peculiar that such different interpretations of a syllogism could produce such similar views about the *logical relationships* among them. If we allow for conflating an argument pattern or form (one traditionalist sense of 'syllogism') and a corresponding conditional sentence expressing such a pattern (the axiomaticist position), the similarity becomes more apparent.[88] The various interpreters hold that reduction amounts to deduction of some syllogisms, taken as derived, from others, taken as primitive, to form a deductive system. The traditionalist R. M. Eaton, for example, holds that reduction shows that "the validity of these [second and third figure] moods is *deducible from* that of moods in the first figure" (Eaton 1959: 123; author's emphasis) and that in general reduction is a process of transforming syllogisms (Eaton 1959: 86, 90, 126). J. Łukasiewicz (1958: 76; cf. 43-44) expresses an axiomaticist view that "reduction here means proof or deduction of a theorem from the axioms"; reduction is an indispensable process of deriving syllogistic theorems from axioms using an implicit propositional logic. J. Corcoran, a deductionist, writes in a similar vein that "'reduce to' *here* means 'deduce by means of'"(1974: 114; author's emphasis).[89]

However, when we consider Aristotle's treatment of reduction at *Prior Analytics A7* we discover that he distinguishes the process of completion from that of reduction: "it is *also* [κὰι] possible to reduce all the syllogisms to [ἀναγαγεῖν πάντας τοὺς συλλογισμοὺς ἐις] the universal syllogisms in the first figure" (29b1-2; cf. 29a30-29b2). Aristotle expressed the distinction more forcefully at *A23*.

> It is clear from what has been said, then, that the syllogisms in these figures [viz. the second and third figures] are both completed by means

[87] J. Łukasiewicz, for example, explicitly considers '*anagein*' (to reduce) and '*analuein*' (to analyze) to be synonyms (1958: 44), and this is the case also with R. Smith (1989: 161), J. Corcoran (1981: 6), J. W. Miller (1938: 25), and L. Rose (1968: 55). It has been customary in all three interpretations to conflate analysis, reduction, and completion. Consider, for example, W. D. Ross' commentary on *A7* in relation to *A4-6* (1949: 314-315) and on *A45* (1949: 417-418); J. N. Keynes' (1906: 318-325) and R. M. Eaton's (1959: 86, 90, 122-124, 128-131) remarks that transforming and deducing syllogisms amount to reduction; and G. Patzig's (1968: 134-137) similar position. Both Patzig (1968: 135) and Eaton (1959: 109) refer to the second and third figure syllogisms as being "disguised" first figure syllogisms. See also Smith's commentaries on *A7* (1989: 118-119; cf. 1986: 58-59), on *A23* (1989: 141), and on *A45* (1989: 177).

[88] Consider A. N. Prior's assessment of J. Łukasiewicz's thinking (1955: 116-117) and J. W. Miller's treatment of the system (1938: ch. 3; cf. 11-14). Miller, in fact, writes (1938: 14, 25, 28) that he is applying the postulational method of Aristotle himself and "merely carr[ying] to its completion an undertaking which Aristotle himself began". Also consider W. T. Parry's treatment of Aristotle's "deductive system" (1991: ch. 20; cf. 520n2, 521n9) and J. Corcoran's remarks (1983) on a connection between a Gentzen-sequent natural deduction system and J. Łukasiewicz's axiomatic system.

[89] J. Corcoran refers to *Pr. An. A7*: 29b1-2 and cites it as follows (1974: 114): "It is possible also to reduce all syllogisms to the universal syllogisms in the first figure". Note Corcoran's change of position on the matter of reduction as expressed in 1974 and later in 1981 and 1983. His earlier position did not especially vary from either the traditionalists' or T. Smiley's views. Later he considered reduction to be "heuristically perverse" insofar as determining validity is concerned (1981: 4-5).

> of [τελειοῦνται τὲ διὰ] the universal syllogisms in the first figure and reduced to them [κὰι ἐις τούτους ἀνάγονται]. (40b17–19)

The topic of reduction is introduced and concluded at *A7* by using the verb '*anagein*', which we translate by 'to reduce'. What seems to have confused interpreters is Aristotle's treating the reduction of syllogisms by using the verbs '*teleiousthai*', '*epiteleisthai*', and 'δέιχνυσθαι' (*deiknusthai*), but not '*sullogizesthai*', exactly as he treats the syllogisms at *A5-6*. Here is what he writes in *A7*, for example, about second figure reduction.

> It is evident that those in the second figure are completed [τελειοῦνται] by means of these [universal first figure] syllogisms, although not all in the same way; the universal syllogisms are completed when the privative premiss is converted, but each of the partial syllogisms is completed through leading away to an absurdity. (29b2-6; see 29b6 8 for the first figure and 29b19-24 for the third figure)

Indeed, Aristotle provides only two actual illustrations of reduction, those of Darii (29b8-11) and Ferio (29b11-15), and these reductions are expressed in *exactly* the same manner as the deductions in *A5-6*, even using the language of completion. Thus, some logicians have taken him not to distinguish two processes but to duplicate in *A7* the project of *A4-6*. However, Aristotle is not here concerned to demonstrate the truth or falsity of a given sentence nor the validity or invalidity of a given argument. Nor is he concerned here to show that a given argument pattern is panvalid as at *A5-6*. Rather, he is now concerned to demonstrate that any sentence fitting any one of the four categorical sentence patterns (i.e., any *problêma*) can be established to be a logical consequence (conclusion) of other categorical sentences by using only the two patterns of the universal *teleioi sullogismoi* as deduction rules. To do this, it is true, he performs deductions as he does at *A5-6*, or he refers to those already performed there. But now he has a different objective in examining the relationships among the patterns which are used by him as rules: namely, to simplify his deduction system.[90]

Aristotle first treats the reduction of the four second figure panvalid patterns, and he treats them in a manner that suggests his readers' familiarity (29b2-6). Again, he does not perform deductions here but refers us to *A5* where he had *already* established their panvalidity by using the first figure *teleioi sullogismoi*

[90]Consider, for example: *On Generation and Corruption* 330a24-25 on reduction of contraries to two pairs, hot-cold, dry-moist (cf. *Physics* 189b26-27, *Meta.* 1004b27-1005a5, 1036b21-22, 1061a1-2, 13-14); *Movement of Animals* 700b18-19 on reducing to thought and desire such sources of movement as intellect, imagination, appetite, etc. We can illuminate Aristotle's use of '*anagein*' at *A7* by citing a rather exact analogue in his discussion of locomotion in *Physics 7.2*. There Aristotle writes that all forms of locomotion caused by something other than the object in motion are reducible to (ἀνάγονται εἰς) four kinds, namely, to pulling, pushing, carrying, and twirling (243a16-18); and he even reduces (ἀνάγειν) carrying and twirling to pulling and pushing (243b15-17). There are numerous instances of similar usage in Aristotle's writings, all of which concern identifying what amount to the special principles (ἴδιαι ἀρχαί) of a given subject matter.

patterns, Barbara and Celarent, as rules in the completion process. He writes only that:

> It is evident that those in the second figure are completed through these syllogisms, although not all in the same way; the universal syllogisms are completed when the privative premiss is converted, but each of the partial syllogisms is completed through leading away to an absurdity. (29b2-6)

At *A7* both Festino and Baroco are completed using *reductio* proof, where in *A5* only Baroco was treated this way. With this established he could proceed to the two partial patterns of the first figure, Darii and Ferio, which are reduced through second figure patterns. Since, in order to show that only the two universal first figure patterns are sufficient, he spends considerably more time on the reduction of the two partial patterns (29b6-19) of the first figure. He concludes (29b19-25) with the reduction of the six third figure patterns in the same manner as he treated the second figure patterns. Table 36 summarizes what Aristotle writes about reduction. Table 35 provides their deductions (completions).

Table 35. Summary of Aristotle's texts on each panvalid pattern in his treatment of reduction

Pattern considered	Manner of the completion	Pattern used in the completion
Barbara:	[completed (*teleiousthai*) probatively through itself δί αὑτόυ].	Barbara
Celarent:	[completed probatively through itself].	Celarent
Camestres:	completed probatively *dia* Celarent (29b2–6; cf. *A5*, 27a9–14).	Celarent
Cesare:	completed probatively *dia* Celarent (19b2–6; cf. *A5*, 27a5–9).	Celarent
Festino:	completed by leading to an impossibility (*reductio*) *dia* Celarent (19b2–6; cf. A5, 27a32–36).	Celarent
Baroco:	completed by *reductio dia* Barbara (29b8–11, 15–19)	Barbara
Darii:	*proved* (*deiknusthai*) by *reductio dia* Camestres; Camestres completed probatively *dia* Celarent (19b11–15, 15 19).	Celarent
Ferio:	*proved* by *reductio dia* Cesare; Cesare completed probatively *dia* Celarent (29b11–15, 15 19).	Celarent
Darapti:	Completed probatively *dia* Darii; Darii proved by *reductio dia* Camestres; Camestres completed probatively *dia* Celarent (19b21–25; cf. *A6*, 28a17–22).	Celarent
Datisi:	same as Darapti (19b21–25; cf. *A6*, 28b11 15).	Celarent
Disamis:	same as Darapti (19b21–25; cf. *A6*, 28b11–15).	Celarent
Felapton:	completely probatively *dia* Ferio; Ferio proved by *reductio dia* Cesare; Cesare completed probatively *dia* Celarent (19b21 25); cf. *A6*, 28a26–30).	Celarent
Ferison:	same as Felapton (29b21 25; cf. *A6*, 28b33–35).	Celarent
Bocardo:	completed by *reductio dia* Barbara (29b21 - 25; cf. *A6*, 28b17–20).	Barbara

Table 36. Completions relating to Aristotle's reduced ded

Barbara		Celarent		Darii 29b6–19		Ferio 2
1. A*a*B		1. A*e*B		1. A*a*B		1. A*e*
2. B*a*C		2. B*a*C		2. B*i*C		2. B*i*C
? A*aC*		? A*e*C		? A*i*C		? A*o*C
3. A*a*C	1,2 **Barb**	3. A*e*C	1,2 **Celar**	3. A*e*C	assume	3. A*a*
				4. C*e*A	3e-con	4. B*e*
				5. A*a*B	1 rep	5. A*a*
				6. C*e*B	4,5 **Celar**	6. B*e*
				7. B*e*C	6 e-con	7. B*e*
				8. B*e*C & B*i*C	7,2 conj; X	8. A*o*
				9. A*i*C		
					3–8 reduct	
Camestres		Cesare		Festino		Barocc
1. B*a*A		1. B*e*A		1. B*e*A		1. B*a*
2. B*e*C		2. B*a*C		2. B*i*C		2. B*o*
? A*e*C		?A*e*C		? A*o*C		?A*o*C
3. C*e*B	2 e-con	3. A*e*B	1 e-con	3. A*a*C	assume	3. A*a*
4. B*a*A	1 rep	4. B*a*C	2 rep	4. B*e*A	1 rep	4. B*a*
5. C*e*A	3,4 **Celar**	5. A*e*C	3,4**Celar**	5. A*a*C	3 rep	5. A*a*
6. A*e*C	5 e-con			6. B*e*C	4,5 **Celar**	6. B*a*
				7. B*e*C & B*i*C	6,2 conj; X	B*a*C&
					3–7 reduct	8. A*o*
				8. A*o*C		

Darapti

1. A*a*B	
2. C*a*B	
? A*i*C	
3. A*e*C	assume
4. C*e*A	3 e-con
5. A*a*B	1 rep
6. C*e*B	4,5 **Celar**
7. C*e*B& C*a*B	6,2 conj; X
8. A*i*C	3–7 reduct

Datisi

1. A*a*B	
2. C*i*B	
? A*i*C	
3. A*e*C	assume
4. C*e*A	ee-con
5. A*a*B	1 rep
6. C*e*B	4,5 **Celar**
7. C*e*B & C*i*B	6,2 conj; X
8. A*i*C	3–7 reduct

Disamis

1. A*i*B	
2. C*a*B	
? A*i*C	
3. A*e*C	assume
4. C*a*B	2 rep
5. A*e*B	3,4 **Cela**
6. A*e*B & A*i*B	5,1 conj;
7. A*i*C	3–6 redu

Ferison

1. A*e*B	
2. C*i*B	
? A*o*C	
3. A*a*C	assume
4. B*e*A	1 e-con
5. A*a*C	3 rep
6. B*e*C	4,5 **Cela**
8. C*e*B & C*i*B	7,2 conj;
9. A*o*C	3–8 redu

By treating *each case* of a panvalid pattern — and this amounts to treating all possible combinations of concludent premiss-pair patterns — Aristotle established the deductive preeminence of the patterns of the two universal syllogisms, Barbara and Celarent, as the only syllogism rules necessary in his deduction system. He demonstrated not only that second and third figure patterns are redundant deduction rules, which was implicitly established at *Prior Analytics A4-6*, but also that the two partial patterns of the first figure are equally redundant. The *same deductive results* are accomplished using only the universal patterns of the first figure. Thus, Aristotle's reduction of syllogistic patterns at *A7* is not a substitute for syllogistic deduction nor a process for axiomatizing a system of logic as axiomaticists hold, but a metalogical process for establishing the independence of a small set of deduction rules.

4.2 Analysis distinct from reduction

Aristotle treated analysis at *Prior Analytics A45*. At the outset of *A45* he used '*anagein*' as a synonym for '*analuein*' (50b5-9) in a way apparently inconsistent with what he wrote at *A7* and *A23*: "not all syllogisms can be reduced ... but only some [οὐχ ἅπαντας δὲ ἀλλ' ἐνίους]". Recall that at *A7* he showed that all the syllogisms could be reduced. We are thus struck by an apparent limitation announced here. However, while Aristotle treated the analyses of first into second and second into first figure syllogisms (50b5-30) by using '*anagein*', he abruptly switched at 50b30-32 (re Baroco) to use '*analuein*' and continued to do so throughout *A45* respecting all the other possible analyses (50b30-51a39). In addition, Aristotle clearly distinguished analysis from reduction at *A45* as he distinguished reduction from completion at *A7*: "... it is evident how one must reduce syllogisms [πῶς μὲν οὖν δεῖ τοὺς συλλογισμοὺς ἀνάγειν], and that the figures are analyzed into one another [καὶ ὅτι ἀναλυέται τὰ σχήματα ἐις ἄλληλα]" (51b3-4).

Aristotle established the scope of analysis in the following way. While every syllogism is reducible (to) (29b1-2), not every syllogism is analyzable (into). Passages in *A45* where Aristotle specifically stated that no analysis of one syllogism into another is possible include: 50b8-9, 50b18, 50b30-32, 50b33-34, 51a1-3, 51a18-19, 51a27, 51a31-32, 51a37-39, and 51a40-41. This is sufficient to establish a difference between the two processes. By examining each case, we can extract Aristotle's two rules for analyzing syllogisms.

1. A given syllogism in one figure is analyzed into a syllogism in another figure whenever both syllogisms prove the same *problêma*, that is, each syllogism proves a sentence fitting the same pattern, whether an *a*, *e*, *i*, or *o* sentence pattern (50b5-8).

Aristotle is writing metalogically here. Thus, for example, there is no analysis of a syllogism fitting Barbara into a syllogism fitting a pattern in any other figure, nor is there of one fitting Darii into one fitting a second figure pattern.

2. A given syllogism fitting a pattern in one figure is analyzed into a syllogism fitting a pattern in another figure by using only conversion and premiss transposition (51a22–25).

Thus, for example, neither a syllogism fitting Baroco nor one fitting Bocardo can be analyzed.

Aristotle treated the possible analysis of almost every syllogism and he identified which are not analyzable, either because conversion does not produce a syllogism or because the same *problêma* is not proved in each figure. Table 37 summarizes Aristotle's results at *Prior Analytics A45*.[91]

Table 37.

Summary of the analyses treated in Prior Analtyics A45	
Barbara	No analysis possible
Celarent	Into Cesare (50b9-13) nto Camestres (not treated)[c]
Darii	Into Disamis (not treated)[c] Into Datisi (50b33-38
Ferio	Into Festino (50b13-16) Into Ferison (50b38-40)
Cesare	Into Celarent (50b17-21)
Camestres	Into Celarent (50b21-25)[c]
Festino	Into Ferio (50b25-30) Into Ferison (51a26-30)
Baroco	No analysis possible
Darapti	Into Darii (51a3-7
Felapton	Into Ferio (51a12-15) Into Festino (51a34-37)
Disamis hline Datisi	Into Darii (51a8-12)[c] Into Darii (51a7-8)
Boardo	No analysis possible
Ferison	Into Ferio (51a15-18) Into Festino (51a34-37)

Aristotle treated analysis in *Prior Analytics A45* quite differently from reduction in *A7* and from completion at *A5-6*, both of which involve a deduction process. Here there is no direct concern with a process of deduction to show that a given premiss-pair generates a syllogism nor that a conclusion follows from premisses

[91]Table 37 uses the following notation: superscript c = premiss transposition and conclusion conversion.

Aristotle treats neither analyzing Darii and Disamis into each other nor Celarent and Camestres into each other. This is simply an oversight.

nor that some patterns are redundant as deduction rules. Neither probative nor *reductio* proofs are cited in relation to analysis, although Aristotle always preserved strict syllogistic syntax through conversion in the case of some conclusion patterns.

Characteristically Aristotle conceived of analysis as one syllogism being transformed into another. In fact, he referred to the process of analysis as a μετάβασις, a transition or *transformation* (51a24-25). Analyses of syllogisms occur between any of the figures. Thus, no syllogistic pattern nor any figure has preeminence in relation to analysis as is the case with completion and reduction. That Aristotle used '*anagein*' in an apparently contrary way at the outset of *A45* is mitigated when once we view the texts of *A7* and *A45* more globally. Aristotle unequivocally distinguished the process of reduction from that of analysis.[92]

5 CONCEPTS IN ARISTOTLE'S LOGIC

While modern logicians believe that Aristotle developed a logic that contains a notion of *formal deducibility* — and their mathematical models establish that this is so — they do not believe that he explicitly formulated this notion, either in general of deductive systems or specifically of his syllogistic system. In addition, it might seem that Aristotle did not define 'following from necessity', τὸ ἐξ ἀνάγκης συμβάινειν, his expression for *logical consequence*, and then show that the syllogisms are true to it. Previous interpreters believed that he only posited the four *teleioi sullogismoi* and then 'reduced' the validity of the others to them as 'principles'. However, modern interpreters have tended not to place Aristotle's logical investigations in *Prior Analytics* into the larger context of his other works relating to logic. Surely what he wrote in *Prior Analytics* is the product of considerable intellectual exploration. Accordingly, then, we might expect to find his thinking about logical consequence elsewhere in the larger corpus and then piece together an account that accommodates modern logicians. In particular, what Aristotle expressed about logical necessity in *Metaphysics* and in *Prior Analytics* comports exactly with his treatment of deducibility in *Prior Analytics*. Below we first extract Aristotle's syntactic notion of formal deducibility (§5.1) and then his semantic notion of logical consequence (§5.3). Two kinds of deduction also are represented (§5.2).

5.1 Aristotle's notion of formal deducibility

The modern notion of formal deducibility, or formal derivability, can be stated as follows:

[92] While it may seem unbecoming to take Aristotle's expression at 50b5-30 as a lapse in precision not later amended, we believe this to be so and that our interpretation does not do violence to Aristotle's meaning.

> A given sentence c is formally deducible from a given set of sentences P when there exists a finite sequence of sentences that ends with c and begins with P such that each sentence in the sequence from P is either a member of P or a sentence generated from earlier sentences solely by means of stipulated deduction rules.

The salient features of this notion pertain to a chain of reasoning from premisses to conclusion such that the chain sequence (1) is finite and (2) might use (a) repetition and (b) stipulated deduction rules. In addition such a chain is cogent in context. Now, it is evident from his discussions in *Prior Analytics* and *Posterior Analytics* that Aristotle subscribed to a notion of formal deducibility remarkably similar to this modern formulation. Moreover, while Aristotle did not express this notion in one rigorously constructed sentence, he nevertheless provided many statements in *Prior Analytics* and in *Posterior Analytics* from which we can extract his own understanding. Here again we take his several statements and organize them according to a modern practice.

Some logicians believe that the closest Aristotle came to defining 'deduction' *per se* is his definition of 'syllogism' at the outset of *Prior Analytics.* He writes that a syllogism (*sullogismos*) is

> a discourse [λόγος] in which, certain things having been supposed [posited or *taken*], something different from the things supposed [posited or *taken*] *follows* of necessity because these things are so [ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβάινει τῷ ταῦτα ἔιναι]. (24b18 20)[93]

Aristotle immediately continues by defining certain aspects of this definition.

> By 'because these things are so' I mean 'resulting through them [τὸ διὰ ταῦτα συμβάινειν]', and by 'resulting through them' I mean 'needing no further term from outside in order for the necessity to come about [τὸ μηδενὸς ἔξωθεν ὅρου προσδεῖν πρὸς τὸ γενέσθαι τὸ ἀναγκαῖον]'. (24b20 22)

However, Aristotle seems here rather more to define 'valid argument' than 'deduction'; there is no strong indication of an epistemic *process* present in a syllogism. We have already seen that he understood his project in *Prior Analytics A5-6* as establishing which patterns of two categorical sentences when taken together as premisses result in a syllogism. Aristotle's logical methodology is to perform deductions in the metalanguage of *Prior Analytics* to establish that certain argument patterns are panvalid patterns.[94] He then proceeds to use these patterns in a deduction process. This being so, we conclude that a syllogism, at least in this

[93]Aristotle defines 'syllogism' in much the same way at three other places: *Top.* 100a25-27, *SR* 164b27-165a2, and *Rh.* 1356b16-18.

[94]In this connection, consider the following passages: in *Pr. An. A5*: 27a12-13, 14, 14-15, 16-18, 36, 27b3, 28a2-7; in *A6*: 28a22, 22-23, 28-30, 28b13-15, 20-21, 34-35, 29a15-16.

discussion, is a valid argument, one that might, to be sure, be used in a deduction process, but which is not itself a deduction. Again, we understand him as securing a set of deduction rules. The corroborating evidence for this interpretation is Aristotle's distinguishing syllogisms into those that are *teleios*, usually translated by 'complete' or 'perfect', and those that are *atelês*, usually translated by 'incomplete' or 'imperfect'; '*dunatos*', translated by 'potential', serves as a synonym for '*atelês*'. His definitions of '*teleios*' and '*atelês*' in *Prior Analytics A1* and his references to the *teleioi sullogismoi* help to secure our interpretation.

> I call a syllogism *complete* [τέλειον] if it stands in need of nothing else besides the things taken in order for the necessity *to be evident* [πρὸς τὸ φανῆναι τὸ ἀναγκαῖον]; I call it *incomplete* [ἀτελῆ] if it still needs either one or several additional things which are necessary because of the terms assumed, but yet were not taken as premisses. (24b22–26)

Of course, '*teleios*' and '*atelês*' are *epistemic* notions and refer to evidency of validity. From *Prior Analytics A4*, *A5-6*, and *A7*, then, we notice that Aristotle, besides using two formal processes of deduction (direct and indirect; §5.2), also identifies two degrees of someone's *recognizing* validity, that is, in connection with the syllogisms, of experiencing a mental act by which someone grasps that extreme terms are mediated by a middle term. (1) A syllogism being *atelês* means that a participant has to go through a number of deductive steps, more than one, to recognize its validity. Being *dunatos* means that it is possible to recognize its validity. (2) A syllogism being *teleios* means that a participant has to go through only one step to recognize the validity. At places Aristotle helps us to understand the meaning of '*teleios*' by referring to the evidency of the necessity of the conclusion following logically from the premisses of a first figure syllogism as 'δἰ αὑτοῦ', or through itself. In *Prior Analytics A7* he writes of the two partial syllogisms of the first figure (viz., Darii and Ferio), and we interpolate to include the two universal syllogisms (Barbara and Celarent) as well: " the partial syllogisms in the first figure *are brought to completion through themselves* [οἱ δ' ἐν τῷ πρώτῳ, οἱ κατὰ μέρος, ἐπιτελοῦνται μὲν καὶ δἰ αὑτῶν]" (29b6–7)[95]

Recognizing validity in the case of a first figure syllogism requires only one step, more steps in the case of a second or third figure syllogism, and, of course, even more steps in a case of a valid argument with more than two premisses. Thus, the manner of Aristotle's discussion in *Prior Analytics A5-6* and *A7* shows that he understands the process of deduction to establish, or to make evident in the mind of a participant, that a given argument is valid — or, as the case may be, that a given argument pattern is panvalid. And, moreover, he uses the syllogism

[95] In contrast, consider: "all [second figure syllogisms] are completed [ἐπιτελοῦνται] by taking in addition certain things"(*Pr. An. A5*: 28a5-6); "all [third figure syllogisms] are completed [τελειοῦνται] by taking certain things in addition"(*A6*: 29a15-16); and "all [first figure syllogisms] are completed by means of the things initially taken [πάντες γὰρ ἐπιτελοῦνται διὰ τῶν ἐξ ἀρχῆς ληφθεντων]"(*Pr. An. A4*: 26b28-33)] or "are completed through themselves" (*Pr. An. A7*: 29b6-8).

patterns in this process. We can illuminate this with the following illustrations (Table 38). We refer to two-premiss arguments, but this applies *mutatis mutandis* to one-premiss arguments.

Table 38.

1	2	3	4
1. Animal [A] belongs to every mammal [M]. 2. Mammal belongs to every dog [D]. ? Animal belongs to every dog.	1. A*a*M 2. M*a*D ? A*a*D	1. P*a*M 2. M*a*S ∴ P*a*S	1. A*a*M 2. M*a*D ? A*a*D 3. A*a*D 1, 2 Barbara
5	6	7	8
1. Animal [A] belongs to every mammal [M] 2. Animal belongs to no line [L]. ? Mammal belongs to no line.	1. A*a*M 2. A*e*L ? M*e*L	1. M*a*P 2. M*e*S ∴ P*e*S (Camestres)	1. A*a*M 2 A*e*L ? M*e*L 3. L*e*A 2*e*-conversion 4. A*a*M 1, repetition 5. L*e*M 3, 4 Celarent 6. M*e*L 5*e*-conversion

Item 1 is an object language argument (premisses numbered and the conclusion indicated by '?') whose validity, while perhaps obvious, let us take as unknown. Item 2 is the same object language argument but whose non-logical constants (terms) have been abbreviated with letters to facilitate recognizing the pattern of and working with the argument. Item 3 is a metalogical object, in this case a schematic representation of the panvalid argument pattern named 'Barbara'.[96]

[96] Here the letters are schematic letters and the line for the 'conclusion' is indicated by the '∴', which signals that a sentence fitting a conclusion pattern has been established always to follow logically from sentences fitting the premiss pattern. Any semantically precise instance of this

Item 4 is an object language deduction, albeit quite simple. In this case, the conclusion of the original argument (item 1) has been established to follow logically, or to be valid; this is indicated by the fourth line (numbered '3') with the reasoning, or explanation, provided to the right. Using Aristotle's nomenclature, item 1 is a *teleios sullogismos.* Likewise in this respect, item 5 is an *atelês sullogismos* and, thus, accordingly, it requires a deduction to establish its validity (item 8).

Of course, a conversion is not a syllogism according to Aristotle because it consists in only one premiss. Still, he understood a conversion necessarily to result in something different. In *Prior Analytics B1*, for example, he writes that each syllogism, save for that whose conclusion is an *o* sentence, has several *different* results:

> If A has been proved to belong to every B or to some, then it is also necessary for B to belong to some A; and if A has been proved to belong to no B, then neither does B belong to any A (and this conclusion is *different* [ἕτερον] from the previous one). (53a10-12)

This, of course, applies also to the subalterns. His taking the conclusions of conversions as different in *Prior Analytics A* is obvious, if not stated as directly as it is here. In any case, it is evident that Aristotle took a conversion and a syllogism equally as species of the same genus, namely, as instances of valid arguments. His purpose in *Prior Analytics* is to identify the panvalid argument patterns relating to converting two terms in a single sentence and to those relating three terms in two sentences because he recognized the epistemic efficacy of such elementary patterns in the deduction process - namely, he recognized the rule nature of such patterns.

We now have some notion of Aristotle's understanding of deduction, but we seem to lack his word for 'deduction'. Jonathan Barnes (1981: 21-25) has suggested 'ἀναγκαῖον' (*anankaion*). While Aristotle frequently uses '*anankaion*' as an adverb, translated by 'necessarily', he also often uses it as a substantive, 'τὸ ἀναγκαῖον', as in the following passage in *Prior Analytics A32*, rendered according to Barnes.

> ... that the syllogism is also a deduction [ὅτι καὶ ὁ συλλογισμὸς ἀναγκαῖόν ἐστιν] since deduction is more extensive than syllogism [ἐπὶ πλέον δὲ τὸ ἀναγκαῖον ἢ ὁ συλλογισμός]; for every syllogism is a deduction, but not every deduction is a syllogism [ὁ μὲν γὰρ συλλογισμὸς πᾶς ἀναγκᾶιον, τὸ δ' ἀναγκᾶιον οὐ πᾶν συλλογισμός]. (47a32–35; see 47a31-40)

His use of '*to anankaion*' here seems to indicate that a conversion and a syllogism are both valid arguments, and perhaps that syllogistic reasoning is only one among other kinds of deductive reasoning.

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argument pattern is a valid argument. This universality, along with its elementary nature, is why such patterns are employed as rules in a deduction process. Cf. above §3.2.

Grasping Aristotle's understanding of deduction can be enhanced by examining instances of his performing deductions and by studying the verbs he used to characterize the deduction process, in particular, the verbs '*epiteleisthai*' or '*teleiousthai*', as well as 'περάινεσθαι' (*perainesthai*), and even '*deiknusthai*'. Examining parts of *Prior Analytics A7* helps in this connection. Consider the following passage (underscoring for comparison).

> It is furthermore evident that all [1] the incomplete deductions [2] are completed through the first figure. For they all [3] come to a conclusion either probatively or through an impossibility, and in both ways the first figure [4] results. For those [5] completed probatively, this results because they all [6] come to a conclusion through conversion, and conversion [7] produces the first figure. And for those [8] proved through an impossibility, it results because, when a falsehood is supposed, the [9] deduction [10] comes about through the first figure. (29a30-36)

This is R. Smith's (1989) translation, which has considerably contributed to raising respect for Aristotle's acumen as a logician. Here is the complete Greek of Aristotle's text.

> Φανερὸν δὲ καὶ ὅτι πάντες [1] οἱ ἀτελεῖς συλλογισμοὶ [2] τελειοῦνται διὰ τοῦ πρώτου σχήματος. ἢ γὰρ δεικτικῶς ἢ διὰ τοῦ ἀδυνάτου [3] περαίνονται πάντες. ἀμφοτέρως δὲ [4] γίνεται τὸ πρῶτον σχῆμα, δεικτικῶς μὲν [5] τελειουμένων, ὅτι διὰ τῆς ἀντιστροφῆς [6] ἐπεραίνοντο πάντες, ἡ δ' ἀντιστροφὴ τὸ πρῶτον [7] ἐπόιει σχῆμα, διὰ δὲ τοῦ ἀδυνάτου [8] δεικνυμένων, ὅτι τεθέντος τοῦ ψεύδους [9] ὁ συλλογισμὸς [10] γίνεται διὰ τοῦ πρώτου σχήματος.

Now, in light of our comments above, we gloss the text to provide a more faithful translation, even if awkward, to render Aristotle's meaning.

> It is also clear that all [1] the valid syllogistic arguments whose validity is not apparent [2] are shown to be valid by means of the first figure. For, either probatively or by means of *reductio*, they are all [3] drawn to a conclusion <by means of a deduction process>. In both cases, <a syllogism in> the first figure [4] arises <in the deduction process to establish validity>. Of those whose [5] evidency of validity is established probatively, because they all are [6] drawn to a conclusion <through a deduction process> by means of conversion, and conversion [7] produces <a syllogism in> the first figure; of those [8] proved by means of *reductio*, because [5] <evidency of validity is established> by assuming a falsehood [9] a syllogism in the first figure [10] arises <in the deduction process>.

In *Prior Analytics A7*: 29b1-8, as well as in *A5-6*, Aristotle used '*teleiousthai*' and '*epiteleisthai*' in exactly the same way: namely, to indicate making evident the validity of a valid argument specifically by means of generating a *teleios sullogismos* in a chain of reasoning. This signals in the mind of a participant, equally today as then, that the chain of reasoning in which a *teleios sullogismos* arises is cogent in context, and thus it links the conclusion sentence as following logically, or necessarily, from the premiss sentences. It is obvious that Aristotle used the first figure panvalid patterns as rules in the deduction process to establish knowledge of validity. While a syllogism is a valid elementary argument, its panvalid pattern can serve as a deduction rule since every instance is valid. The same thinking applies to the conversion rules established in *Prior Analytics A2*: a conversion is a valid one-premiss argument, its panvalid pattern serves as a deduction rule.

The elements of syllogistic deductive reasoning, then, consist in three one-premiss conversion rules and 14 two-premiss syllogism rules, reduced to two in *Prior Analytics A7*. Thus, in respect of his *syllogistic* deduction system, a deduction preeminently involves a chain of reasoning in the mind of a participant that establishes a conclusion sentence to be a logical consequence of a set of premiss sentences — a deduction makes validity, or following necessarily, evident. In the deduction process a participant might use any of the conversion and syllogism rules as well as repetition, an implicit and often used rule.[97] This is what Aristotle means in *Prior Analytics A25*: 42a35-36 by 'ὁ λόγος συλλογισμός', a syllogistic argumentation, or reasoning syllogistically (συλλογιστικῶς). In fact, Aristotle is rather emphatic about this. While it is true that every syllogism has only three terms and two *protaseis*, it is just as true that a syllogistic argumentation is not restricted to two premisses. Rather a syllogistic argumentation consists in chaining syllogisms that are instances of the rules Aristotle articulated in *Prior Analytics A4-6*. Aristotle used 'συστοιχία' (*sustoichia*) to capture the notion of 'chaining' syllogisms.[98] And so, just at the places where he restricts a syllogism to two premisses and three terms, he also writes:

> ... unless the same conclusion comes about [τὸ αὐτὸ συμπέρασμα γίγνηται] through different groups of premisses. (*Pr. An. A25*: 41b37-38)
> ... unless something should be taken in addition for the purpose of completing the deductions [πρὸς τὴν τελέιωσιν τῶν συλλογισμῶν]. (*Pr. An. A25*: 42a33-35)

Aristotle explicitly took up chaining syllogisms in *Prior Analytics A25*: 42b1-26, and he provided many examples of this process in both *Prior Analytics* and *Posterior Analytics*. At *Prior Analytics A25* he wrote about "counting syllogisms" and "prior syllogisms" (προσυλλογισμοί), about counting terms (ὅροι),

[97] See, e.g., in *Pr. An. A5*: 27a7-8, 11 and in *A6*: 28a20-21.

[98] LSJ cites 'series' and 'column' as definitions of 'συστοιχία'. J. Barnes (1994) translates 'συστοιχία' by 'chain': e.g., *Po.An.* 79b7, 8-9, 10, 11, 80b27, and 81a21. Also consider Aristotle's using 'συναπτός', which LSJ defines as 'joined together' or 'linked together'. R. Smith (1989) translates this by 'connected' and 'ἀσύναπτος' by 'unconnected': e.g., *Pr. An.* 41a1, 19, 42a21, 65b14, 33, and 66b27.

counting premisses and intervals (προτάσεις and διαστήματα), and counting conclusions (συμπεράσματα). We cite *Posterior Analytics A25*, where Aristotle treats the superiority of a demonstration having fewer rather than more terms, as an example of his providing an instance of chaining syllogisms.

> ... then let one demonstration show that A holds of E through the middle terms B, C, D, and let the other show that A holds of E through F, G. Thus that A holds of D and that A holds of E are on a level. But that A holds of D is prior to and more familiar than that A holds of E; for the latter is demonstrated through the former, and that through which something is demonstrated is more convincing than it. (86a39-86b4; cf., e.g., in *Pr. An. A25*: 41b36-42a5 & *A28*: 44a11-44b20)

Thus, we understand that central to syllogistic deductive reasoning is generating syllogisms as part of an epistemic process. Aristotle summarizes this notion in *Prior Analytics A29*.

> It is evident from what has been said, then, not only that it is possible for all syllogisms to come about through this route, but also *that this is impossible through any other*. For every syllogism has been proved to come about through some one of the [three] figures stated previously, and these cannot be constructed except through the things each term follows or is followed by (for the premisses and the selection of a middle is from these, so that it is not even possible for a syllogism to come about through other things).[99] (45b36-46a2)

It is useful at this juncture, in connection with the syntactic foundations of syllogistic deduction, to refer back to *Prior Analytics A4-6* where Aristotle established all the panvalid two-premiss patterns. This helps to amplify textually his notion of formal deducibility. Respecting the two-premiss syllogism rules, from which all extended deductive discourses are constructed, Aristotle summarized a constituent part of his notion of deducibility five times in *Prior Analytics A4-6*, in respect of each of the three figures.

> Thus, it is clear when there will and when there will not be a syllogism in this [the first] figure if the terms are universal; and it is also clear both that if there is a syllogism, then the terms must necessarily be related as we have said, and that if they are related in this way, then there will be a syllogism. (*A4*: 26a13-16)

[99]Aristotle had already expressed this in *Pr. An. A27*: "From what had been said, then, it is clear how every syllogism is generated, both through how many terms and premisses and what relationships they are in to one another, and furthermore what sort of problem is proved in each figure, and what sort in more and what in fewer figures.... For surely one ought not only study the origin of syllogisms [43a22-23; cf. 47a2-4], but also have the power to produce them" (43a16-24).

> It is evident from what has been said, then, that if there is a partial syllogism in this [the first] figure, then it is necessary for the terms to be related as we have said (for when they are otherwise, a syllogism comes about in no way). (*A4*: 26b26-28)

> It is evident, then, that if there is a syllogism with the terms universal [in the second figure], then it is necessary for the terms to be related as we said in the beginning. For if they are otherwise, a necessary result does not come about. (*A5*: 27a23-25).

> From what has been said, then, it is evident both that a syllogism comes about of necessity if the terms are related to one another as was stated, and that if there is a syllogism, then it is necessary for the terms to be so related [in the second figure]. (*A5*: 28a1-3)

> It is evident in this [the third] figure, then, when there will and when there will not be a syllogism, and it is evident both that if the terms are related as was said, then a syllogism comes about of necessity, and that if there is a syllogism, then it is necessary for the terms to be so related. (*A6*: 29a11-14)

In each case Aristotle refers his readers to *the necessary and sufficient syntactic conditions* for a syllogism. He alludes to the panvalid patterns that he has identified as deduction rules.

Perhaps the best treatment of deducibility *per se* in Aristotle's logical investigations is contained in *Prior Analytics A23*, a chapter some logicians believe contains Aristotle's attempt at a completeness proof.[100] He begins this chapter by affirming that

> every demonstration, and every deduction, *must prove* something either to belong or not to belong [ἀνάγκη δὴ πᾶσαν ἀπόδειξιν καὶ πάντα συλλογισμὸν ἢ ὑπάρχον τι ἢ μὴ ὑπάρχον δεικνύναι], and this either universally or partially ... either probatively or through an absurdity. (40b23-25)

Sometimes he writes, in this connection, that every syllogism establishes one or another *problêma*. In any case, he then proceeds to describe the deduction process, and his description, while not strictly a definition, amounts, nevertheless, to a kind of stipulative definition of deducibility. We cite this passage at length to provide his complete thinking on this matter. His syntactic treatment of the topic is evident.

> Now, if someone should have to syllogize [συλλογίσασθαι] A of B, either as belonging or as not belonging, then it is necessary for him to take

[100] Some modern logicians believe that what Aristotle writes just preceding this passage (40b23-25) is suggestive of his interest in a completeness proof: "But it will now be evident that this holds for every syllogism without qualification, when every one has been proved to come about through some one of these figures" (40b20-22).

something about something [λαβεῖν τι κατά τινος]. If, then, A should be taken about B, then the initial thing will have been taken. But if A should be taken about C, and C about nothing nor anything else about it, nor some other thing about A, then there will be no syllogism [ὀυδεὶς ἔσται συλλογισμός] (for nothing follows of necessity [οὐδὲν συμβάινει ἐξ ἀνάγκης] through a single thing having been taken about one other). Consequently, another premiss [πρότασιν] must be taken in addition. If, then, A is taken about something else, or something else about it or about C, then nothing prevents there being a syllogism [συλλογισμόν], but it will not be in relation to B through the premisses taken. Nor when C is taken to belong to something else, that to another thing, and this to something else, but it is not connected to B [μὴ συνάπτῃ δὲ πρὸς τὸ B]: there will not be a syllogism [συλλογισμός] in relation to B in this way either. *For, in general, we said that there cannot ever be any syllogism* [συλλογισμὸς] *of one thing about another without some middle term having been taken which is related in some way to each according to the kinds of predications* [ταῖς κατηγορίαις]. *For a syllogism, without qualification, is from premisses* [ὁ μὲν γὰρ συλλογισμὸς ἁπλῶς ἐκ προτάσεων ἐστιν]; *a syllogism* [συλλογισμὸς] *in relation to this term is from premisses in relation to this term; and a syllogism of this term in relation to that is through premisses of this term in relation to that.* And it is impossible to take a premiss in relation to B without either predicating or rejecting anything of it, or again to get a syllogism of A in relation to B without taking any common term, but ⟨only⟩ predicating or rejecting certain things separately of each of them. As a result, something must be taken as a middle term for both which will connect the predications, since the syllogism [συλλογισμὸς] will be of this term in relation to that. If, then, it is necessary to take some common term in relation to both, and if this is possible in three ways (for it is possible to do so by predicating A of C and C of B, or by predicating C of both A and B, or by predicating both A and B of C), and these ways are the figures stated, then it is evident that every syllogism must come about through some one of these figures [φανερὸν ὅτι πάντα συλλογισμὸν ἀνάγκη γίνεσθαι διὰ τούτων τινὸς τῶν σχημάτων]. (40b30 41a18)

Here Aristotle refers to the epistemic value of syllogisms in the deduction process. This passage clearly indicates that a deduction of a conclusion sentence must come from a set of sentences taken as premisses, and, moreover, that the derivation must happen according to prescribed syntactic rules. Thus, according to Aristotle's formal system, there is a derivation of a categorical sentence from other given categorical sentences, either directly or indirectly, whenever "[1] the middle is predicated and a subject of predication, or if it is predicated and something else is denied of it ... [or 2] if it is both predicated of something and denied of something

...[or 3] if others are predicated of it, or one is denied and another is predicated" (*A32*: 47b1-5).

In *Posterior Analytics B4* he writes that "a deduction proves something of something through a middle term [ὁ μὲν γὰρ συλλογισμὸς τὶ κατὰ τινὸς δέικνυσι διὰ τοῦ μέσου]" (91a14-15; cf. *Pr. An. A32*: 47b7-9). Again, consider the following passages from *Posterior Analytics B2*:

> Thus it results that in all our searches we seek either if there is a middle term or what the middle term is. For the middle term is the explanation [αἰτίον], and in all cases it is the explanation that is being sought. (90a5-7, 24)

Again, "it is plain, then, that whatever is sought, it is a search for a middle term" (*Po. An. B3*: 90a35-36). And from *Prior Analytics A28* we read:

> This is because, in the first place, the examination is for the sake of the middle term, and one must take something the same, not something different. (44b38-45a1)

The linchpin, then, in Aristotle's notion of formal deducibility, a concept independent of an intended interpretation, albeit anticipating one for Aristotle, is his notion of the middle term. There is no syllogism of one thing about another without taking a term in common, the *sine qua non* of syllogistic inference. In fact, as we have seen (§2.3), positioning the middle term is a syntactic formation rule of *syllogistic argumentation*. A careful reading of *Prior Analytics* and *Posterior Analytics* reveals Aristotle's preoccupation with the middle term in the deduction process.

Finally, in *Posterior Analytics A19-23*, where he argues that not everything is demonstrable and against reasoning in a circle (cf. *Po. An. A3*, *B12* and *Pr. An. B5-7*, *B16*), Aristotle shows that a deduction cannot contain an infinite chain of reasoning. This position is most strongly argued in *Posterior Analytics A22*.

> Hence if it were possible for this to go on *ad infinitum*, it would be possible for there to be infinitely many middle terms between two terms. But this is impossible if the predicates come to a stop in the upward and the downward directions. And that they do come to a stop we have proved generally earlier and analytically just now. (84a37-84b2)

There Aristotle shows that "one cannot survey infinitely many items in thought" (83b6-7); "there must therefore be some term of which something is predicated primitively, and something else of this" (83b28-29). Again, if demonstration is possible, the predicates in between must be finite (83b38-84a6).

Thus, we see that Aristotle's notion of formal deducibility corresponds exactly with that of a modern logician, point for point. We can now restate a notion of formal deducibility, this time more tailored to Aristotle's system.

> A given categorical sentence c is formally deducible from a given set of categorical sentences P when there exists a finite sequence of categorical sentences that ends with c and begins with P such that each categorical sentence in the sequence from P is either a member of P or a categorical sentence generated from earlier sentences solely by means of stipulated deduction rules such that the terms in c are linked through a series of common terms from P.

Aristotle at no one place expressed this notion in just this manner, but his intentionally subscribing to such a notion, even making statements close to this effect, is unmistakable.

5.2 *Two methods of deduction in Aristotle's system*

Aristotle identified and used two methods of deduction in *Prior Analytics*: (1) direct, or probative, deduction and (2) indirect, or *reductio* (leading to an absurdity or *per impossibile*), deduction. He makes this explicit in *A23*.

> Now, every demonstration, and every deduction [πᾶσαν ἀπόδειξιν καὶ πάντα συλλογισμὸν], must *prove* [ἀνάγκη ... δεικνύναι] something either to belong or not to belong, and this either universally or partially, and in addition either probatively [δεικτικῶς] or from an assumption [ἐξ ὑποθέσεως] (for ⟨deduction⟩ through an absurdity [διὰ τοῦ ἀδυνάτου] is a part of ⟨deduction⟩ from an assumption). (40b23-26)

A direct proof begins a deduction without making an assumption, and an indirect proof begins by assuming the contradictory opposite of the conclusion as an additional premiss and then it deduces a contradiction. Aristotle understood this in exactly the same way as a modern logician, indeed, himself having formulated them for us in ancient times. Here follow two passages from *Prior Analytics* that make this clear, the first is from *A23*.

> For all those which come to a conclusion through an absurdity deduce the falsehood, but prove the original thing from an assumption when something absurd results when its contradiction is supposed [πάντες γὰρ οἱ διὰ τοῦ ἀδυνάτου περαίνοντες τὸ μὲν ψεῦδος συλλογίζονται, τὸ δ' ἐξ ἀρχῆς ἐξ ὑποθέσεως δεικνύουσιν, ὅταν ἀδύνατόν τι συμβαίνῃ τῆς ἀντιφάσεως τεθείσης], ⟨proving,⟩ for example, that the diagonal is incommensurable because if it is put as commensurable, then odd numbers become equal to even ones. It deduces [συλλογίζεται] that odd numbers become equal to even ones, then, but it proves [δέικνυσιν] the diagonal to be incommensurable from an assumption since a falsehood results by means of its contradiction [ἐπεὶ ψεῦδος συμβαίνει διὰ τὴν ἀντίφασιν]. For this is what deducing [συλλογίσασθαι] through an absurdity was: proving something impossible by means of the initial

> assumption [τὸ δεῖξάι τι ἀδυνατον διὰ τὴν ἐξ ἀρχῆς ὑπόθεσιν]. Consequently, since a probative deduction of the falsehood comes about [ὥστ' ἐπεὶ τοῦ ψεύδους γίνεται συλλογισμὸς δεικτικὸς] in those cases which lead away to an absurdity (while the original thing is proved [δείκνυται] from an assumption) ... (41a23-34; cf. *Po.An. A11*: 77a22-25, *A26*: 87a6-12)

The second is from *Prior Analytics B14*.

> A demonstration ⟨leading⟩ into an absurdity differs from a probative demonstration in that it puts as a premiss what it wants to reject by leading away into an agreed falsehood, while a probative demonstration begins from agreed positions. More precisely, both demonstrations take two agreed premisses, but one takes the premisses that the deduction [ὁ συλλογισμός] is from, while the other takes one of these premisses and, as the other premiss, the contradictory of the conclusion [τὴν ἀντίφασιν τοῦ συμπεράσματος]. Also, in the former case it is not necessary for the conclusions to be familiar or to believe in advance that it is so or not, while in the latter case it is necessary to believe in advance that it is not so. It makes no difference whether the conclusion is an affirmation or a denial [φάσιν ἢ ἀπόφασιν], but rather it is similar concerning both kinds of conclusion. (62b29-38)[101]

Aristotle, in fact, treated *reductio* rather fully in *Prior Analytics B11-13*, treating indirect proof in each of the three figures consecutively.

We illustrate each of the two kinds of deduction here by providing an instance of Aristotle's own use of each kind in the metalanguage of *Prior Analytics*. First, in connection with the method of direct deduction, there is Aristotle's text for Camestres in the second figure.

> Next, if M belongs to every N but to no X, then neither will N belong to any X. For if M belongs to no X, neither does X belong to any M; but M belonged to every N; therefore, X will belong to no N (for the first figure has again come about). And since the privative converts, neither will N belong to any X, so that there will be the same syllogism. (It is also possible to prove these results by leading to an impossibility.) (27a9-15)

We provide Aristotle's text for Camestres, organized just as with Aristotle and according to a modern sequencing on the left, and on the right we provide our modern notation that exactly reproduces his meaning (Table 39).

[101] Aristotle continues as follows: "Everything concluded probatively can also be proved through an absurdity, and whatever is proved through an absurdity concluded probatively, through the same terms" (62b38-41). He concludes *Pr. An. B14* with this statement: "it is clear, then, that every problem can be proved in both ways, through an absurdity as well as probatively, and that it is not possible for one of the ways to be separated off" (63b18-21).

Table 39.

An instance of direct deduction					
Aristotle's text		**Modern notation**			
1.	M belongs to every N	1.	M*a*N		
2.	M belongs to no X	2.	M*e*X		
?	neither will N belong to any X	?	N*e*X		
3.	M belongs to no X	3.	M*e*X	2	repetition
4.	neither does X belong to any M	4.	X*e*M	2	*e*-conversion
5.	M belonged to every N	5.	M*a*N	1	repetition
6.	X will belong to no N	6.	X*e*N	4,5	Celarent
7.	neither will N belong to any X	7.	N*e*X	6	3 *e*-conversion

Next, we have Aristotle's text for Baroco in the second figure, followed, again, by a table that organizes this text according to his sequencing on the left and on the right by our modern notation that reproduces his meaning (Table 40).

> Next, if M belongs to every N but does not belong to some X, it is necessary for N not to belong to some X. (For if it belongs to every X and M is also predicated of every N, then it is necessary for M to belong to every X; but it was assumed not to belong to some.) And if M belongs to every N but not to every X, then there will be a deduction that N does not belong to every X. (The demonstration is the same.) (27a36-27b3)

Table 40.

An instance of indirect deduction					
Aristotle's text		**Modern notation**			
1.	M belongs to every N	1.	M*a*N		
2.	M does not belong to some X	2.	M*o*X		
?	N not to belong to some X	?	N*o*X		
3.	N belongs to every X	3.	N*a*X		assume
4.	M is also predicated of every N	4.	M*a*N	1	repetition
5.	M to belong to every X	5.	M*a*N	4,3	Barbara
6.	M not to belong to every X	6.	M*o*X & M*a*X	2,5	conjunction & contradiction
7.	N does not belong to every X	7.	N*o*X	3 6	*reductio*

5.3 Aristotle's notion of logical consequence

Aristotle defines "necessary" in *Metaphysics*, where he defines many philosophical concepts, precisely as he uses the concept in *Prior Analytics* in relation to deciding the concludence and inconcludence of premiss-pair patterns. Our grasping his notion of logical consequence — following from necessity, or τὸ ἐξ ἀνάγκης συμβάινειν — in *Prior Analytics* can be informed by what he writes in *Metaphysics 5.5*: "that which is necessary is that having no other relationship possible [ἔτι τὸ μὴ ἐνδεχόμενον ἄλλως ἔχειν ἀναγκαῖόν φαμεν οὕτως ἔχειν]" (1015a33-35). In *Metaphysics 4.5* he writes much the same: "for it is not possible for what is necessary to be one way and another way, and so if something is of necessity, it cannot be so and not so [τὸ γὰρ ἀναγκαῖον οὐκ ἐνδέχεται ἄλλως καὶ ἄλλως ἔχειν, ὥστ' ἐι τι ἔστιν ἐξ ἀναγκης, οὐχ ἕξει οὕτω τε καὶ οὐχ οὕτως" (1010b28-30). And, thus, in respect of sentences, "opposed statements cannot [both] be true [at the same time] [τὸ μὴ ἔιναι ἀληθεῖς ἅμα τὰς ἀντικειμένας φάσεις]" (1011b13-14).

Aristotle also affirms in this connection a principle of consistency in *Prior Analytics A32*: "for all that is true must in all ways be in agreement with itself [δεῖ γὰρ πᾶν τὸ ἀληθὲς αὐτὸ ἑαυτῷ ὁμολογούμενον ἔιναι πάντῃ]" (47a8-9). A statement of this principle concludes *On Interpretation* in the following way:

> It is evident also that it is not possible either for a true opinion or a true contradictory to be contrary to another true opinion. For contraries relate to their opposites, and concerning these it is possible to assert truly of the same thing, but it is not possible that contraries hold of the same thing [at the same time]. (24b6-9)

And Aristotle expressed this principle somewhat concisely in *On Interpretation 14* a few passages before the one just cited: "a true opinion is never contrary to another true opinion [οὐδέποτε δὲ ἀληθὴς ⟨δόξα⟩ ἀληθεῖ ἐναντία]" (23b37-38).

Returning to his notion of necessity, we find that Aristotle also makes an explicit reference to demonstration at *Metaphysics 5.5*, in connection with the passage cited above, that conforms well with his conception of consistency:

> Demonstration is of necessary things [ἔτι ἡ ἀπόδειξις τῶν ἀναγκαίων], because, if there is a demonstration proper, it is not possible for there to be any other relations [οὐκ ἐνδέχεται ἄλλως ἔχειν]; the reason for this is the premisses [τούτου δ' ἄιτια τὰ πρῶτα], for if there is a syllogism it is [logically] impossible for there to be another relationship among them [ἐι ἀδύνατον ἄλλως ἔχειν ἐξ ὧν ὁ συλλογισμός]. (1015b6-9)[102]

Thus, a premiss-pair that results in a syllogism is such that no other result is possible.[103] Aristotle had established in *Prior Analytics A4-6* that for a syllogism

[102] Cf. *Po. An. A6* on demonstration of that which is necessary. Consider that for Aristotle scientific knowledge, *apodeiktikê epistêmê*, is that which cannot be otherwise.

[103] This holds notwithstanding that a weakened *a* or *e* sentence, i.e., an *i* or *o* sentence, is a different sentence; this is a trivial truth for Aristotle. 'To be otherwise' refers to contrariety and contradiction.

to arise it is *necessary and sufficient* that the terms be related as he stated in a number of syntax rules (§5.1). Likewise, for there not to be a syllogism, it is necessary and sufficient that terms be related in the other ways he covered there (§5.1). As we have seen, Aristotle established a set of formal rules, relating to syllogistic argumentation, for deciding logical consequence.

Another indication of Aristotle's sophistication in respect of logical consequence concerns his discussion of the most certain principle of all, the law of non-contradiction, one statement of which is expressed in *Metaphysics 4.3*: 1005b19-20 (*4.3-8*; cf. *11.5-6*). He then writes that "someone who denies [this principle] would at the same time hold contrary opinions. Hence, everyone who performs a demonstration establishes it on this ultimate principle" (1005b30-33; cf. 1005b22-34). On at least two occasions Aristotle refers to someone not subscribing to this principle as being "no better than a plant" because nothing meaningful can be asserted and rational discourse (*logos*) is destroyed (1006b5-11, 1008b7-9).

It is evident, moreover, from his treatment of the principle of non-contradiction that Aristotle understands every sentence to be a logical consequence of a contradiction. The following passage from *Metaphysics 4.4* illustrates that his grasp of logical consequence in this connection is much the same as that of modern logicians.

> If all contradictories [ἀντιφάσεις] were true at the same time of the same thing, it is evident that everything would be one. For the same thing would be a trireme, a wall, and a man, if it is possible either to affirm or to deny something of everything [ἐι κατὰ παντός τι ἢ καταφῆσαι ἢ ἀποφῆσαι ἐνδέχεται] ... For if someone thinks that a man is not a trireme, then clearly he is not a trireme. But if the contradictory is [just as] true, then he is also a trireme. ... [Such persons are really describing non-being] ... But [then] one must assert an affirmation and a denial about every single thing. For it is absurd that the denial holds of itself and yet excludes other denials that do not belong to it. I mean, for example, that if it were true to say that a man is not a man, it is evident also that he is a trireme and not a trireme. Now, on the one hand, if the affirmation [κατάφασις] is admitted, it is also necessary that the denial [ἀπόφασις] be admitted; on the other hand, if the affirmation does not belong, then the denial will belong more than the denial of itself. Now, if this denial holds, then also the denial of the trireme will belong. And if this, also the affirmation. ... [If this situation holds then again] *any assertion whatever may be denied and any denial whatever may be asserted* [εἰ δὲ περὶ πάσας, πάλιν, ἤτοι καθ' ὅσων τὸ φῆσαι καὶ ἀποφῆσαι καὶ καθ' ὅσων ἀποφῆσαι καὶ φῆσαι]. (1007b18-1008a13)

At *Metaphysics 4.5* Aristotle reiterates his thinking: "for if all opinions and appearances are true, it is necessary that every one is true and false at the same time ... and so it is necessary that the same thing must both be and not be [ὥστ'

ἀνάγκη τὸ αὐτὸ εἶναι τὲ καὶ μὴ εἶναι]. And if this is so, every opinion must be true" (1009a7-13). That every sentence follows from a contradiction is a truth for Aristotle.[104]

In addition to these discussions in *Metaphysics* there is a passage in *Prior Analytics B2* where Aristotle treats "following necessarily" in much the way that modern logicians tend to deny he could if he had not defined "logical consequence". It is worthwhile citing this passage in its entirety, since he states his notion of logical consequence most clearly. He writes:

> First, then, it is clear from the following that it is not possible to deduce [συλλογίσασθαι] a falsehood from truths. For if it is necessary for B to be when A is, then when B is not it is necessary for A not to be. Thus, if A is true, then it is necessary for B to be true, or else it will result that the same thing both is and is not at the same time [συμβήσεται τὸ αὐτὸ ἅμα ἔιναι τὲ καὶ οὐκ ἔιναι]; but this is absurd. (53b11-16)

This passage continues:

> But let it not be believed, because A is set out as a single term, that it is possible for something to result of necessity [ἐξ ἀνάγκης τι συμβάινειν] when a single thing is, for that cannot happen: for what results of necessity is a conclusion [τὸ μὲν γὰρ συμβᾶινον ἐξ ἀνάγκης τὸ συμπέρασμά ἐστι], and the fewest through which this comes about are three terms and two intervals or premisses. (53b16-20)

Aristotle explains his use of schematic letters here in the continuation of this passage, where he addresses logical consequence in connection with syllogistic reasoning by means of a syllogistic discourse. But Aristotle here immediately provides a terse metalogical discourse on syllogistic logical consequence, but *using propositional logic*. He treats a syllogism as fitting a single conditional sentence pattern. First, he conjoins the two sentences in the premiss-set of a syllogism and takes them as a single sentence pattern, A, which is itself the antecedent of a conditional sentence whose consequent is the conclusion pattern, B, of a syllogism: thus, "if A then B". He then affirms that if this is the case, then so must "if not-B then not-A" be the case, otherwise something true would imply something false, and this is absurd. Aristotle affirms the consistency of the following three sentence patterns (Table 41).

Second, he affirms that "if A is true, then it is necessary for B to be true, *or else* it will result that the same thing both is and is not at the same time; but this is absurd". We take the "or else" to refer to taking "if A is [the case] then B is not [the case]", or "A ⊃ -B". Aristotle's test of the consistency of the original three sentence patterns is to substitute "A ⊃ -B" for "A ⊃ B" in the original set and then to deduce a contradiction. We can represent his thinking with a familiar notation of propositional logic as follows (Table 42).

[104] However, G. Priest does not take Aristotle in *Meta. 4* as treating 'explosion'; see, e.g., G. Priest 1998 & 2000.

Table 41.

Aristotle's text	Modern notation
1. If A is [the case], then B is [the case]	1. $A \supset B$
2. A is [the case]	2. A
3. If B is not [the case], then A is not [the case]	3. $-B \supset -A$

Table 42.

Original set of sentences	Modern notation	Test of consistency
1. If A is [the case], then B is [the case]	1. $A \supset B$ Given	1. $A \supset -B$ Substitute
2. If B is not [the case] then A is not [the case].	2. $-B \supset -A$ Given	2. $-B \supset -A$ Given
3. A is [the case].	3. *A* Given	3. A Given
		4. -B 1,3 detachment
		5. -A 2,4 detachment
		6. A & -A 3,5 conjunction, contradiction

This seems to capture Aristotle's thinking, which is regrettably terse. He states that is impossible for a set of true sentences to imply a false sentence without contradiction. His method is not that of providing a model of model sets, but one similar to the method of *reductio* proof. He deduces a contradiction from a set of sentence patterns, one of which has been substituted for one in the original set whose consistency is to be demonstrated. The substituted sentence pattern is the contradictory or contrary of the original sentence pattern, in this case, where 'A is the case', or 'is true'. Thus, if A is true, then for A to imply B, B must also be true; thus, A implying not-B cannot be the case, or is false.

Aristotle continues the quotation cited above from *Prior Analytics B2*, and he now represents the same notion of logical consequence, but this time with syllogistic expressions. He writes:

> But let it not be believed, because A is set out as a single term, that it is possible for something to result of necessity when a single thing is, for that cannot happen: for what results of necessity is a conclusion, and the fewest through which this comes about are three terms and two intervals or premisses. If it is true, then, that A belongs to everything to which B belongs, and B to what C belongs, then it is necessary for A to belong <to what C belongs to>, and this cannot be false (for the same thing would belong and not belong at the same time). Therefore,

A is put as if a single thing, the two premisses being taken together. And similarly also in the case of privative deductions. *For it is not possible to prove a falsehood from truths* [οὐ γὰρ ἔστιν ἐξ ἀληθῶν δεῖξαι ψεῦδος]. (53b16-25)

This, in fact, is a treatment of Barbara to illustrate the truth that true sentences can not imply a false sentence, and, thus, it proves the logical impossibility of an invalid argument fitting this *teleios sullogismos* pattern. We can illustrate rather exactly his thinking, although tersely expressed in this passage, using his schematic letters, his method of deduction (from *Pr. An. A5-6*), and interpolating the steps in his deduction, to set it out in a familiar manner (Table 43).

Table 43.

Aristotle's test of consistency			
Aristotle's text		**Modern notation**	
1.	A belongs to everything to which B belongs	1. A*a*B	
2.	B to [everything to which] what C belongs	2. B*a*C	[1 & 2 = A (53b11-16)]
?	A to belong ⟨to what C belongs to⟩	? A*a*C	[B (53b11-16)]
[3–8]	this cannot be false (for the same thing would belong and not belong at the same time)	3. A*o*C	Assume
		4. A*a*B	1 repetition
		5. A*o*C	3 repetition
		6. B*o*C	4,5 Baroco
		7. B*a*C &B*o*C	2,6 conj, contradiction
		8. A*a*C	3-7 *reductio*

It is evident that Aristotle works with a notion of logical consequence much like Tarski's, with which modern logicians are familiar, namely:

A given sentence c is a logical consequence of a given set of sentences P when every true interpretation of P is a true interpretation of c.[105]

We can express Aristotle's thinking on this matter as follows:

[105] Tarski writes in "On the concept of logical consequence" (Corcoran 1990: 417): "The sentence X *follows logically* from the sentences of the class K if and only if every model of the class K is also a model of the sentence X".

> A given categorical sentence c follows necessarily from a given set of categorical sentences P when every set of term substitutions[106] that makes each sentence in P true makes the sentence in c true.

Now, again, Aristotle does not compose such a sentence, yet it is clear that he formulates other statements that make his understanding clear, especially, for example, those relating to proving inconcludence.

We can now be confident that Aristotle's notion of "following necessarily" expresses his notion of "logical consequence", and that he established that it is logically impossible for true sentences to imply a false sentence. Aristotle frequently writes that it is impossible to deduce a falsehood from truths; this is the theme of *Prior Analytics B2-4*. There he shows that any combination of truth-values for premisses and conclusion can result in a valid argument except in the case of an argument with all true premisses and a false conclusion. In *Prior Analytics B2* he explicitly writes:

> Now, it is possible for circumstances to be such that the premisses by means of which the syllogism comes about are true, or that they are false, or that one premiss is true and the other false. The conclusion, however, is either true or false of necessity. *It is not possible, then, to deduce* [συλλογίσασθαι] *a falsehood from true premisses*, but it is possible to deduce a truth from false ones (except that it is not a deduction of the 'why' but of the 'that', for a deduction [συλλογισμὸς] of the 'why' is not possible from false premisses). (53b4-10)

This translates into affirming that no argument is valid that has true premiss sentences and a false conclusion sentence. Aristotle then follows this passage to provide his reasoning for this condition, which we have treated just above (*Pr. An. B2*: 53b11-25).

6 SUMMARY OF ARISTOTLE'S ACCOMPLISHMENTS IN *PRIOR ANALYTICS*

By the end of *Prior Analytics A6* Aristotle had systematically worked through all possible patterns of two categorical sentences that could serve in the role of premisses to discover "how every syllogism comes about". He established that among these possible patterns there are 14 that result, in their application to an object language, in something following necessarily, that is, that result in syllogisms. To accomplish this project, Aristotle invented a formal language to devise a rudimentary model of his logic in *Prior Analytic*. In this way he was able to describe a deduction system and demonstrate certain logical relationships among syllogistic rules, not the least of which accomplishment was establishing the independence

[106]While substitution and reinterpretation are distinct logical concepts, their application amounts to the same thing.

of a small set of deduction rules (*A7*). The formal language used in *Prior Analytics* built upon the foundation of his linguistic studies in *Categories* and *On Interpretation.* Strictly speaking, Aristotle's formal language does not consist in sentences, as 'sentence' is defined in *On Interpretation* and as '*protasis*' is used in *Prior Analytics.* Rather, his formal language consists in relatively uninterpreted sentence patterns. By substituting non-logical constants — a predicate term and a subject term — for schematic letters, Aristotle could produce any number of object language sentences. We could easily call such sentences interpretations without distortion, as a modern logician understands this notion. This, however, would misrepresent Aristotle's logic. Nevertheless, this closeness to modern practice is not a superficial resemblance, but an indication of Aristotle's genius and originality. Here we summarize some of his accomplishments and insights into logic with a synopsis of his model (§6.1) and with a summary of four proof-theoretic processes he employed in *Prior Analytics* (§6.2).

6.1 Synopsis of Aristotle's model

Aristotle invented his formal language with an aim to model scientific discourse. Such discourse, then, might be taken as its 'intended interpretation'. In any case, using a modern mathematical template, we can re-present Aristotle's own model in the following way (Table 44).

6.2 Four proof-theoretic processes in Prior Analytics

Aristotle did not describe deductions in *Prior Analytics A4-6* but showed how every syllogism comes about. He also explained how syllogisms do not come about and he refined his system. He described a natural deduction system of an underlying logic in ways suggestive of modern methods, and he proved certain properties of this deduction system. His treatment of his logic is thoroughly metalogical. Here we briefly summarize four proof-theoretic processes used in *Prior Analytics.* All four processes have their counterparts in one or another object language.

Deciding concludence: the method of completion

Completion (*teleiôsis*, *teleiousthai*) is a proof-theoretic, deductive process that establishes knowledge that a given argument pattern with places for two premisses having places for three different terms is panvalid by using the patterns of the *teleioi sullogismoi* as deduction rules. Completion is an epistemic process. In *Prior Analytics A4-6* Aristotle established the preeminence of the patterns of the *teleioi sullogismoi* among the panvalid patterns or, conversely, he implicitly established that the patterns of the *ateleis sullogismoi* are redundant rules in his deduction system. The process of completion *per se* does not establish that any rule of deduction is redundant. Nor does completion involve transforming a given argument pattern into another argument pattern, since in the process of deduction a given argument pattern is not itself transformed but shown to be

panvalid through a chain of reasoning cogent in context, which chain of reasoning is generated by means of specified deduction rules. Aristotle's metalogical theorem concerning completion is that "all the *ateleis sullogismoi* are completed by means of the first figure syllogisms using probative and *reductio* proofs" (*A7*: 29a30-33). Aristotle reserved using the verb '*teleiousthai*' specifically in relation to a process by which a valid argument, whose validity is not evident, is made evident by performing a deduction during which a *teleios sullogismos*, one whose validity is obviously evident, is generated; this signals cogency in the deduction process from premisses to conclusion.

Deciding inconcludence: the method of contrasted instances

The method of contrasted instances used in *Prior Analytics A4-6* is the complement of the process of completion. The purpose of this method is to establish which elementary argument patterns are not panvalid. This proof-theoretic method is different from the method of counterargument, since (1) it treats patterns of premisses and argument patterns and not arguments and, thus, it establishes paninvalidity and not invalidity, and (2) it does not produce an argument in the same form as a given argument but with true premisses and a false conclusion. Rather, this method constructs two arguments, each of whose premisses are true sentences fitting the same premiss-pair pattern and whose conclusions also are true sentences, but in the one argument the conclusion is an *a* sentence, in the other an *e* sentence. This establishes that a given premiss pair pattern is inconcludent and that consequently its corresponding four argument patterns are paninvalid. No syllogism is possible in such a case. It is not possible to construct such arguments with a concludent premiss pair pattern: in that case every similar construction that produces true sentences as premisses results in at least one false sentence among the conclusions. Thus, any two sentences of three terms fitting a given inconcludent premiss-pair pattern are shown never to result together in a valid argument. In this way Aristotle was able to eliminate would-be syllogistic deduction rules.

Table 44.

An ancient model of an underlying logic	
Aristotle's own model	**Aristotle's model expressed by a modern notation**
LANGUAGE	**LANGUAGE**
Vocabulary	**Vocabulary**
1. Four fully interpreted logical constants	1. Logical constants
belongs to every	*a*
belongs to no	*e*
belongs to some	*i*
belongs not to every	*o*
2. *n* schematic (upper case) letters intended to hold places for non-logical constants (terms)	2. *n* schematic letters
A, B, C; M, N, X; P, R, S...	A, B, C...
Grammar	**Grammar**
1. Sentences are the elements of a language. A categorical sentence is formed by concatenating a non-logical constant with a logical constant with a different non-logical constant.	1. Categorical sentence patterns
A belongs to every B.	A*a*B
A belongs to no B.	A*e*B
A belongs to some B	A*i*B
A belongs not to every B.	A*o*B
2. Relationships of opposite sentences	2. Sentential relationships
Contradictories	*Contradictories*
A belongs to every B — to — A belongs not to every B.	A*a*B — to — A*o*B
A belongs to no B — to — A belongs to some B	A*e*B — to — A*i*B
Contraries	*Contraries*
A belongs to every B — to — A belongs to no B.	A*a*B — to — A*e*B

3. Premiss formation	3. Premiss formation
One-premiss argument	*One-premiss argument*
Take any one of the four categorical sentences AB	AxB
Two-premiss argument	*Two-premiss argument*
Take any two of the four categorical sentences with three different terms, one in common First figure:PMS	First figure:PMS
1. PM 2. MS	1. PxM 2. MyS
Second figure:MPS	Second figure:MPS
1. MP 2. MS	1. MxP 2. MyS
Third figure: PSM	Third figure: PSM
1. PM 2. SM	1. PxM 2. SyM
4. P-c argument formation	4. P-c argument formation
One-premiss (conversion) argument	*One-premiss argument*
1. AB ?. BA	1. AxB ?. ByA
Two-premiss argument	*Two-premiss argument*
First figure:PMS	First figure:PMS
1. PM 2. MS ?. PS	1. PxM 2. MyS ?. PzS
Second figure:MPS	Second figure: MPS
1. MP 2. MS ?. PS	1. MxP 2. MyS ?. PzS
Third figure: PSM	Third figure: PSM
1. PM 2. SM ?. PS	1. PxM 2. SyM ?. PzS

DEDUCTION SYSTEM	DEDUCTION SYSTEM
1. Deduction rules	1. Deduction rules
One-premiss conversion rules	*One-premiss rules*
If A belongs to every B, then B belongs to some A. If A belongs to some B, then B belongs to some A. If A belongs to no B, then B belongs to no A.	1. AaB 1. AiB 1. AeB $\therefore BiA$ $\therefore BiA$ $\therefore BeA$
Two-premiss syllogism rules (reduced system)	*Two-premiss rules*
If A belongs to every B and B belongs to every C, then A belongs to every C. If A belongs to no B and B belongs to every C, then A belongs to no C.	1. AaB 1. AeB 2. BaC 2. BaC $\therefore AaC$ $\therefore AeC$
2. Types of deduction	2. Types of deduction
Direct deduction	*Direct deduction*
[See section 5.2]	$$\frac{P}{\text{Step 1}}$$ $\vdots$ step $n-1$ step $n = c$
Indirect deduction	*Indirect deduction*
[See Section 5.2]	$$\frac{P}{\text{Step 1: contra of } c}$$ $\vdots$ step $n-1 : X$[conj & contr] step $n = c$

SEMANTICS	SEMANTICS
1. Meanings of sentences	1. Meanings of sentences
A*a*B: universal attributive: Every B has property A A*e*B: universal privative: No B has property A. A*i*B: partial attributive: Some B has property A A*o*B: partial privative: Some B does not have property A.	[Same]
2. Truth-values of sentences	2. Truth-values of sentences
A*a*B is true iff every B has property A. A*e*B is true iff no B has property A. A*i*b is true iff some B has property A. A*o*B is true iff some B does not have property A.	[Same]
3. Logical Consequence	3. Logical consequence
It is impossible that a false sentence follows necessarily from true sentences.	[Same]

Transforming patterns: analysis

Analysis (*analusis*, *analuein*) is a proof-theoretic process that transforms one syllogistic pattern in any one figure into another syllogistic pattern of another figure only if both patterns 'prove' the same *problêma*. Analyses are performed through conversion and premiss transposition. Analysis is not directly concerned with making validity or panvalidity evident, not with a deduction process, nor with establishing whether a given syllogistic pattern is a redundant rule. Rather, Aristotle aimed to promote his students' facility with reasoning syllogistically to establish (τὸ κατασκευάζειν) and to refute (τὸ ἀνασκευάζειν) arguments by studying the logical relationships among their patterns.[107] This is analogous to how modern logicians have studied the relationships among the rules of propositional logic. Aristotle's theorem concerning analysis is that 'the syllogisms in the different figures that prove the same *problêma* are analyzable into each' (see *A45*: 50b5-7).

[107]See *Pr. An. A26-28* and summary at *A30*: 46a3-10. For example, Aristotle writes (A26): "... a universal positive *problêma* is most difficult to establish [κατασκευάσαι] but easiest to refute [ἀνασκευάσαι]" (43a1-2). Cf. Aristotle's projects in writing *SR* and *Top.*

7 CONCLUDING REMARKS

We have represented Aristotle as having modelled his syllogistic as an underlying logic according to the practice of a modern mathematical logician. In *Prior Analytics* he especially articulated the logical syntax of his syllogistic system while, nevertheless, always presupposing its applicability to various axiomatic sciences. Aristotle conceived of his system as a formal calculus, akin to mathematical calculi, since his aim was to establish a reliable deduction instrument for *epistêmê apodeiktikê*. Some modern logicians believe that, since Aristotle did not specifically refer to alternative interpretations or perform operations that suggest his recognizing this, he must have taken his ideal language to be fully interpreted. In this connection, then, they believe that Aristotle could not have conceived of a language apart from its intended interpretation, that Aristotle did not distinguish logical syntax from semantics. However, when we consider Aristotle's accomplishments in *Prior Analytics* along with his other accomplishments in *Categories*, *On Interpretation*, and *Metaphysics*, this interpretation seems not to accord with his having invented a formal language.

One way sufficient for determining whether or not a logician distinguishes logical syntax from semantics is to ascertain whether a logician works with notions of interpretation and reinterpretation. In a reinterpretation one leaves the language fixed but changes its meaning. It is thought that Aristotle's having notions of interpretation and reinterpretation was precluded by his not having distinguished syntax and semantics in his logic. Perhaps, however, it is rather that his distinguishing syntax and semantics is thought to have been precluded by his not having notions of interpretation and reinterpretation because he did not work with model-theoretic and set-theoretic notions. In this connection, then, we can recognize that another equally sufficient way for determining whether or not a logician distinguishes logical syntax from semantics is to ascertain whether a logician works with a notion of *substitution*. In a substitution one changes the language, or the content words and phrases in a given language, while leaving their meanings and the logical form fixed. While we might agree that Aristotle did not have nor work with fully modern notions of interpretation and reinterpretation *per se*, he has nevertheless quite ably distinguished syntax and semantics as is evidenced by his inventing and using a formal language that contains only sentence patterns. And we have already witnessed an instance of Aristotle experimenting with reinterpreting a word in much the same way as a modern logician. While *substitution* and *reinterpretation* are distinct logical concepts, their application amounts to the same thing. In this light, observing Aristotle's pervasive use of schematic letters and his common practice of substitution for establishing inconcludence, we recognize his making a more determinate distinction between semantics and syntax than previous interpreters have allowed. By *substituting* terms for schematic letters, Aristotle was able to produce an unlimited number of sentences according to his definition in *On Interpretation* and his formal grammar in *Prior Analytics*. This method of producing sentences from patterns surely amounts to 'giving an

interpretation', while not itself, of course, strictly an interpretation. Moreover, there are much the same results in relation to recognizing the underlying structures of natural languages and logical languages. Again, he might easily have construed these as interpretations of his formal language. Thus, we believe that there are sufficient *textual* grounds for imputing to Aristotle a belief not only in argument 'forms', but also, then, in distinguishing syntax and semantics, indeed, in a way familiar to A. Church, A. Tarski, and other modern logicians. While it is doubtful that Aristotle had a modern theory of language, and surely not himself a string-theorist, it is nevertheless evident that he recognized different patterns to underlie sentences involving, for example, ambiguity and equivocation. Indeed, identifying these forms or *patterns* and establishing their *logical relationships* were precisely the focus of his project in *Prior Analytics A4-6, 7,* and *45.*

Aristotle's notion of substitution, then, was sufficiently strong for his distinguishing logical syntax and semantics. In this connection he was also able to distinguish validity from deducibility sufficiently to note the completeness of his logic in *Prior Analytics A30.* Consider the following passage:

> Consequently, if the facts concerning any subject have been grasped, we are already prepared to bring the demonstrations readily to light. For if nothing that truly belongs to the subjects has been left out of our collection of facts, then concerning every fact, if a demonstration for it exists, we will be able to find that demonstration and demonstrate it, while if it does not naturally have a demonstration, we will be able to make that evident. (46a22-27)

This statement surely indicates that he believed his deduction system sufficiently strong to deduce every logical consequence of a given set of sentences. And always underpinning his thinking lay his taking such sentences to be the first principles of axiomatizable sciences and his aspiration that his deductive sciences would be universally complete. In *Prior Analytics* Aristotle turned his attention toward objectifying the formal deduction apparatus used to establish scientific theorems. Since the process of deduction is topic neutral and formal, Aristotle was concerned with matters of syntax and deducibility: he treated these matters especially in *Prior Analytics A*. Since Aristotle was concerned with logical consequence and truth, he was occupied also with semantic matters: he treated these matters especially in *Prior Analytics B* among the other places we have examined.

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ARISTOTLE'S MODAL SYLLOGISMS

Fred Johnson

Considering Aristotle's discussion of syllogisms as a whole, the most striking point is that its focus is the modal syllogisms – This is the point on which the logical tradition has diverged most completely from Aristotle, as a rule giving no attention to modal syllogisms .... Paul Henle

Aristotle's system of modal syllogisms, to be found in chapters 3 and 8-22 of the first book of the *Prior Analytics,* has been open to public inspection for over 2300 years. And yet perhaps no other piece of philosophical writing has had such consistently bad reviews. Storrs McCall

... by raising the [completeness] problem, Aristotle earns the right to be considered not only the father of logic, but also the (grand)father of meta- logic. Jonathan Lear

Storrs McCall [1963] developed the first formal system, the L-X-M calculus, for which a decision procedure for assertion or rejection of formal sentences is given that has any chance of matching Aristotle's judgments about which of the n-premised (for $n \geq 2$) "apodeictic syllogisms" are valid or invalid. McCall's remarkable results were achieved by extending Jan Łukasiewicz's [1957] decision procedure for assertion or rejection of expressions in his formal system, ŁA, that is designed to capture Aristotle's judgments about which of the "assertoric (or plain[1]) syllogisms" are valid or invalid.

Łukasiewicz also considers using his four-valued modal system, the ŁM system, to present Aristotle's syllogistic but finds that the match is not very good. Peter Geach also proposes a system for dealing with the apodeictics. But, again, the match is not very good. After examining McCall's L-X-M system and work related to it we shall turn to his work on the "contingent syllogisms". His purely syntactic system, Q-L-X-M, has some unAristotelian features that lead us to develop a modified system, QLXM$'$. A semantics for QLXM$'$ is developed that enables us to provide formal countermodels for a large percentage of the assertoric, apodeictic or contingent syllogisms that Aristotle explicitly considered to be invalid.

1 ŁUKASIEWICZ'S ASSERTORIC SYSTEM, ŁA

For Łukasiewicz, Aristotle's syllogisms are "implicational" rather than "inferential". He says in [1957, p. 21]:

[1] In [1964] P. T. Geach prefers 'plain' over 'assertoric'.

Handbook of the History of Logic. Volume 1
Dov M. Gabbay and John Woods (Editors)

> Syllogisms of the form:
>
> All B is A;
> all C is B;
> therefore
> all C is A
>
> are not Aristotelian. We do not meet them until Alexander. This transference of the Aristotelian syllogisms from the implicational form into the inferential is probably due to the Stoics.

So, Łukasiewicz claims Aristotle construed the above syllogism, with traditional name 'Barbara', as a conditional claim:

> If all B are A then if all C are B then all C are A.

Robin Smith's [1989, p.4] translation of Barbara at *Prior Analytics* 25b37-40 seems to conform with Łukasiewicz's view:

> . . . if A is predicated of every B and B of every C, it is necessary for A to be predicated of every C . . ..

But see [Corcoran, 1972] and [Smiley, 1973] for the view that Aristotle developed natural deduction systems rather than the axiomatic systems of the sort Łukasiewicz envisages.

Łukasiewicz uses Polish notation, a parenthesis-free notation, to express the well-formed formulas (wffs) in his formal system, which we refer to as ŁA. We replace his notation with current "standard" notation when giving the basis for it.[2] So, for example, his Cpq ('If p then q') is our $(p \to q)$. His Np ('not p') and Kpq ('p and q') are our $\neg p$ and $(p \wedge q)$, respectively.

Łukasiewicz's assertions and rejections are marked by $\vdash$ and $\dashv$, respectively. The system that is essentially Łukasiewicz's will be called ŁA.

So, for example, $\vdash(Aba \to (Acb \to Aca))$ says that Barbara is asserted in ŁA, which is true. $\dashv Aba$ says that Aba is rejected in ŁA, which is true. Assertions and rejections are relative to systems. We shall avoid using $\vdash$ŁA , say, and rely on the context to indicate that the assertion is relative to system ŁA.

Primitive symbols

term variables	$a, b, c, \ldots$ (with or without subscripts)
monadic operator	$\neg$
dyadic operator	$\to$
quantifiers	A, I
parentheses	(,)

[2]The manner of presentation of this system is heavily influenced by Hughes and Cresswell's presentations of various systems in [1996].

Formation rules

FR1 If Q_u is a quantifier and x and y are term variables then $Q_u xy$ is a *wff*.
FR2 If p and q are wffs then $\neg p$ and $(p \to q)$ are *wffs*.
FR3 The only wffs are those in virtue of FR1 and FR2.

So, for example, Aab, Iab and $(Abc \to \neg Ibc)$ are wffs. Read them as 'All a are b', 'Some a are b' and 'If all b are c then it is not true that some b are c', respectively.

Definitions

Def ∧ $(p \wedge q) =_{df} \neg(p \to \neg q)$
Def ↔ $(p \leftrightarrow q) =_{df} ((p \to q) \wedge (q \leftarrow p))$
Def E $Exy =_{df} \neg Ixy$
Def O $Oxy =_{df} \neg Axy$

Eab and Oab may be read as 'No a are b' and 'Some a are not b', respectively.

Łukasiewicz's ŁA contains theses that are "assertions" (indicated by $\vdash$) as well as theses that are "rejections"(indicated by $\dashv$). We begin with the former, which are generated by assertion axioms and assertion rules.

Assertion axioms

A0 (PC). If p is a wff that is valid in virtue of the propositional calculus (PC) then $\vdash p$ (that is, p is asserted). (So, for example, $\vdash(Aab \to Aab)$ since it is not possible that the antecedent Aab is true and the consequent Aab is false. And $\vdash((Aab \to Iab) \to (\neg Iab \to \neg Aab))$ since it is not possible that all of these conditions are met: $(Aab \to Iab)$ is true, $\neg Iab$ is true and $\neg Aab$ is false.)
A1 $\vdash Aaa$
A2 $\vdash Iaa$
A3 (Barbara) $\vdash(Abc \to (Aab \to Aac))$
A4 (Datisi) $\vdash(Abc \to (Iba \to Iac))$

Transformation rules for assertions

AR1 (Uniform substitution for assertions, US) From $\vdash p$ infer $\vdash q$ (that is, from the assertion of p infer the assertion of q) provided q is obtained from p by uniformly substituting variables for variables. (So, for example, from $\vdash(Aab \to Iba)$ we may infer $\vdash(Acb \to Ibc)$ and $\vdash(Abb \to Ibb)$, by rule US. But rule US does not permit us to infer that $\vdash(Aab \to Iba)$ given that $\vdash(Abb \to Ibb)$.
AR2 (Modus Ponens, MP) From $\vdash(p \to q)$ and $\vdash p$ infer $\vdash q$.
AR3 (*Definiens* and *definiendum* interchange for assertions, DDI) From $\vdash(\ldots\alpha\ldots)$ and $\alpha =_{df} \beta$ infer $\vdash(\ldots\beta\ldots)$, and vice versa. (So, for example, from $\vdash(\neg Iab \to \neg Aab)$ infer $\vdash(Eab \to Oab)$ by two uses of DDI, given definitions Def E and Def O. Typically a use of DDI will be indicated by simply referring to a definition that is used. So, from $(\neg Iab \to \neg Aab)$ infer $\vdash(Eab \to \neg Aab)$ by Def E. It is to be understood that DDI is also used.)

Given the assertion portion of the basis for ŁA, we shall give some "assertion deductions" — sequences of wffs such that each member of the sequence is either an assertion axiom or is entered from a prior member of the sequence by using a transformation rule for assertions — that capture some Aristotelian principles involving conversions, subordinations, and oppositions.

Theorem 1.1. (Assertoric conversions, Con) $\vdash(Iab \to Iba)$ and $\vdash(Eab \to Eba)$.

Proof.

1. $\vdash(Abc \to (Iba \to Iac))$ (by A4)
2. $\vdash(Abb \to (Iba \to Iab))$ (from 1 by US)
3. $\vdash Abb$ (by A1 and US)
4. $\vdash(Iba \to Iab)$ (from 2 and 3 by MP)
5. $\vdash(Iab \to Iba)$ (from 4 by US)
6. $\vdash((Iba \to Iab) \to (\neg Iab \to \neg Iba))$ (by A0)
7. $\vdash(\neg Iab \to \neg Iba)$ (from 6 and 4 by MP)
8. $\vdash(Eab \to Eba)$ (from 7 by Def E, using DDI) ■

The above reasoning may be presented more succinctly by using the following derived rule for assertions.

DR1 (Reversal, RV) i) From $\vdash(p \to q)$ infer $\vdash(\neg q \to \neg p)$; ii) from $\vdash(p \to (q \to r))$ infer $\vdash(p \to (\neg r \to \neg q))$; and iii) from $\vdash(p \to (q \to r))$ infer $\vdash(\neg r \to (p \to \neg q))$.

Proof. i) Suppose $\vdash(p \to q)$. By A0 $\vdash((p \to q) \to (\neg q \to \neg p))$. By MP $\vdash(\neg q \to \neg p)$. ii) Suppose $\vdash(p \to (q \to r))$. By A0 $\vdash((p \to (q \to r)) \to (p \to (\neg r \to \neg q)))$. By MP $\vdash(p \to (\neg r \to \neg q)))$. Use similar reasoning for iii). ■

So, the annotation for line 7 in the above deduction may read: '(from 4 by RV)'. Line 6 may be deleted.

The following derived rules are useful in generating other principles.

DR2 (Assertion by antecedent interchange, AI) From $\vdash(p \to (q \to r))$ infer $\vdash(q \to (p \to r))$.

Proof. Assume $\vdash(p \to (q \to r))$. By A0 $\vdash((p \to (q \to r)) \to (q \to (p \to r)))$. By MP $\vdash(q \to (p \to r))$. ■

DR3 (Assertion by antecedent strengthening (or equivalence), AS) From $\vdash$(p →(q →r)) and $\vdash$(s →q) infer $\vdash$(p →(s →r)); and from $\vdash$(p →(q →r)) and $\vdash$(s →p) infer $\vdash$(s →(q →r)).[3]

[3] 'Cut' is also used to refer to these rules.

DR4 (Assertion by consequent weakening (or equivalence), CW) From $\vdash(p \rightarrow q)$ and $\vdash(q \rightarrow r)$ infer $\vdash(p \rightarrow r)$; and from $\vdash(p \rightarrow (q \rightarrow r))$ and $\vdash(r \rightarrow s)$ infer $\vdash(p \rightarrow (q \rightarrow s))$.[4]

To prove **DR3** and **DR4** use A0 and MP.

Theorem 1.2. (Assertoric subalternations, Sub-a) i) $\vdash(Aab \rightarrow Iab)$; and ii) $\vdash(Eab \rightarrow Oab)$.

Proof.

1. $\vdash(Abc \rightarrow (Iba \rightarrow Iac))$ (by A4)
2. $\vdash(Iba \rightarrow (Abc \rightarrow Iac))$ (from 1 by AI)
3. $\vdash(Iaa \rightarrow (Aac \rightarrow Iac))$ (from 2 by US)
4. $\vdash Iaa$ (by A2)
5. $\vdash(Aac \rightarrow Iac)$ (from 3 and 4 by MP)
6. $\vdash(Aab \rightarrow Iab)$ (i, from 5 by US)
7. $\vdash(\neg Iab \rightarrow \neg Aab)$ (from 6 by RV)
8. $\vdash(Eab \rightarrow Oab)$ (ii) from 7 by DDI, using Def E and Def O) ■

Theorem 1.3. (Assertoric conversion *per accidens* , Con(pa)) i) $\vdash(Aab \rightarrow Iba)$; and ii) $\vdash(Eab \rightarrow Oba)$.[5]

Proof.

1. $(Aab \rightarrow Iab)$ (by Sub-a)
2. $(Iab \rightarrow Iba)$ (by Con)
3. $(Aab \rightarrow Iba)$ (i, from 1 and 2 by CW)
4. $(Eab \rightarrow Eba)$ (by Con)
5. $(Eba \rightarrow Oba)$ (from 4 by Sub-a and US)
6. $(Eab \rightarrow Oba)$ (ii, from 4 and 5 by CW) ■

The following derived rule, proven by using AO and MP, is useful in proving the next theorem.

DR5 (Biconditional rule, BIC) From $\vdash(p \rightarrow q)$ and $\vdash(q \rightarrow p)$ infer $\vdash(p \leftrightarrow q)$.[6]

Proof. Suppose $\vdash(p \rightarrow q)$ and $\vdash(q \rightarrow p)$. By A0, $\vdash((p \rightarrow q) \rightarrow ((q \rightarrow p) \rightarrow (p \leftrightarrow q)))$. By two uses of MP, $\vdash(p \leftrightarrow q)$. ■

Theorem 1.4. (Assertoric oppositions, Opp) i) $\vdash(\neg Aab \leftrightarrow Oab)$; ii) $\vdash(\neg Eab \leftrightarrow Iab)$; iii) $(\neg Iab \leftrightarrow Eab)$; and iv) $(\neg Oab \leftrightarrow Aab)$.

---

[4] 'Transitivity' and 'Hypothetical syllogism' are also used to refer to the first of these two rules.

[5] I. M. Bocheński, on p. 212 of [1963], states that the "law of accidental conversion of the universal negative is not in Aristotle". He is not saying that Aristotle considered inference ii) to be invalid.

[6] This rule is discussed, but not named, on p. 29 of [Hughes and Cresswell, 1996].

Proof

1. $\vdash(\neg Aab \to \neg Aab)$ (by A0)
2. $\vdash(\neg Aab \to Oab)$ (from 1 by Def O)
3. $\vdash(Oab \to \neg Aab)$ (from 1 by Def O)
4. $\vdash(\neg Aab \leftrightarrow Oab)$ (i, from 2 and 3 by BIC)
5. $\vdash(\neg Eab \to \neg Eab)$ (by A0)
6. $\vdash(\neg Eab \to \neg\neg Iab)$ (by Def E)
7. $\vdash(\neg\neg Iab \to Iab)$ (by A0)
8. $\vdash(\neg Eab \to Iab)$ (from 6 and 7 by CW)
9. $\vdash(\neg\neg Iab \to \neg Eab)$ (from 5 by Def E)
10. $\vdash(Iab \to \neg\neg Iab)$ (by A0)
11. $\vdash(Iab \to \neg Eab)$ (from 10 and 9 by CW)
12. $\vdash(\neg Eab \leftrightarrow Iab)$ (ii, from 11 by BIC)
13. $\vdash((\neg Aab \leftrightarrow Oab) \to (\neg Oab \leftrightarrow Aab))$ (by A0)
14. $\vdash(\neg Oab \leftrightarrow Aab)$ (iv, from 4 and 13 by MP)
15. $\vdash((\neg Eab \leftrightarrow Iab) \to (\neg Iab \leftrightarrow Eab))$ (by A0)
16. $\vdash(\neg Iab \leftrightarrow Eab)$ (iii, from 12 and 15 by MP) ■

The following derived rule is useful in conjunction with the assertoric oppositions.

DR6 (Substitution of equivalents, SE) From $\vdash(p \leftrightarrow q)$ and $\vdash(\ldots p \ldots)$ infer $\vdash(\ldots q \ldots)$.

Proof. Use mathematical induction. ■

So, for example, from $\vdash(Aab \to (Abc \to (\neg Aad \to \neg Acd)))$ infer $\vdash(Aab \to (Abc \to (Oad \to Ocd)))$ by SE, given the oppositions Opp.

On table 1 assertions corresponding to the familiar two-premised syllogisms are listed. In the right column a method of deducing the assertion is given. So, for example, Barbara is trivially asserted by using axiom A3. Celarent is asserted since the assertion of 11 (Disamis) may be transformed into $\vdash(\neg Iac \to (Aba \to \neg Ibc))$ (by RV), which may be transformed into $\vdash(Eac \to (Aba \to Ebc))$ (by SE, since $\vdash(Eac \leftrightarrow \neg Iac)$ and $\vdash(Ebc \leftrightarrow \neg Ibc)$), which may be transformed into 2 (by US, putting 'b' in place of 'a' and 'a' in place of 'b'). Darii is asserted since the assertion of 12 may be transformed into $\vdash(Abc \to (Iab \to Iac))$ (by AS, since $(Iab \to Iba)$).

1.1 Rejection in ŁA

Łukasiewicz uses the notion of "rejection" to develop his formal system.[7] He shows that the invalid syllogistic forms expressed by "elementary wffs" may be rejected by augmenting his formal system for assertions by adding one rejection axiom and four transformation rules that generate rejections. We shall illustrate this claim but not give a full account

[7]Smiley, in his influential article [1996], points out that Carnap and Łukasiewicz were the first logicians to formalize the notion of rejection. Smiley attributes the shunning of rejection by most logicians to Frege's [1960]. Smiley effectively argues that Frege's rejection of rejection, using Occam's razor, was unfortunate, and Smiley shows how rejection may be put to good use in ways other than those envisioned by Carnap or Łukasiewicz. For recent work on rejection that is stimulated by Smiley's article see [Rumfitt, 1997] and [Johnson, 1999b].

Table 1. Deductions in system ŁA

Figure 1	Barbara (1)	$\vdash(Abc \rightarrow (Aab \rightarrow Aac))$	A3
	Celarent (2)	$\vdash(Ebc \rightarrow (Aab \rightarrow Eac))$	11,RV,SE,US
	Darii (3)	$\vdash(Abc \rightarrow (Iab \rightarrow Iac))$	12,AS
	Ferio (4)	$\vdash(Ebc \rightarrow (Iab \rightarrow Oac))$	12,RV,SE,US
Figure 2	Cesare (5)	$\vdash(Ecb \rightarrow (Aab \rightarrow Eac))$	12,RV,SE,AI,US
	Camestres (6)	$\vdash(Acb \rightarrow (Eab \rightarrow Eac))$	3,RV,SE,US
	Festino (7)	$\vdash(Ecb \rightarrow (Iab \rightarrow Oac))$	11,RV,SE,AI,US
	Baroco (8)	$\vdash(Acb \rightarrow (Oab \rightarrow Oac))$	1,RV,SE,US
Figure 3	Darapti (9)	$\vdash(Abc \rightarrow (Aba \rightarrow Iac))$	12,AS
	Felapton (10)	$\vdash(Ebc \rightarrow (Aba \rightarrow Oac))$	20,RV,SE,US
	Disamis (11)	$\vdash(Ibc \rightarrow (Aba \rightarrow Iac))$	12,AI,US,CW
	Datisi (12)	$\vdash(Abc \rightarrow (Iba \rightarrow Iac))$	A4
	Bocardo (13)	$\vdash(Obc \rightarrow (Aba \rightarrow Oac))$	1,RV,SE,US
	Ferison (14)	$\vdash(Ebc \rightarrow (Iba \rightarrow Oac))$	3,RV,SE,US
Figure 4	Bramantip (15)	$\vdash(Acb \rightarrow (Aba \rightarrow Iac))$	20,AI,US,CW
	Camenes (16)	$\vdash(Acb \rightarrow (Eba \rightarrow Eac))$	17,RV,SE,AI,US
	Dimaris (17)	$\vdash(Icb \rightarrow (Aba \rightarrow Iac))$	3,AI,US,CW
	Fresison (18)	$\vdash(Ecb \rightarrow (Iba \rightarrow Oac))$	17,RV,SE,AI,US
	Fesapo (19)	$\vdash(Ecb \rightarrow (Aba \rightarrow Oac))$	15,RV,SE,AI,US
Subalterns	Barbari (20)	$\vdash(Abc \rightarrow (Aab \rightarrow Iac))$	1,CW
	Celaront (21)	$\vdash(Ebc \rightarrow (Aab \rightarrow Oac))$	9,RV,SE,US
	Cesaro (22)	$\vdash(Ecb \rightarrow (Aab \rightarrow Oac))$	9,RV,SE,AI,US
	Camestrop (23)	$\vdash(Acb \rightarrow (Eab \rightarrow Oac))$	20,RV,SE,US
	Camenop (24)	$\vdash(Acb \rightarrow (Eba \rightarrow Oac))$	15,RV,SE,AI,US

of Łukasiewicz's work on rejections, which would require showing that all wffs may be "reduced" to sets of elementary wffs.

Definition 1.5. (elementary wff and simple wff) x is an *elementary wff* iff x has form $(x_1 \rightarrow (x_2 \rightarrow (x_3 \rightarrow \ldots x_n)\ldots)$, where each x_i is a *simple wff*, a wff of form Apq, Ipq, Opq or Epq.

Rejection axioms for ŁA

R1 $\dashv(Acb \rightarrow (Aab \rightarrow Iac))$

Rejection transformation rules for ŁA

RR1 (Rejection by uniform substitution, R-US) If $\dashv x$ and x is obtained from y by uniform substitution of terms for terms, then $\dashv y$.

RR2 (Rejection by detachment (or Modus Tollens), R-D) From $\vdash(x \rightarrow y)$ and $\dashv y$ infer $\dashv x$.

RR3 (Słupecki's rejection rule, R-S) From $\dashv(x \to z)$ and $\dashv(y \to z)$ infer $\dashv(x \to (y \to z))$ provided: i) x and y have form $\neg Apq$ or $\neg Ipq$; and ii) z has form $(x_1 \to (x_2 \to (x_3 \to \ldots x_n) \ldots)$ where each x_i is a simple sentence.

RR4 (*Definiens* and *definiendum* interchange for rejections, R-DDI) From $\dashv(\ldots\alpha\ldots)$ and $\alpha =_{df} \beta$ infer $\dashv(\ldots\beta\ldots)$, and vice versa. (So, for example, from $\dashv(\neg Aab \to \neg Iab)$ infer $\dashv(Eab \to Oab)$ by two uses of R-DDI, given definitions Def O and Def E.)

The following derived rules for rejections, which are counterparts of derived rules for assertions, are useful in simplifying presentations of rejection deductions — sequences of wffs in which each member of the sequence is either an (assertion or rejection) axiom or is entered by an (assertion or rejection) transformation rule, where the last member of the sequence is a rejection.

R-DR1 (Rejection by reversal, R-RV) i) From $\dashv(p \to q)$ infer $\dashv(\neg q \to \neg p)$; ii) from $\dashv(p \to (q \to r))$ infer $\dashv(p \to (\neg r \to \neg q))$; and iii) from $\dashv(p \to (q \to r))$ infer $\dashv(\neg r \to (p \to \neg q))$.

Proof. i) Suppose $\dashv(p \to q)$. By A0 (or PC) $\vdash((\neg q \to \neg p) \to (p \to q))$. By R-D $\dashv(\neg q \to \neg p)$. ii) Suppose $\dashv(p \to (q \to r))$. By A0 $\vdash((p \to (\neg r \to \neg q)) \to (p \to (q \to r)))$. By R-D $\dashv(p \to (\neg r \to \neg q))$. Use similar reasoning for iii). ■

R-DR2 (Rejection by antecedent interchange, R-AI) From $\dashv(p \to (q \to r))$ infer $\dashv(q \to (p \to r))$.

Proof. Assume $\dashv(p \to (q \to r))$. By A0 $\vdash((q \to (p \to r)) \to (p \to (q \to r)))$. By R-D $\dashv(q \to (p \to r))$. ■

R-DR3 (Rejection by antecedent weakening (or equivalence), R-AW) i) From $\dashv(p \to (q \to r))$ and $\vdash(q \to s)$ infer $\dashv(p \to (s \to r))$; and ii) from $\dashv(p \to (q \to r))$ and $\vdash(p \to s)$ infer $\dashv(s \to (q \to r))$.

Proof. Suppose $\dashv(p \to (q \to r))$ and $\vdash(q \to s)$. By AO $\vdash((q \to s) \to ((p \to (s \to r)) \to (p \to (q \to r))))$. By MP $\vdash((p \to (s \to r)) \to (p \to (q \to r)))$. By R-D $\dashv(p \to (s \to r))$. Use similar reasoning for ii). ■

Proofs for the following two derived rules are easily constructed and will be omitted.

R-DR4 (Rejection by consequent strengthening (or equivalence), R-CS) From $\dashv(p \to q)$ and $\vdash(r \to q)$ infer $\dashv(p \to r)$; and from $\dashv(p \to (q \to r))$ and $\vdash(s \to r)$ infer $\dashv(p \to (q \to s))$.

R-DR5 (Rejection by substitution of equivalents, R-SE) From $\vdash(p \leftrightarrow q)$ and $\dashv(\ldots p \ldots)$ infer $\dashv(\ldots q \ldots)$.

R-DR6 (Rejection by implication introduction, R-II) From $\vdash p$ and $\dashv q$ infer $\dashv(p \to q)$.

Proof. Suppose $\vdash p$ and $\dashv q$. By A0 $\vdash (p \to ((p \to q) \to q))$. By MP $\vdash ((p \to q) \to q)$. By R-D $\dashv(p \to q)$. ■

Given the above apparatus we are able to show how the four syllogisms referred to at *Prior Analytics* 26a2-9 are rejected in ŁA. This is Łukasiewicz's translation from [1957, p. 67].

> If the first term belongs to all the middle [Aba], but the middle to none of the last [Ecb], there will be no syllogism of the extremes; for nothing necessary follows from the terms being so related; for it is possible that the first should belong to all as well as to none of the last, so that neither a particular nor a universal conclusion is necessary. But if there is no necessary consequence by means of these premises, there cannot be a syllogism. Terms of belong to all: animal, man, horse; to none: animal, man, stone.

The four syllogisms are $(Aba \to (Ecb \to x))$, where x is Ica, Oca, Aca or Eca. We shall give rejection deductions to establish the rejection of the first two (AEI-1 and AEO-1) and then use derived rule R-CS to show the last two (AEA-1 and AEE-1) are rejected.[8]

Theorem 1.6. (Rejection of AEI-1) $\dashv(Aba \to (Ecb \to Ica))$.

Proof.

1. $\dashv(Acb \to (Aab \to Iac))$ (by R1)
2. $\vdash(Iac \to (Acb \to (Aab \to Iac)))$ (by A0)
3. $\dashv Iac$ (from 1 and 2 by R-D)
4. $\vdash Acc$ (by A1 and US)
5. $\dashv(Acc \to Iac)$ (from 3 and 4 by R-II)
6. $\dashv(Acb \to Iab)$ (from 5 by R-US)
7. $\dashv(Eab \to Ocb)$ (from 6 by R-RV and R-SE)
8. $\dashv(Acb \to Iac)$ (from 5 by R-US)
9. $\dashv(Eac \to Ocb)$ (from 8 by R-RV and R-SE)
10. $\dashv(Eab \to (Eac \to Ocb))$ (from 7 and 9 by R-S)
11. $\dashv(Acb \to (Eac \to Iab))$ (from 10 by R-RV)
12. $\dashv(Aba \to (Ecb \to Ica))$ (from 11 by R-US) ■

[8] In the above passage Aristotle uses the semantic counterpart of this two-stage syntactic process. First, he shows by his counterexample that $\{Aba, Ecb, Aca\}$ and $\{Aba, Ecb, Eca\}$ are semantically consistent, from which it follows that neither of the particulars Oca and Ica is a semantic consequence of $\{Aba, Ecb\}$. Secondly, since the universal claims Eca and Aca are stronger than Oca and Ica, respectively, they cannot be a semantic consequence of $\{Aba, Ecb\}$. Aristotle is using what W. D. Ross [1949, p. 302] calls a "proof by contrasted instances," to show a pair of premises is, in Jonathan Lear's [1980, p. 54] terms, "semantically sterile".

Theorem 1.7. (Rejection of AEO-1) $\dashv(\mathrm{Aba} \rightarrow(\mathrm{Ecb} \rightarrow\mathrm{Oca}))$.

Proof.

1. $\dashv(Acb \rightarrow (Aab \rightarrow Iac))$ (By R1)
2. $\dashv((Acb \rightarrow (Eac \rightarrow Oab))$ (from 1 by R-RV and SE)
3. $\dashv(Aba \rightarrow (Ecb \rightarrow Oca))$ (from 2 by R-US) ■

Theorem 1.8. (Rejection of AEA-1 and AEE-1) i) $\dashv(Aba \rightarrow (Ecb \rightarrow Aca))$; and ii) $\dashv(Aba \rightarrow (Ecb \rightarrow Eca))$.

Proof.

1. $\dashv(Aba \rightarrow (Ecb \rightarrow Ica))$ (by theorem 1.6)
2. $\vdash(Aca \rightarrow Ica)$ (by Sub-a, US)
3. $\dashv(Aba \rightarrow (Ecb \rightarrow Aca))$. (i, from 1 and 2 by R-CS)
4. $\dashv(Aba \rightarrow (Ecb \rightarrow Oca))$ (by theorem 1.7)
5. $\vdash(Eca \rightarrow Oca)$ (by Sub-a, US)
6. $\dashv(Aba \rightarrow (Ecb \rightarrow Eca))$ (ii, from 4 and 5 by R-CS) ■

The following passage clearly shows that Ross favors Łukasiewicz's method of rejecting the AEx-1s over Aristotle's. On p. 302 of [1949] Ross says:

> ... [Aristotle] gives *no reason* (my italics) for this [claim that no conclusion is yielded by the premises of AEx-1], e.g. by pointing out that an undistributed middle or an illicit process is involved; but he often points to an empirical fact. ... instead of giving the reason why All B are A, No C is B yields no conclusion, he simply points to one set of values for A, B, C (animal, man, horse) for which, all B being A and no C being B, all C is in fact A, and to another set of values (animal, man, stone) for which, all B being A and no C being B, no C is in fact A. Since in the one case all C is A, a negative conclusion cannot be valid; and since in the other case no C is A, an affirmative conclusion cannot be valid. Therefore there is no valid conclusion (with C as subject and A as predicate).

Aristotle is reasoning as follows. It is true that all men are animals, it is true that no horses are men, and it is true that all horses are animals (and thus false that no horses are animals and false that some horses are not animals). So neither Eca nor Oca is a logical consequence of Aba together with Ecb. Since it is true that all men are animals, it is true that no stones are men, and it is true that no stones are animals (and thus false that all stones are animals and false that some stones are not animals), it follows that neither Aca nor Ica is a logical consequence of Aba together with Ecb.

Łukasiewicz also objects to Aristotle's reasoning, claiming in [1957, p. 72] that it:

> introduces into logic terms and propositions not germane to it. 'Man' and 'animal' are not logical terms, and the proposition 'All men are animals' is not a logical thesis. Logic cannot depend on concrete terms and statements. If we want to avoid this difficulty, we must reject some forms axiomatically.

But Aristotle's procedures have support among modern logicians. Robin Smith [1989, p. 114] regards Aristotle's reference to animals, men and horses as a reference to a "countermodel" and says "countermodels are the paradigmatic means of proving invalidity for modern logicians." In the surrounding text Smith refers to Jonathan Lear [1980, pp. 54-61 and pp. 70-75] who defends Aristotle's techniques against criticisms by Łukasiewicz and Geach [1972]. In the following sections we shall make extensive use of formal countermodels to show the invalidity of apodeictic and contingent syllogisms. Such models may also be used to show the invalidity of assertoric syllogisms.

The following passage from the *Prior Analytics 27b12-23,* quoted and discussed by Łukasiewicz on p. 70 of [1957], illustrates another method Aristotle uses to reject inferences. Ross [1949, p. 304] calls it an argument "from the ambiguity of a particular proposition." A better name for the reasoning is "rejection by premise weakening". Ross points out that this method of rejection is also used by Aristotle at 26b14-20, 27b27-28, 28b28-31, 29a6 and 35b11.

> Let M belong to no N, and not to some X. It is possible then for N to belong either to all X or to no X. Terms of belonging to none: black, snow, animal. Terms of belonging to all cannot be found, if M belongs to some X, and does not belong to some X. For if N belonged to all X, and M to no N, then M would belong to no X; but it is assumed that it belongs to some X. In this way, then, it is not possible to take terms, and the proof must start from the indefinite nature of the particular premise. For since it is true that M does not belong to some X, even if it belongs to no X, and since if it belongs to no X a syllogism is not possible, clearly it will not be possible either.

Given the semantic consistency of {No snow is black, Some animals are not black, No animal is snow } we know by half of the "contrasted instances" argument that neither 'Some animal is snow' nor 'All animals are snow' is a logical consequence of 'No snow is black' together with 'Some animals are not black.' So, a "countermodel" is given for the inferences from Enm and Oxm to Ixn or Axn. To show that neither Oxn nor Exn is a semantic consequence of Enm and Oxm, Aristotle relies on two facts: i) neither Oxn nor Exn is a semantic consequence of Enm and Exm; and ii) Oxm is a semantic consequence of Exm.

In ŁA a purely syntactic rejection of the "implicational syllogisms" $(Enm \to (Oxm \to Oxn))$ and $(Enm \to (Oxm \to Exn))$ is given by using R-AW.

Theorem 1.9. (Rejection of EOO-2 and EOE-2) i) $\dashv(Enm \to (Exm \to Oxn))$; and ii) $\dashv(Enm \to (Oxm \to Exn))$.

Proof.

1. $\dashv(Aba \to (Ecb \to Ica))$ (by theorem 1.6)
2. $\dashv(Eca \to (Ecb \to Oba))$ (from 1 by R-RV and R-SE)
3. $\vdash(Ecb \to Obc)$ (by Con(pa) and US)
4. $\dashv(Eca \to (Obc \to Oba))$ (from 2 and 3 by R-AW)
5. $\vdash(Eca \to Eac)$ (by Con and US)

6. $^{\dashv}(Eac \to (Obc \to Oba))$ (from 4 and 5 by R-AW)
7. $^{\dashv}(Enm \to (Oxm \to Oxn))$ (i, from 4 by R-US)
8. $^{\vdash}(Exn \to Oxn)$ (by Sub-a and US)
9. $^{\dashv}(Enm \to (Oxm \to Exn))$ (ii, from 7 and 8 by R-CS) ■

Up to this point we have rejected elementary wffs of form $(x_1 \to (x_2 \to \ldots (x_n \to y) \ldots)$ where $n \leq 2$. For Łukasiewicz's system to be fully Aristotelian he must show how elementary sentences, where $n > 2$, are rejected. We illustrate such a rejection.

Theorem 1.10. (Rejection of an AAAA mood) $^{\dashv}$(Aab →(Abc →(Adc →Aad))).

Proof.

1. $^{\dashv}(Acb \to (Aab \to Iac))$ (by R1)
2. $^{\vdash}(Acb \to (Aba \to Iac))$ (by Bramantip)
3. $^{\dashv}((Acb \to (Aba \to Iac))mc(Acb \to (Aab \to Iac))$ (from 2 and 1 by R-II)
4. $^{\vdash}((Aba \to Aab) \to ((Acb \to (Aba \to Iac))mc(Acb \to (Aab \to Iac)))$ (by A0)
5. $^{\dashv}(Aba \to Aab)$ (from 3 and 4 by R-D)
6. $^{\vdash}Aaa$ (by A1)
7. $^{\dashv}(Aaa \to (Aba \to Aab))$ (from 6 and 5 by R-II)
8. $^{\dashv}(Aaa \to (Aaa \to (Aba \to Aab)))$ (from 7 and 5 by R-II)
9. $^{\dashv}(Aaa \to (Aaa \to (Ada \to Aad)))$ (from 8 by R-US)
10. $^{\dashv}(Aaa \to (Aac \to (Adc \to Aad)))$ (from 9 by R-US)
11. $^{\dashv}(Aab \to (Abc \to (Adc \to Aad)))$ (from 10 by R-US) ■

Łukasiewicz's system for the assertoric syllogistic has "100% Aristotelicity", to use McCall's expression. This means that every 2-premised syllogism deemed valid by Aristotle is asserted in Łukasiewicz's system, and every 2-premised syllogism deemed invalid by Aristotle is rejected in Łukasiewicz's system. We shall see below that McCall's L-X-M calculus also has 100% Aristotelicity though his Q-L-X-M calculus does not.

2 ŁUKASIEWICZ'S MODAL SYSTEM, ŁM

Łukasiewicz developed his system for the assertoric syllogistic by using the non-modal propositional calculus, what he calls the "theory of deduction," as a "base logic". Following the procedure used in Hughes and Cresswell's [1968] and [1996], we simplified Łukasiewicz's presentation of his system by simply using axiom A0 to provide his "basis". Łukasiewicz's approach to Aristotle's modal logic is to develop a modal propositional logic (with quantifiers), which we refer to as the "ŁM system", that will enable him to present Aristotle's work on the modal syllogisms.[9]

The following sentences are tautologies in ŁM, modifying Łukasiewicz's notation in a natural way: 1) $((p \to q) \to (Mp \to Mq))$ and 2) $((p \to q) \to (Lp \to Lq))$, reading M and L as 'it is possible that' and 'it is necessary that', respectively. The following passages on p. 138 of [Łukasiewicz, 1957] attempt to show that the "M-law of extensionality" (1) and the "L-law of extensionality" (2) are endorsed by Aristotle.

[9] See [1961] for Smiley's extensions of Łukasiewicz's work on ŁM.

> First it has to be said that if (if α is, β must be), then (if α is possible, β must be possible too). [34a5-7]

> If one should denote the premises by α, and the conclusion by β, it would not only result that if α is necessary, then β is necessary, but also that if α is possible, then β is possible. [34a22-24]

> It has been proved that if (if α is, β is), then (if α is possible, then β is possible). [34a29-31]

A more natural reading of these passages is that they show that Aristotle endorsed both 3) $(L(p \rightarrow q) \rightarrow (Mp \rightarrow Mq))$ and 4) $(L(p \rightarrow q) \rightarrow (Lp \rightarrow Lq))$.[10]

That 1) - 4) are tautologies in ŁM is seen by considering the following four-valued truth tables.

Table 2. Four-valued truth tables for $\rightarrow$, $\neg$, L and M

$\rightarrow$	1	2	3	4	$\neg$	M	L
*1	1	2	3	4	4	1	2
2	1	1	3	3	3	1	2
3	1	2	1	2	2	3	4
4	1	1	1	1	1	3	4

Among the four truth values 1 to 4, 1 is the only designated value, marked with an asterisk in its entry in the first column on the table. A sentence x in the ŁM-system is a *tautology* iff for every input of values the output value is always the designated value 1.

Theorem 2.1. (L-law of extensionality) $((p \rightarrow q) \rightarrow (Lp \rightarrow Lq))$ is a tautology.

Proof. Suppose $((p \rightarrow q) \rightarrow (Lp \rightarrow Lq))$ is assigned a value other than 1. Then i) $(p \rightarrow q)$ is not assigned 4 and ii) $(Lp \rightarrow Lq)$ is not assigned 1, and iii) the value assigned to $(p \rightarrow q)$ is not the value assigned to $(Lp \rightarrow Lq)$. By i) p is not assigned 1 and q is not assigned 4. By ii) Lp is not assigned 4 and thus p is assigned neither 3 nor 4. And by ii) Lp is not assigned the same value as Lq. So p is assigned the value 2 and q is assigned the value 3. Then $(p \rightarrow q)$ and $(Lp \rightarrow Lq)$ are assigned the same value, which conflicts with iii). ■

Proofs that 1), 3) and 4) are tautologies are not required for our purposes, and we omit the straightforward proofs.

McCall [1963, pp. 31-32] points out that Łukasiewicz's use of the L-law of extensionality yields highly unAristotelian results. For example, using McCall's notation, Camestres LXL ('Necessarily all c are b; no a are b; so (necessarily) necessarily no a

[10] See [Hughes and Cresswell, 1968, pp. 29-30] for a discussion of this sentence, an axiom in Robert Feys's *System T*.

are c'), Baroco LXL ('Necessarily all c are b; some a are not b; so necessarily some a are not c'), Barbara XLL ('All b are c; necessarily a are b; so necessarily all a are c') and Ferio XLL ('No b are c; necessarily some a are b; so necessarily some b are c'), when construed as "implicational syllogisms", are asserted in Łukasiewicz's Ł-system even though Aristotle rejects all of them.

Following McCall we use 'XXX' after the name of a syllogism to indicate that the syllogism is a plain, assertoric syllogism. So, for example, Camestres XXX has form 'All c are b; no a are b; so no a are c'. Camestres XXX, Baroco XXX, Barbara XXX and Ferio XXX are asserted in Łukasiewicz's assertoric system. So, given the following theorem, Camestres LXL, Baroco LXL, Barbara XLL and Ferio XLL are asserted in Łukasiewicz's Ł-system.

Theorem 2.2. i) $\vdash((p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow (Lq \rightarrow Lr)))$; and ii) $\vdash((p \rightarrow (q \rightarrow r)) \rightarrow (Lp \rightarrow (q \rightarrow Lr)))$.

Proof.

1. $\vdash((q \rightarrow r) \rightarrow (Lq \rightarrow Lr))$ (by theorem 2.1)
2. $\vdash(((q \rightarrow r) \rightarrow (Lq \rightarrow Lr)) \rightarrow ((p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow (Lq \rightarrow Lr)))$ (by A0)
3. $\vdash((p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow (Lq \rightarrow Lr)))$ (i, from 1 and 2 by MP)
4. $\vdash(((p \rightarrow (q \rightarrow r)) \rightarrow (p \rightarrow (Lq \rightarrow Lr)) \rightarrow ((q \rightarrow (p \rightarrow r)) \rightarrow (Lq \rightarrow (p \rightarrow Lr)))$ (by A0)
5. $\vdash((q \rightarrow (p \rightarrow r)) \rightarrow (Lq \rightarrow (p \rightarrow Lr)))$ (from 3 and 4 by MP)
6. $\vdash((p \rightarrow (q \rightarrow r)) \rightarrow (Lp \rightarrow (q \rightarrow Lr)))$ (ii, from 5 by US) ■

One of the virtues of McCall's L-X-M calculus, discussed below, is that Camestres LXL, Baroco LXL, Barbara XLL and Ferio XLL are rejected in it. But before we examine McCall's system we look briefly at some recent systems of modal predicate logic that have been used to attempt to understand Aristotle's work on the modal syllogisms.

3 MODERN MODAL PREDICATE LOGIC

It is natural to try to view Aristotle's modal logic through the eyes of modern modal monadic first order predicate logic.[11] On pp. 18-22 McCall refers to Albrecht Becker's [1933][12] and works by others who have tried to do this. On pp. 176-181 Patterson discusses Ulrich Nortmann's [1990] attempt to do this. Patterson points out that the Kripkean "possible worlds semantics" used by Nortmann does not conform with Aristotle's ontological principles. I agree. McCall argues that all uniform readings of Aristotle's modal propositions as sentences in a modal first order predicate logic will make some valid Aristotelian syllogisms invalid or will make some invalid Aristotelian syllogisms valid. I also agree with McCall and will give some examples that support his position.

[11]For recent books that contain sections on modal predicate logic see [Hughes and Cresswell, 1996], [Fitting and Mendelsohn, 1998], [Girle, 2000] and [Bell *et al.*, 2001].

[12]See [Bocheński, 1963, pp. 57-62] for a useful discussion of Becker's work.

To illustrate how invalid Aristotelian inferences may be made valid consider Bocardo LXL, (that is, '$LObc, Aba$; so $LOac$', using McCall's notation). Suppose we translate it into modal predicate logic as: '$\exists x(Bx \wedge \Box\neg Cx); \forall x(Bx \rightarrow Ax)$; so $\exists x(Ax \wedge \Box\neg Cx)$' (that is, 'There is an x such that x is a B and x is necessarily not a C; for all x if x is a B then x is an A; so there is an x such that x is an A and x is necessarily not a C'). We are using one of Becker's two methods for translating LO sentences. Using "singular sentences" such as Bm (read as m is a B, for 'Max is a bear', for example) and familiar rules such as Existential Instantiation (EI)[13], Universal Instantiation (UI) and Existential Generalization (EG) together with propositional calculus (PC) inferences we may construct a deduction for Bocardo LXL, which Aristotle considered to be invalid.[14]

Proof.

1. $\exists x(Bx \wedge \Box\neg Cx)$ (premise)
2. $\forall x(Bx \rightarrow Ax)$ (premise)
3. $(Bm \wedge \Box\neg Cx)$ (from 1 by EI)
4. $(Bm \rightarrow Am)$ (from 2 by UI)
5. $(Am \wedge \Box\neg Cx)$ (from 3 and 4 by PC)
6. $\exists x(Ax \wedge \Box\neg Cx)$ (from 5 by EG) ■

To illustrate how valid Aristotelian inferences may be made invalid, consider Bocardo LLL, (that is, 'LObc; LAba; so LOac', using McCall's notation). Using another Becker translation of LO sentences and a Becker translation of LA sentences the argument amounts to this: '$\forall x(Cx \rightarrow \Box Bx); \exists x(\Box Ax \wedge \Box\neg Bx)$; so $\exists x(\Box Ax \wedge \Box\neg Cx)$, call it the "the MPredC argument". Aristotle at [30a6-14] gives a proof by *ecthesis* to show that Bocardo LLL is valid. But using the semantics for the modal system, S5, the translated argument is S5-invalid. For suppose there are only two possible worlds w_1 and w_2, where each world "sees" each world (including itself). If "the MPredC argument" is S5-valid then the following modal propositional calculus argument is S5-valid, call it the "the MPropC argument": '$((Cm \rightarrow \Box Bm) \wedge (Cn \rightarrow \Box Bn)); ((\Box Am \wedge \Box\neg Bm) \vee (\Box An \wedge \Box\neg Bn))$; so $((\Box Am \wedge \Box\neg Cm) \vee (\Box An \wedge \Box\neg Cn))$'. But then a countermodel is constructed by: i) letting Am, Bn and Cn be true in world w_1; ii) letting Bm, Cm and An be false in w_1; iii) letting Am, Cm and Bn be true in world w_2; and iv) letting Bm, An and Cn be false in world w_2. Then in w_1 $(Cm \rightarrow \Box Bm)$ is true, $(Cn \rightarrow \Box Bn)$ is true, $(\Box Am \wedge \Box\neg Bm)$ is true, $\Box Am \wedge \Box\neg Cm)$ is false, and $(\Box An \wedge \Box\neg Cn)$ is false. So "the MPropC argument" is S5-invalid. So "the MPredC argument" is invalid.

The same countermodel may be used to invalidate the argument that results by replacing the premise $\forall x(Cx \rightarrow \Box Bx)$ in "the MPredC" argument with $\forall x(Cx \rightarrow \Box Bx)$.

Geach [1964, p. 202] makes the following remarks about McCall's comments list of seven "Becker-type interpretations":

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[13]In [Johnson, 1993] Aristotle's proofs by *ecthesis* are treated as essentially proofs by Existential Instantiation. For alternative accounts of proofs by *ecthesis* see [Thom, 1993] and [Smith, 1982].

[14]Paul Thom in [1991] argues that Aristotle made a mistake in regarding Bocardo LXL as valid. Thom contrasts his views with those in [Johnson, 1989], [Patterson, 1989], [Patterson, 1990] and [van Rijen, 1989].

> Here McCall has not proved what he claims: namely that no Becker-type interpretation will secure simultaneously the validity of Barbara LLL and LXL, the invalidity of Barbara XLL, and the simple conversion of LI propositions (C LIab LIba). For all of these results are obtained if we combine reading (i) of LA from McCall's list with reading (iii) or equivalently (iv) of LI.

McCall's list on p. 21 of Becker type interpretations is given on table 3.

Table 3. Seven Becker-type interpretations

	Universal	*Particular*
(i)	$\forall x(Ax \rightarrow \Box Bx)$	$\exists x(Ax \wedge \Box Bx)$
(ii)	$\Box\forall x(Ax \rightarrow Bx)$	$\Box\exists x(Ax \wedge Bx)$
(iii)	$\forall x\Box(Ax \rightarrow Bx)$	$\exists x\Box(Ax \wedge Bx)$
(iv)	$\forall x(\Box Ax \rightarrow \Box Bx)$	$\exists x(\Box Ax \wedge \Box Bx)$
(v)	$\forall x(\Diamond Ax \rightarrow \Box Bx)$	$\exists x(\Box Ax \wedge \Box Bx)$
(vi)	$\forall x(\Diamond Ax \rightarrow Bx)$	$\exists x(\Box Ax \wedge \Box Bx)$
(vii)	$\forall x(\Box Ax \rightarrow Bx)$	$\exists x(\Box Ax \wedge Bx)$

McCall finds interpretations (i) and (ii) in [Becker-Freyseng, 1933], (ii) in [von Wright, 1951], (i) to (v) in [Sugihara, 1957a] and [Sugihara, 1957b], and all but (v) in [Rescher, 1963].

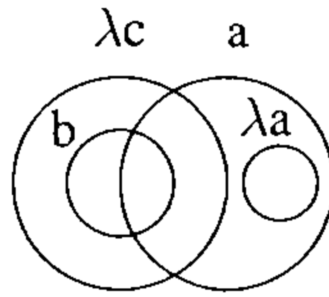
This is what McCall says about these seven interpretations:

> None of these interpretations does justice to Aristotle's system. Not one of them even simultaneously provides for the validity of Barbaras LLL, the invalidity of Barbara XLL, and the convertibility of the particular premise 'Some A is necessarily B' into 'Some B is necessarily A'.

And McCall is correct. Geach is in effect proposing two more interpretations in addition to the seven on the list. Let us call one of them (viii), where $LAab$ is translated as $\forall x(Ax \rightarrow \Box Bx)$ and $LIab$ is translated as $\exists x\Box(Ax \wedge Bx)$. As Geach says, the other one is essentially the same as it. But interpretation (viii) produces results that are not Aristotelian. For example, if Darii-LXL, valid for Aristotle, is translated using interpretation (viii) the resulting argument is S5-invalid. McCall is looking for an interpretation that provides "100% Aristotelicity". Geach (p. 202) invites the reader to consider an interpretation of McCall's LAab and LOab as sentences of an extended assertoric syllogistic, call it the "G-system", that allows sentences to be formed by using complex terms, terms of form λp (necessarily p) and μp (possibly p), where p is a simple term. McCall's $LAab$, $LEab$, $LIab$ and $LOab$ are translated into the G-system as $Aa\lambda b$, $Ea\mu b$, $I\lambda a\lambda b$ and $Oa\mu b$ respectively. Geach (p. 202) says:

> A decision procedure for this calculus can easily be devised: write every formula so that λ-terms and μ-terms appear instead of categoricals prefaced

Figure 1. The invalidity of Darii LXL in the G-system



> with L, add an antecedent of the form $CA\lambda aa$ [that is, $(A\lambda aa \rightarrow]$ for each λ-term and one of the form $CAa\mu a$ [that is, $(Aa\mu a \rightarrow]$ for each μ-term, and apply Łukasiewicz's decision procedure for the plain syllogistic to the resulting formula.

So, for example, to determine whether Bocardo LXL (that is, 'LObc; Aba; so LOac') is syntactically accepted or syntactically rejected we form the following sentence in the G-system: $(Ac\mu c \rightarrow (Ob\mu c \rightarrow (Aba \rightarrow Oa\mu c)))$. Following Łukasiewicz's decision procedure on pp. 121-126 of [1957], we form an elementary sentence consisting of affirmative simple sentences that is deductively equivalent to it: $(Ac\mu c \rightarrow (Aa\mu c \rightarrow (Aba \rightarrow Ab\mu c)))$ or (by interchanging terms) $(Ab\mu b \rightarrow (Ac\mu b \rightarrow (Aac \rightarrow Aa\mu b)))$. The latter sentence fits subcase (d) of the fifth case (p. 124):

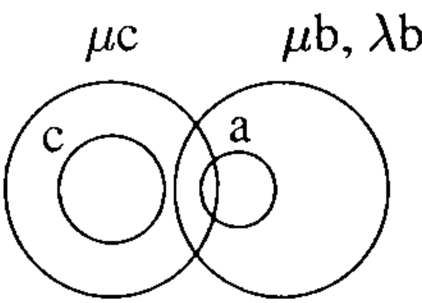
> The consequent is Aab, and there are antecedents of the type Aaf with f different from a. If there is a chain leading from a to b the expression is asserted on the ground of axiom 3 [our A3, above], the mood Barbara; if there is no such chain, the expression is rejected.

Since a is linked to b by the chain $\{Aac, Ac\mu b\}$, $(Ab\mu b \rightarrow (Ac\mu b \rightarrow (Aac \rightarrow Aa\mu b)))$ is accepted. So Bocardo LXL is accepted in the G-system. But for Aristotle Bocardo LXL is valid.

Since questions of validity in the G-system are reduced to questions of validity in the assertoric syllogistic, the familiar Euler diagrams provide a technique for determining whether or not arguments are valid. So, for example, the diagram in figure 1 displays the invalidity of Darii LXL, $(LAbc \rightarrow (Iab \rightarrow LIac))$. Since circle b is included in circle λc, $LAbc$ is true. Since circle a overlaps circle b, Iab is true. Since circle λa does not overlap λc, $LIac$ is false. When constructing such diagrams these conditions must be met: for every term x, the λx circle is included in or equal to the x circle, which is included in or equal to the μx circle. These conditions are natural since whatever is necessarily x is x, and whatever is x is possibly x.

The diagram in figure 2 displays the invalidity of Cesare LLL, $(LEcb \rightarrow (LAab \rightarrow LEac))$. $LEcb$ is true since circle c does not overlap circle μb; $LAab$ is true since circle a is included in circle λb, which is identical to circle μb; and $LEac$ is false since circle a overlaps circle μc.

Figure 2. The invalidity of Cesare LLL in the G-system



Geach does not claim that his G-system has "100 percent Aristotelicity". He says on p. 203 of [1964] that it "can fit in most of Aristotle's results about syllogisms *de necessario* ". But table 4 shows that the G-system does not get high marks. "V" occurs in a cell if and only if the relevant syllogism is valid for Aristotle, and "Gc" occurs in a cell if and only if the G-system's judgment about the acceptance or rejection of the relevant syllogism is in conflict with Aristotle's. So, for example, the "Gc" in the Darii/LXL cell means that Darii LXL is rejected in the G-system though Aristotle accepts it. The "Gc" in the Bocardo/LXL cell means that Bocardo LXL is accepted in the G-system though Aristotle rejects it. The G-system's Aristotelicity is $((3 \times 14) - 13) \div (3 \times 14)$ or about 69%.

Table 4. Aristotle's system vs. the G-system

		LLL	LXL	XLL
Figure 1	Barbara	V	V	
	Celarent	V	V	
	Darii	V	V,Gc	
	Ferio	V	V	
Figure 2	Cesare	V,Gc	V,Gc	
	Camestres	V,Gc		V,Gc
	Festino	V,Gc	V,Gc	
	Baroco	V,Gc		
Figure 3	Darapti	V	V,Gc	V,Gc
	Felapton	V	V	
	Disamis	V		V,Gc
	Datisi	V	V,Gc	
	Bocardo	V	Gc	
	Ferison	V	V	

Geach's G-system and Łukasiewicz's ŁM illustrate two approaches to understanding Aristotle's work on modal logic. Martha Kneale on p. 91 of [Kneale and Kneale, 1962] poses a dilemma for students of Aristotle given her belief that there are only two approaches to Aristotle's work.

> If modal words modify predicates [Geach's *de re* approach is taken], there is no need for a special theory of *modal* syllogisms. For these are only ordinary assertoric syllogisms of which the premises have peculiar predicates. On the other hand, if modal words modify the whole statements to which they are attached [Łukasiewicz's *de dicto* approach is taken], there is no need for a special modal *syllogistic* since the rules determining the logical relations between modal statements are independent of the character of the propositions governed by the modal words.

McCall agrees with Kneale that the two approaches described above are inadequate. And he devises a third approach that is designed to "catch the fine distinctions Aristotle makes between valid and invalid syllogisms (p. 96)".

4 MC CALL'S L-X-M SYSTEM

The basis for L-X-M includes that of ŁA together with the following primitive symbols, formation rules, definitions, axioms and transformation rules. Only some of the rejection axioms are given here. The partial list is big enough to illustrate how rejection deductions are constructed in L-X-M. For the full list of rejection axioms see [McCall, 1963] or [Johnson, 1989].

Primitive symbols

monadic operator L

Formation rules

FR1′ If Q_u is a quantifier and x and y are term variables then Q_uxy is a *categorical expression* .
FR2′ If p is a categorical expression then $\neg p$ is a *categorical expression* and Lp is a *wff* .
FR3′ Categorical expressions are *wffs* .
FR4′ If p and q are wffs then $\neg p$ and $(p \to q)$ are *wffs* .
FR5′ The only *wffs* are those in virtue of FR1′ to FR4′.

So, for example, Aab is a categorical expression by FR1′, so $\neg Aab$ is a categorical expression by FR2′, so $\neg\neg Aab$ is a categorical expression by FR2′, so $L\neg\neg Aab$ is a wff by FR2′, so $\neg L\neg\neg Aab$ is a wff by FR4′. Note that $LLAab$ is not a wff.

Definitions

Def M $Mp =_{df} \neg L\neg p$

Assertion axioms

Use A0, A1, A3 and A4 from system ŁA. Change A2 for ŁA from $\vdash Iaa$ to $\vdash LIaa$. Then add the following axioms.

A5 (Barbara LXL) $\vdash(LAbc \to (Aab \to LAac))$
A6 (Cesare LXL) $\vdash(LEcb \to (Aab \to LEac))$
A7 (Darii LXL) $\vdash(LAbc \to (Iab \to LIac))$
A8 (Ferio LXL) $\vdash(LEbc \to (Iab \to LOac))$
A9 (Baroco LLL) $\vdash(LAcb \to (LOab \to LOac))$
A10 (Bocardo LLL) $\vdash(LObc \to (LAba \to LOac))$
A11 (LI conversion) $\vdash(LIab \to LIba)$
A12 (LA subordination) $\vdash(LAab \to Aab)$
A13 (LI subordination) $\vdash(LIab \to Iab)$
A14 (LO subordination) $\vdash(LOab \to Oab)$

Assertion transformation rules

Use the assertion transformation rules AR1 to AR3 from ŁA and add the following rule.

AR4 (Assertions involving doubly negated categorical expressions, DN) From $\vdash(\ldots p \ldots)$ infer $\vdash(\ldots \neg\neg p \ldots)$ and vice versa, if p is a categorical expression. (So, for example, from $\vdash(LAab \to LAab)$ infer $\vdash(LAab \to L\neg\neg Aab)$ by DN. By using SE we may infer that $\vdash(LAab \to \neg\neg LAab)$ given $\vdash(LAab \to LAab)$.)

Rejection axioms

Use R1 from system ŁA and add the following rejection axioms.

R2 (*5.21, p. 58) $\dashv$(LAbb →(MAab →(Aac →(LAca →(LAbc →LAac)))))
R3 (*5.3, p. 58) $\dashv$(LAaa →(LAcc →(MAac →(LAca →Aac))))[15]
R4 (*5.6, p. 64) $\dashv$(LAaa →(LAbb →(LAcc →(LAab →(MAba →(MAbc →(LAcb →Iac)))))))

Page references are to McCall's [1963]. McCall uses asterisks to refer to rejections.

Rejection transformation rules

Use rejection transformation rules RR1-RR4 as well as the following rule.

RR5 (Rejections involving doubly negated categorical expressions, R-DN) From $\dashv(\ldots p \ldots)$ infer $\dashv(\ldots \neg\neg p \ldots)$ and vice versa, if p is a categorical expression. (So, for example, from $\dashv L\neg\neg Iab$ infer $\dashv LIab$.)

We imitate the discussion of Łukasiewicz's ŁA system by proving various "immediate inferences". Oppositions, conversions, subalternations and subordinations are listed.

Theorem 4.1. (Apodeictic oppositions, Ap-opp) i) $\vdash(\neg LAab \leftrightarrow MOab)$; ii) $\vdash(\neg MOab \leftrightarrow LAab)$; iii) $\vdash(\neg LEab \leftrightarrow MIab)$; iv) $\vdash(\neg MIab \leftrightarrow LEab)$; v)

[15] Correction: on p. 273 of [1989] change $\to LAac$ in *5.3 to $\to Aac$.

$\vdash(\neg LIab \leftrightarrow MEab)$; vi) $\vdash(\neg MEab \leftrightarrow LIab)$; vii) $\vdash(\neg LOab \leftrightarrow MAab)$; and viii) $\vdash(\neg MAab \leftrightarrow LOab)$.

Proof.

1. $\vdash(\neg LAab \leftrightarrow \neg LAab)$ (by A0)
2. $\vdash(\neg LAab \leftrightarrow \neg L\neg\neg Aab)$ (from 1 by DN)
3. $\vdash(\neg LAab \leftrightarrow MOab)$ (i, from 2 by DDI, given Def M and Def O)
4. $\vdash(\neg MOab \leftrightarrow \neg\neg LAab)$ (from 3 by RV)
5. $\vdash(\neg\neg LAab \leftrightarrow LAab)$ (by A0)
6. $\vdash(\neg MOab \leftrightarrow LAab)$ (ii, from 4 and 5 by SE)
7. $\vdash(\neg LEab \leftrightarrow \neg LEab)$ (by A0)
8. $\vdash(\neg LEab \leftrightarrow \neg L\neg\neg Eab)$ (from 7 by DN)
9. $\vdash(\neg LEab \leftrightarrow \neg L\neg\neg\neg Iab)$ (from 8 by DDI, given Def E)
10. $\vdash(\neg LEab \leftrightarrow \neg L\neg Iab)$ (from 9 by DN)
11. $\vdash(\neg LEab \leftrightarrow MIab)$ (iii, from 10 by DDI, given Def M)
12. $\vdash(\neg MIab \leftrightarrow \neg\neg LEab)$ (from 11 by RV)
13. $\vdash(\neg\neg LEab \leftrightarrow LEab)$ (by A0)
14. $\vdash(\neg MIab \leftrightarrow LEab)$ (iv, from 12 and 13 by SE) ■

Use similar reasoning for the other four asserted biconditionals.

Theorem 4.2. (Apodeictic conversions, Ap-con) i) $\vdash(LEab \rightarrow LEba)$; ii) $\vdash(LIab \rightarrow LIba)$; iii) $\vdash(MEab \rightarrow MEba)$; and iv) $\vdash(MIab \rightarrow MIba)$.

Proof.

1. $\vdash(LIab \rightarrow LIba)$ (ii, by A11)
2. $\vdash(\neg LIba \rightarrow \neg LIab)$ (from 1 by RV)
3. $\vdash(MEba \rightarrow MEab)$ (from 2 by SE, Ap-opp)
4. $\vdash(MEab \rightarrow MEba)$ (iii, from 3 by US)
5. $\vdash(LEcb \rightarrow (Aab \rightarrow LEac))$ (A6)
6. $\vdash Aaa$ (A1)
7. $\vdash(LEca \rightarrow LEac)$ (from 5 and 6 by AI, US, MP)
8. $\vdash(LEab \rightarrow LEba)$ (i, from 7 by US)
9. $\vdash(MIab \rightarrow MIba)$ (iv, from 8 by RV, SE, US) ■

Theorem 4.3. (Apodeictic subalternations, Ap-sub-a) i) $\vdash(LAab \rightarrow LIab)$; ii) $\vdash(LEab \rightarrow LOab)$; iii) $\vdash(MAab \rightarrow MIab)$; and iv) $\vdash$(MEab →MOab).

Proof.

1. $\vdash(LAab \rightarrow LIba)$ (by A11)
2. $\vdash(LIba \rightarrow LIab)$ (by Ap-con, US)
3. $\vdash(LAab \rightarrow LIab)$ (i, from 1 and 2 by CW)

4. $\vdash(MEab \to MOab)$ (iv, from 3 by RV, SE)
5. $\vdash(LEbc \to (Iab \to LOac))$ (by A8)
6. $\vdash(LEac \to (Iaa \to LOac))$ (from 5 by US)
7. $\vdash(LEab \to LOab)$ (ii, from 6 by AI, MP, US)
8. $\vdash(MAab \to MIab)$ (iii, from 7 by RV, SE) ■

Theorem 4.4. (Apodeictic conversions *per accidens*, Ap-con(pa)) i) $\vdash(LAab \to LIba)$; ii) $\vdash(MAab \to MIba)$; iii) $\vdash(LEab \to LOba)$; and iv) $\vdash(MEab \to MOba)$.

Proof.

1. $\vdash(LAab \to LIab)$ (by Ap-sub-a)
2. $\vdash(LIab \to LIba)$ (by Ap-con)
3. $\vdash(LAab \to LIba)$ (i, from 1 and 2 by CW)
4. $\vdash(LEab \to LEba)$ (by Ap-con)
5. $\vdash(LEba \to LOba)$ (by Ap-sub-a, US)
6. $\vdash(LEab \to LOba)$ (iii, from 4 and 5 by CW)
7. $\vdash(MAab \to MIba)$ (ii, from 6 by RV, SE, US)
8. $\vdash(MEab \to MOba)$ (iv, from 3 by RV, SE, US) ■

Theorem 4.5. (Subordinations, Sub-o) i) $\vdash(LAab \to Aab)$; ii) $\vdash(Aab \to MAab)$; iii) $\vdash(LEab \to Eab)$; iv) $\vdash(Eab \to MEab)$; v) $\vdash(LIab \to Iab)$; vi) $\vdash(Iab \to MIab)$; vii) $\vdash(LOab \to Oab)$; and viii) $\vdash(Oab \to MOab)$.

Proof.

1. $\vdash(LOab \to Oab)$ (vii, by A14)
2. $\vdash(Aab \to MAab)$ (ii, from 1 by RV, SE)
3. $\vdash(Aaa \to MAaa)$ (from 2 by US)
4. $\vdash Aaa$ (by A1)
5. $\vdash MAaa$ (from 3 and 4 by MP)
6. $\vdash(LEbc \to (Iab \to LOac))$ (by A8)
7. $\vdash(MAac \to (Iab \to MIbc))$ (from 6 by RV, SE)
8. $\vdash(MAaa \to (Iab \to MIba))$ (from 7 by US)
9. $\vdash(Iab \to MIba)$ (from 5 and 8 by MP)
10. $\vdash(MIba \to MIab)$ (by Ap-con, US)
11. $\vdash(Iab \to MIab)$ (vi, from 9 and 10 by CW)
12. $\vdash(LEab \to Eab)$ (iii, from 11 by RV, SE, US)

Proofs of the other four subordinations are straightforward and are omitted. ■

We show that all of the entries marked with 'V' on table 4 and all of the entries marked with a blank on table 5 correspond to asserted wffs in L-X-M. Proofs are streamlined by assuming immediate inferences established above and any immediate inferences obtainable from them by US. So, for example, in the proof of Barbari LXL from Barbara LXL by CW in theorem LXL the subalternation $\vdash(LAac \rightarrow LIac)$ is assumed.

Theorem 4.6. All unmarked LXL and XLL cells on table 5 represent asserted wffs.

Proof.

1. $\vdash(LAbc \rightarrow (Aab \rightarrow LAac))$ (Barbara LXL, by A5)
2. $\vdash(LAbc \rightarrow (Aab \rightarrow LIac))$ (Barbari LXL, from 1 by CW)
3. $\vdash(Acb \rightarrow (LAba \rightarrow LIac))$ (Bramantip XLL, from 2 by AI, CW, US)
4. $\vdash(LAbc \rightarrow (Iab \rightarrow LIac))$ (Darii LXL, by A7)
5. $\vdash(LAbc \rightarrow (Iba \rightarrow LIac))$ (Datisi LXL, from 4 by AS)
6. $\vdash(LAbc \rightarrow (Aba \rightarrow LIac))$ (Darapti LXL, from 5 by AS)
7. $\vdash(Ibc \rightarrow (LAab \rightarrow LIac))$ (Disamis XLL, from 5 by AI, CW, US)
8. $\vdash(Icb \rightarrow (LAba \rightarrow LIac))$ (Dimaris XLL, from 4 by AI, CW, US)
9. $\vdash(Acb \rightarrow (LAba \rightarrow LIac))$ (Darapti XLL, from 7 by AS)
10. $\vdash(LEbc \rightarrow (Iab \rightarrow LOac))$ (Ferio LXL, by A8)
11. $\vdash(LEcb \rightarrow (Iab \rightarrow LOac))$ (Festino LXL, from 9 by AS)
12. $\vdash(LEbc \rightarrow (Iba \rightarrow LOac))$ (Ferison LXL, from 9 by AS)
13. $\vdash(LEbc \rightarrow (Aba \rightarrow LOac))$ (Felapton LXL, from 11 by AS)
14. $\vdash(LEcb \rightarrow (Iba \rightarrow LOac))$ (Fresison LXL, from 9 by AS)
15. $\vdash(LEcb \rightarrow (Aba \rightarrow LOac))$ (Fesapo LXL, from 13 by AS)
16. $\vdash(LEcb \rightarrow (Aab \rightarrow LEac))$ (Cesare LXL, by A6)
17. $\vdash(LEbc \rightarrow (Aab \rightarrow LEac))$ (Celarent LXL, from 15 by AS)
18. $\vdash(Acb \rightarrow (LEab \rightarrow LEac))$ (Camestres XLL, from 16 by AI, CW, US)
19. $\vdash(Acb \rightarrow (LEba \rightarrow LEac))$ (Camenes XLL, from 17 by AS)
20. $\vdash(LEbc \rightarrow (Aab \rightarrow LEac))$ (Celaront LXL, from 16 by CW)
21. $\vdash(LEcb \rightarrow (Aab \rightarrow LOac))$ (Cesaro LXL, from 15 by CW)
22. $\vdash(Acb \rightarrow (LEab \rightarrow LEac))$ (Camestrop XLL, from 18 by CW)
23. $\vdash(Acb \rightarrow (LEba \rightarrow LEac))$ (Camenop XLL, from 19 by CW) ■

Theorem 4.7. All unmarked LLL cells on table 5 represent asserted wffs.

Proof. Use A9, A10 and AS with theorem 4.6. So, for example, Barbara LLL is asserted, since Barbara LXL is asserted and $\vdash(LAab \rightarrow Aab)$. Disamis LLL is assserted, since Disamis XLL is asserted and $\vdash(LIab \rightarrow Iab)$. ■

Theorem 4.8. All unmarked MXM, XMM, LMX and MLX cells on table 5 represent asserted wffs.

Proof. Use theorem 4.6 and RV. So, for example, the assertion of Darii MXM is generated from the assertion of Ferison LXL as follows. $\vdash(MAbc \rightarrow (Iab \rightarrow MIac))$ since $\vdash(LEac \rightarrow (Iab \rightarrow LObc))$ (by RV and SE), since $\vdash(LEbc \rightarrow (Iba \rightarrow LOac))$ (by US). The assertion of Festino LMX is generated from the assertion of Celarent LXL as follows. $\vdash(LEcb \rightarrow (MIab \rightarrow Oac))$ since $\vdash(LEcb \rightarrow (Aac \rightarrow LEab))$ (by RV and SE), since $\vdash(LEbc \rightarrow (Aab \rightarrow LEac))$ (by US). The assertion of Camenes MLX is generated from the assertion of Fresison LXL as follows. $\vdash(MAcb \rightarrow (LEba \rightarrow Eac))$, since $\vdash(Iac \rightarrow (LEba \rightarrow LOcb))$ (by RV and SE), since $\vdash(LEba \rightarrow (Iac \rightarrow LOcb))$ (by AI), since $(LEcb \rightarrow (Iba \rightarrow LOac))$ (by US). ■

4.1 *Rejections in L-X-M*

To reject the syllogisms not marked with a "V" on table 4, as well as other invalid inferences, McCall adds twelve rejection axioms to the list of rejection axioms for the ŁA-system. We shall illustrate how some of these rejection axioms are used to reject some wffs.

Theorem 4.9. (Rejection of Barbara XLL) $\dashv$(Abc →(LAab →LAac)).

Proof. Recall that R2 = $\dashv\sigma$, where $\sigma = (LAbb \rightarrow (MAab \rightarrow (Aac \rightarrow (LAca \rightarrow (LAbc \rightarrow LAac)))))$.

1. $\dashv\sigma$ (by R2)
2. $\vdash((Aac \rightarrow (LAca \rightarrow LAac)) \rightarrow \sigma)$ (by A0)
3. $\dashv(Aac \rightarrow (LAca \rightarrow LAac))$ (from 1 and 2 by R-D)
4. $\vdash(Aac \rightarrow (LAca \rightarrow LAaa))$ (by A5 and US)
5. $\vdash((Aac \rightarrow (LAca \rightarrow LAaa)) \rightarrow ((Aac \rightarrow (LAaa \rightarrow LAac)) \rightarrow (Aac \rightarrow (LAca \rightarrow LAac))))$ (by A0)
6. $\vdash((Aac \rightarrow (LAaa \rightarrow LAac)) \rightarrow (Aac \rightarrow (LAca \rightarrow LAac)))$ (from 4 and 5 by MP)
7. $\dashv(Aac \rightarrow (LAaa \rightarrow LAac))$ (from 3 and 6 by R-D)
8. $\dashv(Abc \rightarrow (LAab \rightarrow LAac))$ (from 7 by R-US) ■

Theorem 4.10. Baroco XMM and Bocardo MLX are rejected.

Proof.

1. $\dashv(Abc \rightarrow (LAab \rightarrow LAac))$ (by theorem 4.9)
2. $\dashv(Abc \rightarrow (MOac \rightarrow MOab))$ (from 1 by theorem R-RV and R-SE)
3. $\dashv(Acb \rightarrow (MOab \rightarrow MOac))$ (Baroco XMM, from 2 by R-US)
4. $\dashv(MOac \rightarrow (LAab \rightarrow Obc))$ (from 1 by R-RV and R-SE)
5. $\dashv(MObc \rightarrow (LAba \rightarrow Oac))$ (Bocardo MLX, from 4 by R-US) ■

Theorem 4.11. Barbara LMX, Baroco LXL and Bocardo XMM are rejected.

Proof. Recall that R3 = $\dashv\sigma$, where σ is $(LAaa \rightarrow (LAcc \rightarrow (MAac \rightarrow (LAca \rightarrow Aac))))$.

1. $^{\dashv}\sigma$ (by R3)
2. $^{\vdash}((LAcc \to (MAac \to Aac)) \to \sigma)$ (by A5)
3. $^{\dashv}(LAcc \to (MAac \to Aac))$ (from 1 and 2 by R-D)
4. $^{\dashv}(LAbc \to (MAab \to Aac))$ (Barbara LMX, from 3 by R-US)
5. $^{\dashv}(LAbc \to (Oac \to LOab))$ (from 4 by R-RV and R-SE)
6. $^{\dashv}(LAcb \to (Oab \to LOac))$ (Baroco LXL, from 5 by R-US)
7. $^{\dashv}(Oac \to (MAab \to MObc))$ (from 4 by R-RV)
8. $^{\dashv}(Obc \to (MAba \to MOac))$ (Bocardo XMM, from 7 by R-US) ■

Theorem 4.12. Barbari MLX, Bramantip LMX, Felapton XLL and Baroco XLL are rejected.

Proof. Recall that R4 = $^{\dashv}\sigma$, where σ = (LAaa →(LAbb →(LAcc →LAab →(MAba →(MAbc →(LAcb →Iac))))) .

1. $^{\dashv}\sigma$ (by R4)
2. $^{\vdash}((MAbc \to (LAab \to Iac)) \to \sigma)$ (by A0)
3. $^{\dashv}(MAbc \to (LAab \to Iac))$ (Barbari MLX, from 1 and 2 by R-US)
4. $^{\dashv}(LAab \to (MAbc \to Iac))$ (from 3 by R-AI)
5. $^{\vdash}(Ica \to Iac)$ (by Con)
6. $^{\dashv}(LAab \to (MAbc \to Ica))$ (from 4 and 5 by R-CS)
7. $^{\dashv}(LAcb \to (MAba \to Iac))$ (Bramantip LMX, from 6 by R-US)
8. $^{\dashv}(Eac \to (LAab \to LObc))$ (from 3 by R-RV and SE)
9. $^{\dashv}(Ebc \to (LAba \to LOac))$ (Felapton XLL, from 8 by R-US)
10. $^{\dashv}(Ebc \to Obc)$ (by Sub-a and US)
11. $^{\dashv}(Obc \to (LAba \to LOac))$ (Baroco XLL, 9 and 10 by R-AW) ■

Our purpose in this section has been to illustrate how McCall's rejection apparatus works. In the next section we discuss this result: whatever is rejected by using McCall's rejection apparatus may be shown invalid by using countermodels. McCall's [1963] contains no discussion of models.

5 SEMANTICS FOR L-X-M

In [Johnson, 1989] a semantics for McCall's L-X-M is given. Validity is defined by using models, asserted wffs in L-X-M are shown to be valid (that is, system L-X-M is *sound*), and rejected sentences are shown to be invalid. So valid wffs in X-L-M are shown to be accepted (that is, system L-X-M is *complete*) since, as McCall shows, every wff in L-X-M is either accepted or rejected. The presentation of the semantics here will benefit from comments about it in Thom's [1996] and Thomason's [1993] and [1997].[16]

[16]For example, I borrow Thom's use of "base conditions" and "superstructural conditions" to present what he calls a "two-layered semantics". And I borrow Thomason's use of "V_M" to refer to a valuation relative to a model.

The semantics for L-X-M extends the familiar semantics for the assertoric syllogistic that assigns non-empty sets of objects to terms. To define the semantic notion of validity we refer to models and valuations relative to models.

Definition 5.1. (model) $\mathcal{M}$ is a *model* iff $\mathcal{M} = \langle W, n^+, q^+, n^-, q^- \rangle$, where W is a non-empty set and n^+, q^+, n^-, and q^- are functions that map terms into subsets of W and satisfy the following "base conditions", where $^+(x)$ is short for $n^+(x) \cup q^+(x)$, and $x \circ y$ (x overlaps y) is short for $x \cap y \neq \emptyset$:

B1 If f and g are any of the functions n^+, q^+, q^- or n^- and $f \neq g$, then, for every term x, $f(x) \cap g(x) = \emptyset$; and for every x, $n^+(x) \cup q^+(x) \cup q^-(x) \cup n^-(x) = W$
B2 For every x, $n^+(x) \neq \emptyset$
B3 (For every x, y and z) if $^+(z) \subseteq n^-(y)$ and $^+(x) \subseteq {}^+(y)$ then $^+(x) \subseteq n^-(z)$
B4 If $^+(y) \subseteq n^+(z)$ and $^+(x) \circ {}^+(y)$ then $n^+(x) \circ n^+(z)$
B5 If $^+(y) \subseteq n^-(z)$ and $^+(x) \circ {}^+(y)$ then $n^+(x) \circ n^-(z)$
B6 If $^+(z) \subseteq n^+(y)$ and $n^+(x) \circ n^-(y)$ then $n^+(x) \circ n^-(z)$

For an intuitive grasp of the notion of a model think of W as the world, $n^+(a)$ as the set of things in W that are essentially a, $q^+(a)$ as the set of things in W that are contingently a and are a, $n^-(x)$ as the set of things in W that are essentially non-a, and $q^-(a)$ as the set of things in W that are contingently not a and are not a.

Definition 5.2. (valuation) A *valuation* V is a function that assigns t or f, but not both, to sentences, where: i) $V(\neg p) = t$ iff $V(p) = f$; and ii) $V(p \to q) = t$ iff $V(\neg p) = t$ or $V(q) = f$; and iii) $V(L\neg\neg p) = t$ iff $V(Lp) = t$.

Definition 5.3. (valuation relative to model M) Let V_M, a *valuation relative to a model* M, be a valuation that satisfies the following "superstructural conditions":

S1 (For every x and y) $V_M(Axy) = t$ iff $^+(x) \subseteq {}^+(y)$
S2 $V_M(Ixy) = t$ iff $^+(x) \circ {}^+(y)$
S3 $V_M(LAxy) = t$ iff $^+(x) \subseteq n^+(y)$
S4 $V_M(LIxy) = t$ iff $n^+(x) \circ n^+(y)$
S5 $V_M(L\neg Axy) = t$ iff $n^+(x) \circ n^-(y)$
S6 $V_M(L\neg Ixy) = t$ iff $^+(x) \subseteq n^-(y)$

Definition 5.4. (valid) Let σ be an L-X-M sentence. σ is *valid* ($\models \sigma$) iff, for every model M, every valuation relative to M assigns t to σ. σ is *invalid* iff σ is not valid.

In this section we shall construct models that show the invalidity of all of the syllogisms that correspond to marked cells on table 5. Exactly four models suffice to show the invalidity of the invalid LXL and XLL models marked on these tables. Models constructed by interchanging rows in these four models suffice to invalidate the remaining invalid syllogisms mentioned on the table.

Table 5 agrees with table 7 on p. 43 of [McCall, 1963]. A cell on the former is marked if and only if it is unmarked on the latter. The marks on McCall's table indicate the relevant syllogism is syntactically asserted in L-X-M. McCall's discussion of L-X-M is

totally syntactic. He gives no formal semantics and thus no formal definition of validity. But as shown in [1989], the syllogisms that are syntactically asserted in L-X-M are the syllogisms that are valid in L-X-M and vice versa. The above theorems 4.6 and 4.7 pertain to the unmarked cells on table 5.

Table 5. Countermodels for L-X-M syllogisms

		LLL	LXL	XLL	MXM	XMM	LMX	MLX
Figure 1	Barbara			1ac	4bc	4ab	3ab	2bc
	Celarent			2ac	1ba	2ab		
	Darii			1ac			3ac	2bc
	Ferio			2ac		2ab		1bc
Figure 2	Cesare			2ac	1ba	2ab		
	Camestres		3ac		2ba	1ab		
	Festino			2ac		2ab		1bc
	Baroco		3ac	4ac	2ba	1ab		4ba
Figure 3	Darapti						2cb	2bc
	Felapton			2ac		3bc		1bc
	Disamis		1ca				2cb	2bc
	Datisi			1ac			2cb	2bc
	Bocardo		4ac	2ac		3cb	4cb	1bc
	Ferison			2ac		3cb		1bc
Figure 4	Bramantip		1ca				2cb	3ba
	Camenes		3ac		2ab	1ab		
	Dimaris		1ca				2cb	3ba
	Fresison			2ac		3cb		1bc
	Fesapo			2ac		3cb		1bc
Subalterns	Barbari			1ac			3ab	2bc
	Celaront			2ac		2ab		
	Cesaro			2ac		2ab		
	Camestrop		3ac		2ba	1ab		
	Camenop		3ac		2ba	1ab		

We begin by constructing a model $\mathcal{M}_1$, presented by table 6, that shows the invalidity of Barbara LXL. When giving such tables we use the following conventions: set brackets are omitted when giving the range of a function, a blank cell indicates the range of the relevant function is the empty set, for terms x other than those explicitly mentioned on the table, $n^+(x) = n^+(a)$, $q^+(x) = q^+(a)$, $n^-(x) = n^-(a)$ and $q^-(x) = q^-(a)$, and $W = n^+(a) \cup q^+(a) \cup n^-(a) \cup q^-(a)$.

So, for example, given table 6 the set of things that are essentially a has only one member, namely 1. The set of things that are c and are contingently c has two members: 1 and 2. The set of things that are essentially not b has no members. And the set of things that are not d and are contingently not d has 3 as its only member. $W = \{1, 2, 3\}$.

Table 6 expresses a model. Base conditions **B1** and **B2**, here and below, do not require a comment. **B3**, **B5** and **B6** are trivially satisfied since, for every x and y, $^+(x) \cap n^-(y) = \emptyset$.

Table 6. Model $\mathcal{M}_1$

	n^+	q^+	n^-	q^-
a	1	2		3
b	1,2,3			
c	3	1,2		

Suppose $^{(}y) \subseteq n^+(z)$. Then $z = b$. For all x, $n^+(x) \circ n^+(b)$. So **B4** is satisfied.

Given model $\mathcal{M}_1$: i) $V_{\mathcal{M}_1}(Abc) = t$ since $^+(b) \subseteq {}^+(c)$; ii) $V_{\mathcal{M}_1}(LAab) = t$ since $^+(a) \subseteq n^+(b)$; and iii) $V_{\mathcal{M}_1}(LAac) = f$ since $^+(a) \not\subseteq n^+(c)$. So $V_{\mathcal{M}_1}(Abc \rightarrow (LAaB \rightarrow LAac)) = f$. So $\not\models (Abc \rightarrow (LAab \rightarrow LAac))$. So Barbara XLL is invalid. The invalidity of Barbara XLL is marked on table 5 by putting '1ac' in the Barabara/XLL cell.

Aristotle's informal counterexample for Barbara XLL at 30a28-30 uses terms 'motion', 'animal' and 'man'. For Aristotle, Barbara XLL, construed as an inferential syllogism, is invalid given the inference 'All animals are (accidentally) in motion; all men are necessarily animal; so all men are necessarily in motion'. Aristotle takes the premises to be true and the conclusion false, making Barbara XLL invalid.

By interchanging rows a and b in table 6 we may construct a model $\mathcal{M}_{1bc}$ expressed by table 7 that shows that Ferio MLX invalid.

Table 7. Model $\mathcal{M}_{1bc}$

	n^+	q^+	n^-	q^-
a	1,2,3			
b	1	2		3
c	3	1,2		

In general, if a table satisfies conditions **B1** to **B6** so will a table that results from the interchanging of its rows. For none of these conditions requires a particular ordering of rows. Note that $V_{\mathcal{M}_{1bc}}(MEac) = t$, $V_{\mathcal{M}_{1bc}}(LIab) = t$ and $V_{\mathcal{M}_{1bc}}(Obc) = f$. So $\not\models (MEac \rightarrow (LIab \rightarrow Oac))$. That is, Ferio MLX is invalid.

This is the recipe for constructing a table T_2 for model $\mathcal{M}_{Nxy}$ (where x and y are a, b or c) from a table T_1 for model $\mathcal{M}_N$ (where T_1 has rows a, b and c): make row a in T_1 be row x in T_2, make row c in T_1 be row y in T_2, and make row b in T_1 be the third row in T_2. Every row in T_2 must be an a-row, a b-row or a c-row. So, for example, consider the Baroco/XMM cell on table 5, which is marked with '1ab'. Use the recipe to construct table 8 for model $\mathcal{M}_{1ab}$, which invalidates Baroco XMM. (The a-row of table 6 becomes the a-row of table 8; the c-row of 6 becomes the b-row of table 8; and the b-row of 6 becomes the c-row of table 8.) Since $\mathcal{M}_{1ab}(Acb) = t$, $\mathcal{M}_{1ab}(MOab) = t$ and $\mathcal{M}_{1ab}(MOac) = f$, $\not\models (Acb \rightarrow (MOab \rightarrow MOac))$.

Table 8. Model $\mathcal{M}_{1ab}$

	n^+	q^+	n^-	q^-
a	1	2		3
b	3	1,2		
c	1,2,3			

Model $\mathcal{M}_2$ expressed by table 9 may be used to show that Celarent XLL is invalid.

Table 9. Model $\mathcal{M}_2$

	n^+	q^+	n^-	q^-
a	1		2	3
b	1		2	3
c	2	3		1

Table 9 expresses a model. For all x and y, $^+x \not\subseteq n^-(y)$. So conditions **B3** and **B5** are trivially satisfied. For all x and y if $^+(x) \circ \ ^+(y)$ then $n^+(x) \circ \ ^+(y)$. So **B4** is satisfied. For all x and y, if $^+(x) \subseteq n^+(y)$ then $n^-(y) \subseteq n^-(x)$. So **B6** is satisfied.

Celarent XLL is invalid since: i) $V_{\mathcal{M}_2}(Ebc) = t$ since $^+(b)$ does not overlap $^+(c)$; ii) $V_{\mathcal{M}_2}(LAab) = t$ since $^+(a) \subseteq n^+(b)$; and iii) $V_{\mathcal{M}_2}(LEac) = f$ since $^+(a) \not\subseteq n^-(c)$. So $V_{\mathcal{M}_2}(Ebc \to (LAab \to LEac)) = f$. So $\not\models (Ebc \to (LAab \to LEac))$.

Model $\mathcal{M}_3$ expressed by table 10 may be used to show that Camestres LXL is invalid.

Table 10. Model $\mathcal{M}_3$

	n^+	q^+	n^-	q^-
a	1	2		3
b	3		2	1
c	3		2	1

Table 10 expresses a model. **B3** and **B5** are trivially satisfied since, for every x and y, $^+(x) \not\subseteq n^-(y)$. For **B4** note that if $^+(x) \circ \ ^+(y)$ then $n^+(x) \circ \ ^+(y)$. For **B6** note that if $^+(z) \subseteq n^+(y)$ then $n^-(y) \subseteq n^-(z)$.

Camestres LXL is invalid since: i) $V_{\mathcal{M}_3}(LAcb) = t$ since $^+(c) \subseteq n^+(b)$; ii) $V_{\mathcal{M}_3}(Eab) = t$ since $^+(a) \cap ^+(b) = \emptyset$; and iii) $V_{\mathcal{M}_3}(LEac) = f$ since $^+(a) \not\subseteq n^-(c)$. So $V_{\mathcal{M}_3}(LAcb \to (Eab \to LEac)) = f$. So $\not\models (LAcb \to (Eab \to LEac))$.

For Aristotle, Camestres LXL is invalid since Celarent XLL is invalid. A "semantic rule" that underwrites this reduction of an invalidity to an invalidity may be stated as follows.

$\mathbf{R}^{\neq}$-DR3 i) From $\not\models (p \to (q \to r))$ and $\models (p \to s)$ infer $\not\models (s \to (q \to r))$; and ii) from $\not\models (p \to (q \to r))$ and $\models (q \to s)$ infer $\not\models (p \to (s \to r))$.

Proof. For i) Suppose a) $\not\models (p \to (q \to r))$ and b) $\models (p \to s)$. By a) there is a model M such that $V_M(p) = t$, $V_M(q) = t$ and $V_M(r) = f$. By b) $V_M(s) = t$. So $\not\models (s \to (q \to r))$. Use similar reasoning for ii). ■

$\mathbf{R}^{\neq}$-AW is the semantic counterpart of the syntactic rule **R-DR3**, which is called rejection by antecedent weakening (R-AW). Given the rejection of Celarent XLL $(Ebc \to (LAab \to LIac))$ and the conversion principle $^{\vdash}(Ebc \to Ecb)$, Camestres LXL is rejected by R-AW. Likewise, given the invalidity of Celarent XLL and the semantic conversion principle $\models (Ebc \to Ecb)$, Camestres LXL is invalid by **$\mathbf{R}^{\not\models}$-AW**.

Semantic counterparts of other syntactic rejection rules may be put to use to establish invalidities. We illustrate this point by considering the semantic counterpart of R-RV.

$\mathbf{R}^{\neq}$-RV i) From $\not\models (p \to q)$ infer $\not\models (\neg q \to \neg p)$; ii) from $\not\models (p \to (q \to r))$ infer $\not\models (p \to (\neg r \to \neg q))$; and iii) from $\not\models (p \to (q \to r))$ infer $\not\models (\neg r \to (p \to \neg q))$.

Proof. For i) suppose $\not\models (p \to q)$. So there is a model M such that $V_M(p) = t$ and $V_M(q) = f$. So $V_M(\neg q) = t$ and $V_M(\neg p) = f$. So $\not\models (\neg q \to \neg p)$. Use similar reasoning for ii) and iii). ■

By **$\mathbf{R}^{\not\models}$-RV**, since $\not\models (Ebc \to (LAab \to LEac))$ (Celarent XLL is invalid), $\not\models (Ebc \to (\neg LEac \to \neg LAab))$. By using semantic counterparts of other syntactic principles stated above it is easy to conclude that Festino XMM is invalid.

To show that Baroco XLL is invalid we use model $\mathcal{M}_4$, presented on table 11.[17]

Table 11. Model $\mathcal{M}_4$

	n^+	q^+	n^-	q^-
a	1,2	3,4		
b	1,3	4	2	
c	1	4	3	2

Table 11 expresses a model. Base conditions **B3** and **B5** are trivially satisfied since, for every x and y, $^+(x) \not\subseteq n^-(y)$. **B4** and **B6** are trivially satisfied since, for every x and y, $n^+(x) \not\subseteq n^+(y)$.

Given model $\mathcal{M}_4$: i) $V_{\mathcal{M}_4}(Acb) = t$ since $^+(c) \subseteq\ ^+(b)$; ii) $V_{\mathcal{M}_4}(LOab) = t$ since $n^+(a) \circ\ n^-b$; and iii) $V_{\mathcal{M}_4}(LOac) = f$ since $n^+(a)$ does not overlap $n^-(c)$. So $V_{\mathcal{M}_4}((Acb \to (LOab \to LOac))) = f$. So $\not\models (Acb \to (LOab \to LOac))$. Following the pattern indicated above we record on table 5 the invalidity of Baroco XLL by

[17] Thomason [1993, p. 127] uses this table to invalidate Baroco XLL and Bocardo LXL though his definition of "validity" is not identical to that which we are currently discussing. Thomason models are discussed below.

putting '4ac' in the Baroco/XLL cell, where '4' refers to model $\mathcal{M}_4$ and 'ac' indicates that 'a' and 'c' are taken as minor and major terms, respectively.

Aristotle's counterexample for Baroco XLL is controversial. According to Thom on p. 148 of [1991] Aristotle used terms 'animal', 'man' and 'white', generating the purported counterexample: 'All men are animals; some whites are necessarily not animals; so some whites are necessarily not men.' Thom says:

> The problem with this counter-example is not (as van Rijen supposes [1989]) that the major premise is necessarily true. It is that, if the minor is taken to be true then the conclusion will be true also.

In agreement with Thom, Aristotle did not provide a good counterexample for Baroco XLL. A better informal counterexample is found in [Johnson, 1993, p. 179]: 'All things that are chewing are bears (Acb); some animals (dogs, say) are necessarily not bears ($LOab$); so some animals are necessarily not chewing ($LOac$)'. We do not follow Thom in developing formal systems that take Baroco XLL to be invalid.

Though models $\mathcal{M}_1$ to $\mathcal{M}_4$ and variants of them constructed by interchanging rows in them suffice to give countermodels for the invalid syllogisms marked on table 5, other models are needed to invalidate all of McCall's rejection axioms and thus all of the invalid wffs. The model used in [1989] to invalidate McCall's (LAbb →(LAff →(Aad →(LAda →(MAae →(LAcb →(LAbd →(LAce →(Aec →(LAfc →(MAdf →MAac)))))))))))) (*5.41 on p. 59) has four members. It is presented on table 12.

Table 12. Model for *5.41

	n^+	q^+	n^-	q^-
a	1,2,3,4			
b	3,4			1,2
c	4	3	1	2
d	3,4	1,2		
e	3,4			1,2
f	4		2	1,3

An implication of [Johnson, 1989] is that every invalid L-X-M wff of form $(p_1 \rightarrow (p_2 \rightarrow \ldots \rightarrow p_n)\ldots)$, where each p_i (for $1 \leq i \leq n$) is a simple wff or the negation of a simple wff, may be shown invalid by using a model $\langle W, \ldots\rangle$ in which W has no more than 6 members. [18]

In the next section we shall examine valuable attempts by Thomason to improve on the semantics discussed in this section.

[18] [Johnson, 1991] shows that W does not require more than 3 members if all simple sentences are assertoric and all terms are "chained".

5.1 Thomason models

In [Thomason, 1993] three notions of models are defined that enable Thomason to obtain soundness and completeness results for McCall's L-X-M calculus. In contrast to the soundness and completeness proofs given in [Johnson, 1989] no use is made of rejection axioms and rejection rules. One of these models comes close to the notion of a model defined above. We call it a "T3-model" (his "model$_3$") and define it as follows.

Definition 5.5. (T3-model) $\mathcal{M}$ is a *T3-model* iff $\mathcal{M} = \langle W, n^+, q^+, n^-, q^- \rangle$, where W is a non-empty set and n^+, q^+, n^-, and q^- are functions that map terms into subsets of W and satisfy the following "base conditions", where $^+(x)$ is short for $n^+(x) \cup q^+(x)$:

B1 If f and g are any of the functions n^+, q^+, q^- or n^- and $f \neq g$, then, for every term x, $f(x) \cap g(x) = \emptyset$; and for every x, $n^+(x) \cup q^+(x) \cup q^-(x) \cup n^-(x) = W$

B2 For every x, $n^+(x) \neq \emptyset$

BT3 (For every x and y) if $^+(x) \circ {}^+(y)$ then $^+(x) \circ n^+(y)$

BT4 If $^+(x) \subseteq n^-(y)$ then$^+(y) \subseteq n^-(x)$

BT5 If $^+(x) \subseteq n^+(y)$ then $n^-(y) \subseteq n^-(x)$

To define "valuation relative to a model" and "validity" Thomason uses the same superstructural conditions as used above.

Thomason, on p. 133 of [1997], says that in his [1993] he "tried to find simpler, and apparently weaker, requirements for models" than those given in [Johnson, 1989]. In the motivating section of [Thomason, 1993] he says "Johnson ... provided a semantics that has the right validities, but the latter is in some sense contrived." No doubt conditions **BT3**, **BT4** and **BT5** are more easily understood than **B3**, **B4**, **B5** and **B6** but Thomason is not correct in saying that the former, taken collectively, are weaker than the latter, taken collectively. We use the following theorem to show the relationship between T3-models and "J-models", the models defined above that satisfy base conditions **B1** to **B6**.

Theorem 5.6. i) Every T3-model is a J-model, but ii) there are J-models that are not T3-models.

Proof. For i) suppose $\mathcal{M}$ is a T3-model. First, suppose $^+(z) \subseteq n^-(y)$ and $^+(x) \subseteq {}^+(y)$. Then, by **BT4**, $^+(y) \subseteq n^-(z)$. Then $^+(x) \subseteq n^-(z)$. Then $\mathcal{M}$ satisfies **B3**. Next, suppose $^+(y) \subseteq n^+(z)$ and $^+(x) \circ {}^+(y)$. Then $^+(y) \circ {}^+(x)$ and, by **BT3**, $^+(y) \circ n^+(x)$. Then $n^+(x) \circ n^+(z)$. Then $\mathcal{M}$ satisfies **B4**. Next, suppose $^+(y) \subseteq n^-(z)$ and $^+(x) \circ {}^+(y)$. Then, by **BT3**, $n^+(x) \circ n^+(z)$. Then $\mathcal{M}$ satisfies B5. Next, suppose $^+(z) \subseteq n^+(y)$ and $n^+(x) \circ n^-(y)$. Then, by **BT5**, $n^-(y) \subseteq n^-(z)$. Then $n^+(x) \circ n^-(z)$. Then $\mathcal{M}$ satisfies **B6**.
For ii) note that $\mathcal{M}_4$, specified in table 11, is a J-model but not a T3-model since condition **BT3** is not satisfied. Note that $^+(a) \circ +(b)$ but $^+(a)$ does not overlap $n^+(b)$. ■

Though both T3-models and J-models, with the superstructural conditions defined above, will reveal the invalidity of any invalid syllogism with any finite number of antecedents (or premises), it is not clear that **BT3** and **BT5** are Aristotelian principles. Certainly **BT4** is Aristotelian, given 25a27-28. And J-models may be simplified by replacing **B3** with **BT4**, given the following theorem.

Theorem 5.7. i) **B3** is derivable from **BT4**; and ii) **BT4** is derivable from **B3**.

Proof. For i) suppose that a) if $^+(z) \subseteq n^-(y)$ then $^+(y) \subseteq n^-(z)$ and that b) $^+(z) \subseteq n^-(y)$ and $^+(x) \subseteq {}^+(y)$. Then $n^+(y) \subseteq n^-(z)$. Then $^+(x) \subseteq n^-(z)$.
For ii) suppose that c) if $^+(z) \subseteq n^-(y)$ and $^+(x) \subseteq {}^+(y)$ then $^+(x) \subseteq n^-(z)$ and d) $^+(z) \subseteq n^-(y)$. Since $+(y) \subseteq^+ (y)$, $^+(y) \subseteq n^-(z)$. ■

B4 (Darii LXL), **B5** (Ferio LXL) and **B6** (Baroco LLL) are Aristotelian given 30a37-b2 and 30a6-14 of the *Prior Analytics* .

5.2 *Variants of the L-X-M system*

Paul Thom in [1991, p. 137] points out that condition B2, used in the definitions of J-models and T3-models to guarantee that McCall's axiom $LIaa$ is valid, is unAristotelian. He says that it is unAristotelian to think that there are walkers that are essentially walkers and whites that are essentially white. Johnson's [1993] and [1995] provide variants of McCall's L-X-M that are sound and complete systems, where condition **B2** is omitted. Both systems have 100% Aristotelicity. The systems deviate from McCall's in that lines in deductions need not be axioms or lines that are ultimately derived from axioms by rules of inference. The systems are "natural deduction systems" rather than "axiomatic systems". Proofs of completeness assume that the inferences under discussion satisfy what Smiley calls the "chain condition" in [Smiley, 1994, p. 27]. And the systems attempt to accommodate Aristotle's proofs by *ecthesis* .[19] In the remainder of this section we illustrate proofs by ecthesis and then discuss the chain condition in the next section.

In addition to sentences such as Abc and $Labc$ discussed above we count $m \in a$ (m is an a), $m \in_n a$ (m is necessarily an a), $m \notin_n a$ (m is necessarily not an a), etc. The latter are singular sentences. In contrast to Thom's view, to present proofs by *ecthesis* singular sentences are required.[20] Consider this proof of Darapti XXX taken from Smith's [1989, p. 9] with my additions in square brackets:

> When they [terms] are universal, then when both P [that is, c] and R [that is, a] belong to every S [that is, b], it results of necessity that P will belong to some R. ... It is ... possible to carry out the demonstration through ... the setting out [that is, by *ecthesis*]. For if both terms belong to every S, then if some one of the S's is chosen (for instance N [that is, m], then both P and R will belong to this; consequently, P will belong to some R. (28a18-26)

Aristotle's proof by *ecthesis* may be formalized as follows:

1. Abc (Premise)
2. Aba (Premise)

[19] The systems proposed by [Łukasiewicz, 1957], [Corcoran, 1972] and [Smiley, 1973] do not attempt to accommodate Aristotle's proofs by *ecthesis* . According to Thom's [Thom, 1991] account of *ecthesis* both Baroco XLL and Bocardo LXL are valid, though Aristotle regarded them as invalid.

[20] For an alternative method of working with singular sentences in the context of syllogistic reasoning see [Johnson, 1999a].

3. $m \in b$ (By *ecthesis* from 1. Since all b are c there must be a b that may be referred to as m.)
4. $m \in c$ (From 1 and 3. Since all b are c and m is a b it follows that m is a c.)
5. $m \in a$ (From 2 and 3 by the reasoning for line 4.)
6. Iac (From 4 and 5 by "Existential Generalization" – if a particular object m is both an a and a c then something is both an a and a c.)

Aristotle proves that Baroco LLL is valid in the following passage, taken from Smith's [1989, p. 13]:

> ... it is necessary for us to set out that part [m] to which each term [b and c] does not belong and produce the deduction about this [m]. For it will be necessary in application to each of these; and if it is necessary of what is set out, then it will be necessary of some part [a] of the former term (for what is set out is just a certain "that". (30a9-15)

His proof by *ecthesis* may be formalized as follows:

1. LAcb (Premise. Whatever is c is necessarily b.)
2. LOab (Premise. There is something that is necessarily a but necessarily not b.)
3. $m \in_n a$
4. $m \notin_n b$ (Lines 3 and 4 come from line 2 by *ecthesis* . This is a use of "Existential Instantiation".)
5. $m \notin_n c$ (From 1 and 4. If whatever is c is necessarily b and m is necessarily not in c then m is necessarily not in c.)
6. LOac (From 3 and 5 by Existential Generalization.)

6 THE CHAIN CONDITION, RELEVANCE LOGIC AND THE AP SYSTEM

The following remarks by Smiley from two of his papers show that Aristotle held views endorsed by contemporary "relevance logicians".[21]

> By building onto the propositional calculus Łukasiewicz in effect equates syllogistic implication with strict implication and thereby commits himself to embracing the novel moods corresponding to such theorems as

[21] It is very surprising that Aristotle is scarcely mentioned in [Anderson and Belnap, 1975 and 1992], which provides authoritative discussions of relevance logic. See McCall's discussion of "connexive implication" [1975 and 1992, pp. 434-452] for the one reference to Aristotle. In [Johnson, 1994] a syllogistic logic is developed that is a "connexive logic". Pleasing relevance logicians, the logic satisfies both *Aristotle's thesis* (If y is the logical consequence of a non-empty set of premises, X, then X is semantically consistent) and *Boethius's Thesis* (If z is the logical consequence of a set of premises, $X \cup y$, then z is not the logical consequence of a set of premises $X \cup y'$, where y' contradicts y). Ironically, neither Aristotle's nor Boethius's thesis holds for what is now known as the "classical propositional calculus". In [Johnson, 1994] a theorem is proven that has as a corollary this interesting result due to C. A. Meredith in [1953]: The number of valid n-premised assertoric syllogisms (for $n \geq 2$) is $3n^2 + 5n + 2$. There is no question that in Chapter 25 of Book I of the *Prior Analytics* Aristotle was looking for such a general result. Given the chain condition such counts are possible.

$((Aab \wedge Oab) \to Icd)$ or $((Aab \wedge Acd) \to Aee)$. On the other hand Aristotle's own omission of these syllogisms of strict implication, as they may be called can hardly be written off as an oversight. For they violate his dictum that a syllogism relating this to that proceeds from premises which relate this to that' (41a6). This dictum is part of a principle which is absolutely fundamental to his syllogistic, namely the principle that the premises of a syllogism must form a chain of predications linking the terms of the conclusion. Thus his doctrine of the figures, which provides the framework for his detailed investigation of syllogistic, is founded on this principle (40b30 ff.) Not less important is that the chain principle is essential to the success of his attempt at a completeness proof for the syllogistic. By this I mean his attempt to show that every valid syllogistic inference, regardless of the number of premises, can be carried out by means of a succession of two-premise syllogisms. [Smiley, 1973, pp. 139-140]

Probably the easiest way to formulate this 'chain condition' is to use the notation AB to denote any of the forms a, e, i, o regardless whether the subject is A or B. Then the condition is that a valid argument must be of the form '$AC, CD, DE, EF, \ldots GH, HB$; therefore AB'. The chain condition dramatically alters the character of the completeness problem (for a start, it excludes the possibility of anything following from an infinite number of premises) and it permits simple strategies for the proof that would otherwise be inconceivable. It is therefore not surprising that Aristotle's proof should fail to fit the same picture as, for example, Corcoran's own completeness proof for syllogistic logic without the chain condition [Corcoran, 1972].[22] [Smiley, 1994, p. 27]

Aristotle's case for the chain condition is redolent of relevance — the need for some overt connection of meaning between premises and conclusion as a prerequisite for deduction. [Smiley, 1994, p. 30]

Since McCall's presentation of the L-X-M calculus imitates Łukasiewicz's, it also embraces "novel moods" of the sort mentioned by Smiley. (LAab →(¬ LAab →Icd)) is asserted in L-X-M even though neither c nor d occurs in the antecedent (and thus the consequent is irrelevant to the antecedents). This follows from the completeness result, mentioned above, for L-X-M. Note that for every model $\mathcal{M}$ either $V_{\mathcal{M}}(LAab) = f$ or $V_{\mathcal{M}(\neg LAab} = f$. So $\models (LAab \to (\neg LAab \to Ide))$. And $(LEab \to (LEcd \to (LEef \to Igg)))$ is asserted in L-X-M. For in every model $\mathcal{M}$, $V_{\mathcal{M}}(Igg) = t$. So $\models (LEab \to (LEcd \to (LEef \to Igg)))$. So, by completeness, $\vdash$(LEab →(LEcd →(LEef →Igg))) even though g does not occur in any of the antecedents.

[22]Corcoran gives a Henkin-style completeness proof for the assertoric syllogistic. His system validates inferences such as 'Eab; so Acc', inferences eschewed by relevance logicians. This inference is valid for Corcoran since the conclusion is logically true, even though the premise is irrelevant to the conclusion.

By using the chain condition in [1973], Smiley formulates an elegant decision procedure for the assertoric syllogistic. In [Johnson, 1994] a system is developed for Aristotle's apodeictic syllogisms, call it the "AP system", that uses the chain condition. A decision procedure is given for it that yields Smiley's decision procedure as a corollary.[23] Both decision procedures are given below.

Definition 6.1. (chain condition) Let Pr_i refer to "prefixes" of assertoric or apodeictic sentences: A, E, I, O, LA, LE, LI, LO, MA, ME, MI and MO. A *chain* is a set of sentences whose members can be arranged as a sequence $\langle Pr_1[x_1x_2], Pr_2[x_2x_3], \ldots, Pr_n[x_nx_1]\rangle$, where $Pr_i[x_ix_j]$ is either $Pr_ix_ix_j$ or $Pr_ix_jx_i$ and $x_i \neq x_j$ if $i \neq j$.

So, for example, $\{LAab, MAcb, LIcd, Ead\}$ and $\{Oba, LEbc, LEdc, LAda\}$ are chains. But neither $\{LAaa\}$ nor $\{LAab, Aba, MAac, Aca\}$ is a chain.

Definition 6.2. (abbreviations for subsets of chains) $X/LAxy$ refers to Axy or $LAxy$. $X/LAx - y$ refers to $\emptyset$ if $x = y$; otherwise, it refers to $\{X/LAz_1z_2, X/LAz_2z_3, \ldots, X/LAz_{n-1}z_n\}$, a subset of a chain, where $x = z_1$, $y = z_n$ and $n > 1$. $LAx - y$ refers to $\emptyset$ if $x = y$; otherwise it refers to $X/LAx - z, LAzy$, a subset of a chain. $X/LExy$ refers to Exy or $LExy$. $X/LIxy$ refers to Ixy or $LIxy$. And $X/LOxy$ refers to Oxy or $LOxy$.

So, for example, $LAab, LAbc, Acd$ has form $X/LAa - d$, but does not have form $LAa - d$. $LAab, LAbc, Acd, LAd - e$ has form $X/LAa - e$ and form $LAa - e$.

Definition 6.3. (contradictory of, cd) Let $cd(Axy) = Oxy$ where 'cd' may be read as 'the contradictory of'. Let $cd(Ixy) = Exy$, $cd(LAxy) = MOxy$, $cd(LExy) = MIxy$, $cd(LIxy) = MExy$, and $cd(LOxy) = MAxy$. And let $cd(cd(x)) = x$. So, for example, $cd(Exy) = Ixy$.

Theorem 6.4. (Johnson [1994], decision procedure for "AP-validity") Suppose "valid$_{AP}$" (apodeictic syllogistic validity) is defined as in [1994]. Consider an inference in the "AP system" from premises $P_1, P_2, \ldots, P_n$ to conclusion C. This inference is valid$_{AP}$ if and only if $\{P_1, P_2, \ldots, P_n, cd(C)\}$ is a chain that has one of the following eleven forms:

1. X/LAx-y, X/LOxy
2. LAx-z, MAzu, LAu-y, LOxy
3. X/LAx-z, LAzy, MOxy
4. X/LAz-x, X/LAz-y, X/LExy
5. X/LAz-x, X/LAz-u, MAuv, X/LAv-y, LExy (or LEyx)
6. X/LAz-x, X/LAz-u, LAuy, MExy (or MEyx)
7. X/LAz-x, X/LAu-y, X/LIzu, X/LExy (or X/LEyx)
8. X/LAz-x, X/LAu-v, MAvw, X/LAw-y, X/LIzu (or X/LIuz), LExy (or LEyx)
9. X/LAz-x, X/LAu-y, MIzu, LExy (or LEyx)
10. LIxy, MExy (or MEyx)

[23]See [Johnson, 1994] and [Johnson, 1997] for other systems that yield Smiley's decision procedure as a special case of a more general decision procedure.

11. X/LAz-x, X/LAu-v, LAvy, X/LIzu (or X/LIuz), MExy (or MEyx)

So, for example, $\{LAab, LAbc, Acd, cd(Aad)\}$ has form 1. So '$LAab$, $LAbc$, Acd; so Aad' is valid. $\{LAab, LAbc, Acd, cd(MAad)\}$ has form 1. So '$LAab, LAbc, Acd$; so $MAad$' is valid. $\{Aab, cd(Obc), Acd, Oad\}$ has form 1. So 'Aab, Acd, Oad; so Obc' is valid. $\{MAac, LAcb, cd(MAab)\}$ has form 2. So '$MAac, LAcb$; so $MAab$' (Barbara LMM) is valid. $\{LAcb, Iac, cd(LIab)\}$ has form 11. So '$LAcb, Iac$; so $LIab$' (Darii LXL) is valid.

Notice that since 'E' occurs at most once in any of the forms, it follows that no valid syllogism, regardless of the number of premises, is such that 'E' occurs in two or more of its premises. A similar comment applies to occurrences of 'M'.

The following result is a corollary of theorem 6.4.

Theorem 6.5. (Smiley [1973], decision procedure for "AS-validity") Suppose "valid$_{AS}$" (assertoric syllogistic validity) is defined as in [1973]. Consider an inference in the assertoric syllogistic from premises $P_1, P_2, \ldots, P_n$ to conclusion C. This inference is valid$_{AS}$ if and only if $\{P_1, P_2, \ldots, P_n, cd(C)\}$ is a chain that has one of the following three forms:

1. Ax-y, Oxy (restriction of form 1 of theorem 6.4)
2. Az-x, Az-y, Exy (restriction of form 4 of theorem 6.4)
3. XAz-x, Au-y, Izu, Exy (or Eyx) (restriction of form 7 of theorem 6.4)

So, for example, given form 2 of the corollary both 'Aca, Acb; so Iab' (Darapti) and 'Aca, Eab; so Ocb' (Celaront) are valid.

On table 13 a syllogism is marked as valid by referring by number to the form listed in theorem 6.4 in virtue of which it is valid. So, for example, the first occurrence of '1' on the table indicates that Barbara XXX, XXM, XLX, XLM, LXX, LXM, LLX and LLM are valid in virtue of their relationship to $\{X/LAx-y, X/LOxy\}$. The 333 valid syllogisms marked on the table exactly match the 333 syllogisms that McCall accepts in his L-X-M system. See p. 46 of [McCall, 1963].

7 CONTINGENT SYLLOGISMS

A. N. Prior [1962, p. 188] gives a simple account of "the usual meaning of 'contingent'" in the following passage:

> In the *De Interpretatione* Aristotle remarks that the word 'possible' is ambiguous; we should sometimes say that 'It is possible that p' follows from 'It is necessary that p', but sometimes that it is inconsistent with it. In the former sense 'possible' means simply 'not impossible'; in the latter sense, 'neither impossible nor necessary'. It is for 'possible' in this second sense that the word 'contingent' is generally used. That is, 'It is contingent that p' means 'Both p and not-p are possible', KMpMNp [or $(Mp \wedge M\neg p)$]. Contingency in this sense stands between necessity and impossibility, but in quite a different way from that in which the simply factual stands between the

Table 13. Valid_{AP} 2-premised syllogisms

	X/L	L	L	X	L	M	M	X	L	M	
	X/L	L	X	L	M	L	X	M	M	L	
	X/M	L	L	L	M	M	M	M	X	X	
Barbara	1	3	3		2	2					
Celarent	7	9	9		8	11			8	11	
Darii	7	11	11		9	8	8	9			
Ferio	7	8	8		9	11	11		9		
Cesare	7	9	9		8	11			8	11	
Camestres	7	9		9	11	8			11	8	
Festino	7	8	8		9	11	11		9		
Baroco	1	2			3	2			3		
Darapti	4	6	6	6	5	5	5	5			
Felapton	4	5	5		5	6	6		5		
Disamis	7	11		11	8	9	9	8			
Datisi	7	11	11		9	8	8	9			
Bocardo	1	2			2	3	3				
Ferison	7	8	8		9	11	11		9		
Bramantip	4	6		6	5	5	5	5			
Camenes	7	9		9	11	8			11	8	
Dimaris	7	11		11	8	9	9	8			
Fresison	7	8	8		9	11	11		9		
Fesapo	4	5	5		5	6	6		5		
Barbari	4	6	6		5	5	5	5			
Celaront	4	5	5		5	6	6		5	6	
Cesaro	4	5	5		5	6	6		5	6	
Camestrop	4	5		5	6	5			6	5	
Camenop	4	5		5	6	5			6	5	
Total	8×24	24	15	8	24	24	16	7	15	8	= 333

necessary and the possible. It is not that necessity implies contingency, and contingency impossibility; rather we have three mutually exclusive alternatives which divide the field between them — either a proposition is necessary, or it is neither-necessary-nor-impossible (i.e. contingent), or it is impossible ....

On p. 190 of [1962] Prior introduces the symbol 'Q' and reads 'Qp' as 'It is contingent that p'. McCall adopts Prior's use of 'Q' to refer to Aristotle's contingency operator and Thom [1994, p. 91] refers to [McCall, 1963] to support his use of 'Q' in his discussions of contingency. In the discussion below, we shall also use 'Q'.[24]

[24]The following symbols are also found in the literature that formalizes contingency: 'E_2' [Becker-Freyseng, 1933], '$\mathcal{T}$' [Łukasiewicz, 1957] and 'P()' [Smith, 1989]. Smith's P(Aab) is McCall's QAab, and Smith's PAab is McCall's MAab. Łukasiewicz used '$\mathcal{T}$' instead of 'Q' since earlier in his book he used 'Q' for 'is equivalent to'. McCall's Barbara LQM is Ross's [1949] $A^n A^c A^p$. Montgomery and Routley use ∇ for contingency in [1966] and [1968]. And Cresswell uses ∇ for contingency and $\triangle$ for non-contingency in [1988].

Thom makes the following remarks about contingency at the beginning of his article (p. 91):

> Aristotle's contingency syllogistic deals with the logic of derivations involving propositions that contain an expressed mode of contingency. The contingent is defined at I. 13, 32^{a}18-20, as that which is not necessary, but which being supposed does not result in anything impossible, i.e. as two-sided possibility.

Fitting Prior's remarks, the two sides of contingency (Q) are necessity and impossibility. The one side of possibility (M) is impossibility.

McCall in [1963] diminishes and extends the L-X-M calculus, formulating the Q-L-X-M calculus. We give the basis for it.

Primitive symbols

Use the primitive symbols for L-X-M together with

monadic operator Q

Formation rules

Use the formation rules for L-X-M, amending FR2′ as follows.

FR2′ If p is a categorical expression then $\neg p$ is a *categorical expression* and Lp and Qp are *wffs*.

Assertion axioms

Use A0-A4 from system ŁA and A5-A14 from system L-X-M. So A2 is IAA. Add the following axioms.

A15 (Barbara QQQ)	$\vdash(QAbc \to (QAab \to QAac))$
A16 (Darii QQQ)	$\vdash(QAbc \to (QIab \to QIac))$
A17 (QXQ-AAE, figure 1)	$\vdash(QAbc \to (Aab \to QEac))$
A18 (Darii QXQ)	$\vdash(QAbc \to (Iab \to QIac))$
A19 (Barbara XQM)	$\vdash(Abc \to (QAab \to MAac))$
A20 (Celarent XQM)	$\vdash(Ebc \to (QAab \to MEac))$
A21 (Ferio XQM)	$\vdash(Ebc \to (QIab \to MOac))$
A22 (complementary conversion, QE-QA)	$\vdash(QEab \to QAab)$
A23 (complementary conversion, QI-QO)	$\vdash(QIab \to QOab)$
A24 (complementary conversion, QO-QI)	$\vdash(QOab \to QIab)$
A25 (QI conversion)	$\vdash(QIab \to QIba)$
A26 (QE-ME subordination)	$\vdash(QEab \to MEab)$
A27 (QI-MI subordination)	$\vdash(QIab \to MIab)$

A28 (QO-MO subordination) ⊢(QOab →MOab)

Assertion transformation rules

Use the assertion tranformation rules for L-X-M.

On p. 76 of [1963] McCall gives the following reason for changing A2 from $LIaa$ to Iaa.

> If we retain the axiom $LIaa$, we may, by means of the substitution $CKQAacLIaaIac$ $[((QAac \wedge LIaa) \to Iac))]$ of Darii QLX (proved below), derive the implication $CQAacIac$ $[(QAac \to Iac)]$, which is un-Aristotelian.

We shall present this reasoning systematically.

Proof.

1. $\vdash(Ebc \to (QAab \to MEac))$ (by A20)
2. $\vdash(QAab \to (Ebc \to MEac))$ (from 1 by AI)
3. $\vdash(QAab \to (LIac \to Ibc))$ (from 2 by RV and SE)
4. $\vdash(QAab \to (LIaa \to Iba))$ (from 2 by US)
5. $\vdash LIaa$ (by A2 for L-X-M)
6. $\vdash(QAab \to Iba)$ (from 4 and 5 by AI and MP)
7. $\vdash(QAab \to Iab)$ (from 6 by CW, given Con)
8. $\vdash(QAac \to Iac)$ (from 7 by US) ■

McCall devised his system Q-L-X-M so that it has this feature: $(QEab \to QEba)$ is not accepted. He wishes to reflect Aristotle's view that universally negative contingent propositions are not convertible.[25] McCall puts Aristotle's argument for the non-convertibility of such propositions as follows:

> ... in 36b35-37a3, Aristotle gives what is in essence the following argument. We know that $QAab$ implies $QEab$, and that $QEba$ implies $QAba$ [by complementary conversion]. Therefore if $QEab$ implied $QEba$, $QAab$ would imply $QAba$, which it does not. Hence $QEab$ is not convertible.

But, unfortunately, McCall's Q-L-X-M system is too strong. It forces us, for example, to accept $(QAbc \to (LAab \to LAde))$, which is clearly unAristotelian. It does not satisfy the chain condition mentioned above. After showing this, we shall lay out a system that is semantically consistent and maximizes Aristotelicity.

[25]On p. 198 of [1957] Łukasiewicz calls Aristotle's view a 'grave mistake'. Łukasiewicz says 'He [Aristotle] does not draw the right consequences from his definition of contingency, and denies the convertibility of universally-negative contingent propositions, though it is obviously admissible.' But, following McCall, one can attempt to formulate Aristotle's contingency syllogistic without, in effect, defining $QEab$ as $(\neg LEab \wedge \neg L\neg Eab)$.

7.1 *Overlooked acceptances in the Q-L-X-M system*

McCall claims that Barbara QLX is not a thesis in his Q-L-X-M system. See table 13 on p. 92 of [1963]. But this result is a corollary of the following theorem.

Theorem 7.1. $\vdash$(Q Abc $\rightarrow$(LAab $\rightarrow x$)), where x is any wff.

Proof.

1. $\vdash(Eca \rightarrow (QAbc \rightarrow MEba))$ (by A20 and US)
2. $\vdash(LAab \rightarrow LIba)$ (by Ap-sub-a)
3. $\vdash(MEba \rightarrow MOab)$ (from 2 by RV and SE)
4. $\vdash(Eca \rightarrow (QAbc \rightarrow MOab))$ (from 1 and 3 by CW)
5. $\vdash(QAbc \rightarrow (LAab \rightarrow Ica))$ (from 4 by AI, RV and SE)
6. $\vdash(LAab \rightarrow (Ica \rightarrow LIcb))$ (A7 and US)
7. $\vdash((QAbc \rightarrow (LAab \rightarrow Ica)) \rightarrow ((LAab \rightarrow (Ica \rightarrow LIcb)) \rightarrow (QAbc \rightarrow (LAab \rightarrow LIcb))))$ (by AO)
8. $\vdash(QAbc \rightarrow (LAab \rightarrow LIcb))$ (from 5, 6 and 7 by MP)
9. $\vdash(QAbc \rightarrow QEbc)$ (by CC and US)
10. $\vdash(QEbc \rightarrow MEbc)$ (by A26 and US)
11. $\vdash(MEbc \rightarrow MEcb)$ (by Ap-con and US)
12. $\vdash(QAbc \rightarrow MEcb)$ (from 9, 10 and 11 by CW)
13. $\vdash(QAbc \rightarrow \neg LIcb)$ (from 12 by SE)
14. $\vdash((QAbc \rightarrow (LAab \rightarrow LIcb)) \rightarrow ((QAbc \rightarrow \neg LIcb) \rightarrow (QAbc \rightarrow (LAab \rightarrow x))))$ (by A0)
15. $\vdash$(QAbc $\rightarrow$(LAab $\rightarrow x$)) (from 8, 13, and 14 by MP) ■

The following theorem provides additional evidence that McCall's Q-L-X-M system is too strong to be Aristotelian.

Theorem 7.2. $\vdash$(LAbc $\rightarrow$(QAab $\rightarrow x$)), where x is any sentence.

Proof.

1. $\vdash(Eac \rightarrow (QIba \rightarrow MObc))$ (by A21 and US)
2. $\vdash(QAab \rightarrow QIba)$ (by A18, US, A2, MP)
3. $\vdash(Eac \rightarrow (QAab \rightarrow MObc))$ (from 1 and 2 by AS)
4. $\vdash(LAbc \rightarrow (QAab \rightarrow Iac))$ (from 3 by RV and SE)
5. $\vdash(Iac \rightarrow Ica)$ (by Con)
6. $\vdash(LAbc \rightarrow (QAab \rightarrow Ica))$ (from 4 and 5 by CW)
7. $\vdash(QAab \rightarrow (Ica \rightarrow QIcb))$ (by A18)
8. $\vdash((LAbc \rightarrow (QAab \rightarrow Ica)) \rightarrow ((QAab \rightarrow (Ica \rightarrow QIcb)) \rightarrow (LAbc \rightarrow (QAab \rightarrow QIcb))))$ (by AO)
9. $\vdash(LAbc \rightarrow (QAab \rightarrow QIcb))$ (from 6, 7 and 8 by MP)
10. $\vdash(QIcb \rightarrow QIbc)$ (by A25 and US)
11. $\vdash(QIbc \rightarrow QObc)$ (by A23 and US)

12. $\vdash(QObc \to MObc)$ (by A28 and US)
13. $\vdash(LAbc \to (QAab \to MObc))$ (from 9, 10, 11 and 12 by MP)
14. $\vdash(LAbc \to (QAab \to \neg LAbc))$ (from 13 by SE)
15. $\vdash((LAbc \to (QAab \to \neg LAbc)) \to (LAbc \to (QAab \to x)))$ (by A0)
16. $\vdash$(LAbc $\to$(QAab $\to x$)) (from 14 and 15 by MP) ■

According to McCall's table 13 on p. 92 of [McCall, 1963], sentences representing Barbara QLX, Barbara LQX, Barbara LQQ, Baroco QXM and Bocardo XQM are not accepted in the Q-L-X-M system, though they correspond to inferences that Aristotle considered to be valid. But it is an immediate consequence of theorems 7.1 and 7.2 that the first three sentences are accepted. That the last two are accepted may be seen as follows:

1. $\vdash(LAbc \to (QAab \to Aac))$ (by theorem 7.2)
2. $\vdash(Oac \to (QAab \to MObc))$ (from 1 by RV and SE)
3. $\vdash(Obc \to (QAba \to MOac))$ (Bocardo XQM, from 2 by US)
4. $\vdash(QAbc \to (LAab \to Aac))$ (by theorem 7.1)
5. $\vdash(QAbc \to (Oac \to MOab))$ (from 4 by RV and SE)
6. $\vdash(QAcb \to (Oab \to MOac))$ (Baroco QXM, from 5 by US)

So McCall's claim on p. 93 of [1963] that Q-L-X-M has 85% Aristotelicity needs to be modified. Instead of 24 "non-Aristotelian moods" out of 154 moods marked on his table 13, there are 29 out of 154. So the Aristotelicity of the Q-L-X-M system is about 81%.

When determining the Aristotelicity of a system, McCall only uses figures 1, 2 and 3 and none of the "subaltern moods" such as Barbari. Given theorems 7.1 and 7.2, the following wffs are accepted in Q-L-X-M, though they are not marked as accepted on McCall's table 13: Bramantip QLQ, Camenes LQQ, Fesapo QLQ and Barbari LQQ.

In the following section we modify Q-L-X-M so that the resulting system, QLXM$'$, does not have the unAristotelian features that result from theorems 7.1 and 7.2. Given the data – that Aristotle regarded Barbara LQM as invalid and Bocardo QLM as valid, for example – it is a virtue of the modified system that it does not have 100% Aristotelicity. Note that if $\vdash(QObc \to (LAba \to MOac))$ (Bocardo QLM) then $\vdash(LAac \to (QObc \to MOba))$ (Baroco LQM) by Reversal. In system QLXM$'$ both Barbara LQM and Bocardo QLM are invalid. In contrast, in system Q-L-X-M both are valid.

8 QLXM$'$

To ensure that theorems 7.2 and 7.1 may not be proven in system QLXM$'$ we exclude axioms A20 (Celarent XQM) and A21 (Ferio XQM). This decision is not difficult to make since, as McCall points out, Aristotle's proofs of Celarent XQM and Ferio XQM are flawed. McCall shows that one who endorses such reasoning, thinking that "what is impossible cannot follow from what is merely false, but not impossible", is committed to the absurd consequence that 'Some B are A; all C are A; so some C are A' is valid.

Only one other Q-L-X-M axiom is excluded to form QLXM′: delete axiom A28, $\vdash(QOab \to MOab)$. This decision is a result of semantic considerations. For A28 to be validity preserving $QOab$ and $LAab$ must be semantically inconsistent. Since $LAab$ is true iff $^+(a) \subseteq n^+(b)$, we could make $QOab$ and $LAab$ contraries by fixing the semantics so that $QOab$ is true iff $^+(a) \circ q(b)$. But then we are forced to say that Bocardo QLQ, for example, is valid even though Aristotle considered it to be invalid. (Suppose that $V_{\mathcal{M}}(QOab) = t$ and $V_{\mathcal{M}}(LAac) = t$. Then $^+(a) \circ q(b)$ and $^+(a) \subseteq n^+(c)$. Then $n^+(c) \circ q(b)$. Then $^+(c) \circ q(b)$. Then $V_{\mathcal{M}}(QOcb) = t$.) Note, also, that if QOab is true iff $^+(a) \circ q(b)$ then we would want to ensure that $QIab$ is true iff $^+(a) \circ q(b)$ to guarantee the soundness of the complementary conversion principles that Aristotle clearly supported. But then we would be forced to say that Disamis QLQ is valid even though Aristotle considered it to be invalid. (Suppose that $V_{\mathcal{M}}(QIbc) = t$ and $V_{\mathcal{M}}(LAba) = t$. Then $^+(b) \circ q(c)$ and $^+(b) \subseteq n^+(a)$. Then $n^+(a) \circ q(c)$. Then $^+(a) \circ q(c)$. Then $V_{\mathcal{M}}(QIac) = t$.) Similar remarks may be made about Disamis QXQ, Datisi LQQ and Datisi XQQ.

Rather than fixing the conditions for the truth of $QOab$ as indicated above we may let $QOab$ be true iff either $QIab$ or $QIba$ is true.[26] To make A28 truth preserving we must ensure that if $LAab$ is true then both $QIab$ and $QIba$ are false. Such a position does not fit the sorts of examples Aristotle uses. Suppose, for example, that all things that are sleeping are necessarily men. It does not follow that it is not true that some men are contingently sleeping.

We avoid the above difficulties by deleting axiom A28 when defining QLXM′.

In this system, as in Q-L-X-M, there are no rejection axioms and no rejection rules.

Before giving a semantics for QLXM′ we shall establish some immediate inferences that are conversions, subalternations or subordinations. With them we shall show the acceptance of various two-premised syllogisms indicated on table 15 by leaving a cell unmarked. After the semantics is given we shall show that sentences corresponding to the other cells, those in which numerals occur, are invalid. An occurrence of the "hat sign" in a cell in the table means the entry conflicts with Aristotle's judgments about validity as recorded on McCall's authoritative table 12 of [McCall, 1963].[27]

Theorem 8.1. (Ordinary Q-conversions, Q-con) i) $\vdash(QIab \to QIba)$; and ii) $\vdash(QOab \to QOba)$.

Proof. i) is A25. For ii) use A23, A24 and CW. ■

Theorem 8.2. (Contingency subalternations, Q-sub-a) i) $\vdash(QAab \to QIab)$; ii) $(QAab \to QOab)$; iii) $\vdash(QEab \to QIab)$; and iv) $\vdash(QEab \to QOab)$.

Proof. For i) use A18, AI, A2 and MP. For ii) use i), A23 and CW. For iii) use i), A22 and AS. For iv) use ii), A22 and AS. ■

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[26]Thom evaluates $QOab$ in this way in [1993] and [1994].

[27]In the notes for table 12 McCall comments on tables in [Becker-Freyseng, 1933, p. 88] and [Ross, 1949, after p. 286].

Table 14. McCall's Table 12 and RV inconsistencies

	Q Q Q	Q X Q	X Q Q	Q L Q	L Q Q	Q X M	X Q M	Q L X	L Q X	Q L M	L Q M
Barbara	V	V		V		V^{1}	V			V^{13}	V
Celarent	V	V		V		V^{2}	V^{7}		V	V^{14}	V
Darii	V	V		V		V	V^{8}	4	12	V	V
Ferio	V	V		V		V^{3}	V^{9}		V	V^{15}	V
Cesare							V^{10}	3	V	15	V
Camestres						V^{4}		V	6	V	17
Festino							V^{11}	2	V	14	V
Baroco								1	5	13	16
Darapti	V	V		V		V	V			V	V
Felapton	V	V		V		V	V		V	V	V
Disamis	V		V		V	V	V	11	7	V	V
Datisi	V	V		V		V	V	10	9	V	V
Bocardo	V					V^{5}			V	V^{16}	V
Ferison	V	V		V		V^{6}	V^{12}		8	V^{17}	

McCall follows Ross's use of "complementary conversion" to refer to A22 to A24. On p. 298 of [Ross, 1949] Ross, in his discussion of 35a29-b1, identifies the following entailments, endorsed by Aristotle, as "complementary conversions":

> 'For all B, being A is contingent' [QAba] entails 'For all B, not being A is contingent' [QEba] and 'For some B, not being A is contingent' [QOba]. 'For all B, not being A is contingent' [QEba] entails 'For all B, being A is contingent' [QAba] and 'For some B, being A is contingent' [QIba]. 'For some B, being A is contingent' [QIba] entails 'For some B, not being A is contingent' [QOba]. 'For some B, not being A is contingent' [QOba] entails 'For some B, being A is contingent' [QIba].

Given the following theorem and US, Ross's six complementary conversions are asserted in QLXM$'$.

Theorem 8.3. (Complementary conversion, CC) i) $\vdash(QAab \rightarrow QEab)$; ii) $\vdash(QAab \rightarrow QOab)$; iii) $\vdash(QEab \rightarrow QAab)$; iv) $\vdash(QEab \rightarrow QIab)$; v) $\vdash(QIab \rightarrow QOab)$; and vi) $\vdash(QOab \rightarrow QIab)$.

Proof. For i) use A17, US, AI, A1 and MP. For ii) use Q-sub-a, A23 and CW. iii) is A22. For iv) use iii), Q-sub-a and CW. v) is A23. vi) is A24. ■

Theorem 8.4. (Complementary conversions *per accidens*, CC(pa)) i) $\vdash(QAab \rightarrow QIba)$; ii) $\vdash(QAab \rightarrow QOba)$; iii) $\vdash(QEab \rightarrow QIba)$; and iv) $\vdash(QEab \rightarrow QOba)$.

Table 15. QLXM′ countermodels

	Q Q Q	Q X/L Q	X/L Q Q	Q X M	X Q M	Q L X	L Q X	Q L M	L Q M
Barbara			5ac			7ab	8ac		
Celarent			6ac		$\widehat{7ac}$				
Darii			5ac			7ab	8ac		
Ferio			6ac		$\widehat{7ac}$	5cb			
Cesare	7ac	9ca	6ac	5ca	$\widehat{7ac}$				
Camestres	7ac	6ca	9ac	$\widehat{7ac}$	5ac				
Festino	7ac	9ca	6ac		$\widehat{7ac}$				
Baroco	7ac	6ca	9ac	7ac	11bc		11bc		11bc
Darapti						7cb	7bc		
Felapton			9bc		$\widehat{8bc}$	1bc			
Disamis		5ca				7cb	7bc		
Datisi			5ca			7cb	7bc		
Bocardo		5ca	9bc	$\widehat{11ac}$	8bc	11ac		$\widehat{11ac}$	
Ferison			9bc		$\widehat{8bc}$	5cb			
Bramantip		5ca				8ca	7ba		
Camenes	10ac	6ca	7bc	7ac					
Dimaris		5ca				8ca	7ba		
Fresison	7ac	5ca	6bc		8bc				
Fesapo		5ca	6bc		8bc				
Barbari			5ac			7ab	8ac		
Celaront			6ac		7ac				
Cesaro	7ac	9ca	6ac		7ac				
Camestrop	7ac	6ca	9ac	7ac	1ab				
Camenop		6ca		7ac					

Proof. For i) use Q-sub-a, Q-con, US and CW. For ii) use i), A23, US and CW. For iii) use i), A22 and AS. For iv) use iii), A23, US and CW. ■

Theorem 8.5. (Contingency subordinations, Q-sub-o) i) $\vdash(QAab \rightarrow MAab)$; ii) $\vdash(QEab \rightarrow MEab)$; and iii) $\vdash(QIab \rightarrow MIab)$.

Proof. For i) use A19, A1, US and MP. ii) is A26. iii) is A27. ■

Uses of AS or CW in proofs of the following theorems involve only those immediate inferences that have been proven above. So, for example, in the proof that Celarent QQQ is asserted AS is used with Q-sub-a and US ($\vdash(QEbc \rightarrow QAbc)$) and CW is used with Q-sub-a and US ($\vdash(QAac \rightarrow QEac)$).

Theorem 8.6. (asserted QQQs) The non-numbered QQQ cells on table 15 correspond to asserted sentences.

Proof.

1. ⊢(QAbc →(QAab →QAac)) (Barbara QQQ, by A17)
2. ⊢(QEbc →(QAab →QEac)) (Celarent QQQ, from 1 by AS, US, CW)
3. ⊢(QAbc →(QIab →QIac)) (Darii QQQ, by A16)
4. ⊢(QEbc →(QIab →QOac)) (Ferio QQQ, from 3 by AS, US, CW)
5. ⊢(QAbc →(QAba →QIac)) (Darapti QQQ, from 3 by AS, US)
6. ⊢(QEbc →(QAba →QOac)) (Felapton QQQ, from 5 by AS, US, CW)
7. ⊢(QAbc →(QIba →QIac)) (Datisi QQQ, from 3 by AS, US)
8. ⊢(QEbc →(QIba →QOac)) (Ferison QQQ, from 7 by AS, US, CW)
9. ⊢(QIbc →(QAba →QIac)) (Disamis QQQ, from 7 by AI, CW, US)
10. ⊢(QObc →(QAba →QOac)) (Bocardo QQQ, from 9 by AS, US, CW)
11. ⊢(QIcb →(QAba →QIac)) (Dimaris QQQ, from 9 by AS, US)
12. ⊢(QAcb →(QAba →QIac)) (Bramantip QQQ, from 11 by AS, US)
13. ⊢(QEcb →(QAba →QOac)) (Fesapo QQQ, from 12 by AS, US, CW)
14. ⊢(QAbc →(QAab →QAac)) (Barbari QQQ, from 1 by CW, US)
15. ⊢(QEbc →(QAab →QEac)) (Celaront QQQ, from 2 by CW, US)
16. ⊢(QAcb →(QEba →QOac)) (Camenop QQQ, from 12 by AS, US, CW) ■

Theorem 8.7. (asserted QXQs and XQQs) The non-numbered QXQ and XQQ cells on table 15 correspond to asserted sentences.

Proof.

1. ⊢(QAbc →(Aab →QAac)) (Barbara QXQ, by A17, US, CW)
2. ⊢(QEbc →(Aab →QEac)) (Celarent QXQ, from 1 by AS, US, CW)
3. ⊢(QAbc →(Iab →QIac)) (Darii QXQ, by A18)
4. ⊢(QEbc →(Iab →QOac)) (Ferio QXQ, from 3 by AS, US, CW)
5. ⊢(QAbc →(Iba →QIac)) (Datisi QXQ, from 3 by AS, US)
6. ⊢(QEbc →(Iba →QOac)) (Ferison QXQ, from 5 by AS, US, CW)
7. ⊢(QAbc →(Aba →QIac)) (Darapti QXQ, from 5 by AS, US)
8. ⊢(QEbc →(Aba →QOac)) (Felapton QXQ, from 7 by AS, US, CW)
9. ⊢(QAbc →(Aab →QIac)) (Barbari QXQ, from 1 by US, CW)
10. ⊢(QEbc →(Aab →QOac)) (Celaront QXQ, from 2 by US, CW)
11. ⊢(Ibc →(QAba →QIac)) (Disamis XQQ, from 5 by AI, CW, US)
12. ⊢(Abc →(QAba →QIac)) (Darapti XQQ, from 11 by AS)
13. ⊢(Icb →(QAba →QIac)) (Dimaris XQQ, from 11 by AS)
14. ⊢(Acb →(QAba →QIac)) (Bramantip XQQ, from 13 by AS)
15. ⊢(Acb →(QEba →QOca)) (Camenop XQQ, from 14 by AS, CW) ■

Theorem 8.8. (asserted QLQs and LQQs) The non-numbered QLQ and LQQ cells on table 15 correspond to asserted sentences.

Proof. Use theorem 8.7 and Sub-o. ■

So, for example, ⊢(QAbc →(LAab →QAac)) since ⊢(QAbc →(Aab →QAac)) by theorem 8.7 and since ⊢(LAab →Aab) by Sub-o.

Theorem 8.9. (asserted QXMs and XQMs) The non-numbered QXM and XQM cells on table 15 correspond to asserted sentences.

Proof. For non-numbered QXM and XQM cells referred to by names that do not end with 'o' use theorem 8.7 wherever possible with Q-sub-o and CW. So Barbara QXM is asserted since Barbara QXQ is assserted. And, by this reasoning, Celarent QXM, Darii QXM, Darapti QXM, Datisi QXM, Barbari QXM, Darapti XQM, Disamis XQM, Bramantip XQM, Camenes XQM, Dimaris XQM and Barbari XQM. For the remaining non-numbered cells use asserted MXM syllogisms from table 13 wherever possibile with Q-sub-o and AS. So, Ferio QXM is accepted since Ferio MXM is accepted. And, by this reasoning, Festino QXM, Felapton QXM, Disamis QXM, Ferison QXM, Bramantip QXM, Dimaris QXM, Fresison QXM, Fesapo QXM, Celaront QXM, Cesaro QXM, Darii XQM, Datisi XQM and Barbari XQM. The only remaining non-numbered QXM and XQM cells correspond to the axiom Barbara XQM (A21) and Camenop XQM, which is deduced from Camenes XQM by CW given Ap-sub-a. ■

Theorem 8.10. (asserted QLXs and LQXs) The non-numbered QLX and LQX cells on table 15 correspond to asserted sentences.

In the following proof the asterisks mark inconsistencies in the data as reported on McCall's table 12 on pp. 84-85 of [1963].

Proof. Use theorem 8.9 with RV and SE. So i) Celarent QLX is asserted since Festino QXM is asserted; ii) Celaront QLX (is asserted) since Cesaro QXM (is asserted); iii)* Cesare QLX since Ferio QXM; iv) Camestres QLX since Darii QXM; v)* Festino QLX since Celarent QXM; vi)* Baroco QLX since Barbara QXM; vii) Cesaro QLX since Celaront QXM; viii) Camestrop QLX since Barbari QXM; ix) Camenes QLX since Dimaris XQM; x) Fresison QLX since Camenes XQM; xi) Fesapo QLX since Camenop XQM; xii) Camenop QLX since Bramantip XQM; xiii) Celarent LQX since Disamis XQM; xiv) Ferio LQX since Datisi XQM; xv) Celaront LQX since Barbari XQM; xvi) Cesare LQX since Datisi QXM; xvii)* Camestres LQX since Ferison QXM; xviii) Festino LQX since Disamis QXM; xix) Cesaro LQX since Darapti QXM; xx) Camestrop LQX since Felapton QXM; xxi) Felapton LQX since Barbari XQM; xxii) Bocardo LQX since Barbara XQM; xxiii)* Ferison LQX since Darii XQM; xxiv) Camenes LQX since Fresison QXM; xxv) Fresison LQX since Dimaris QXM; xxvi) Fesapo LQX since Bramantip QXM; and xxvii) Camenop LQX since Fesapo QXM. ■

Theorem 8.11. The non-numbered QLM and LQM cells on table 15 correspond to asserted sentences.

Proof. For the QLMs use: i) results for the accepted QXM syllogisms stated in theorem 8.9, Sub-o and AS; or ii) results for the accepted QLX syllogisms stated in theorem 8.10, Sub-o and CW. So, for example, ⊢$(QAbc \rightarrow (LAab \rightarrow MAac))$ (Barbara QLM is asserted) since ⊢$(QAbc \rightarrow (Aab \rightarrow MAac))$ and ⊢$(LAab \rightarrow Aab)$ given AS. ⊢$(QEcb \rightarrow$

$(LAab \to MEac))$ (Cesare QLM is asserted) since $^{\vdash}(QEcb \to (LAab \to Eac))$ and $^{\vdash}(Eac \to MEac)$.
For the LQMs use results for the XQMs in theorem 8.9 or the LQXs in theorem 8.10 together with Sub-o, AS or CW. So, for example, $^{\vdash}(LAbc \to (QIab \to MIac))$ (Darii LQM is asserted) since $^{\vdash}(LAbc \to (QIab \to Iac))$ and $^{\vdash}(Iac \to MIac)$. $^{\vdash}(LEbc \to (QIab \to MOac))$ (Ferio LQM is asserted) since $^{\vdash}(LEbc \to (QIab \to Oac))$ and $^{\vdash}(Oac \to MOac))$. ■

Theorem 8.12. The non-numbered QQMs on table 15 correspond to asserted sentences.

Proof. Obtain the assertion of Barbara QQM from the assertion of Barbara QQQ by using CW with Q-sub-o. Use similar reasoning for Celarent, Darii, Barbari, Darapti, Disamis, Datisi, Bramantip and Dimaris. We generate the remaining four QQMs as follows.

1. $^{\vdash}(QEbc \to (QAab \to QEac))$ (Celarent QQQ)
2. $^{\vdash}(QEac \to MEac)$ (by Q-sub-o)
3. $^{\vdash}(MEac \to MOac)$ (by Ap-sub-a)
4. $^{\vdash}(QEac \to MOac)$ (from 2 and 3 by CW)
5. $^{\vdash}(QEbc \to (QAab \to MOac))$ (Celaront QQM, from 1 and 4 by CW)
6. $^{\vdash}(QAab \to (QEbc \to QEac))$ (from 1 by AI)
7. $^{\vdash}(MEac \to MEca)$ (by Ap-con)
8. $^{\vdash}(QEac \to MEca)$ (from 2 and 7 by CW)
9. $^{\vdash}(QAab \to (QEbc \to MEca))$ (from 6 and 8 by CW)
10. $^{\vdash}(QAcb \to (QEba \to MEac))$ (Camenes QQM, from 9 by US)
11. $^{\vdash}(MEac \to MOac)$ (Ap-sub-a and US)
12. $^{\vdash}(QAcb \to (QEba \to MOac))$ (Camenop QQM, from 10 and 11 by CW)
13. $^{\vdash}(QEcb \to (QAba \to MOac))$ (Fesapo QQM, from 12 by CC and AS) ■

8.1 *Semantics for QLXM′*

The semantics for QLXM′ is given by referring to Q-models.

Definition 8.13. (Q-model) $\mathcal{M}$ is a *Q-model* iff $\mathcal{M} = \langle W, n^+, q^+, n^-, q^- \rangle$, where W is a non-empty set and n^+, q^+, n^-, and q^- are functions that map terms into subsets of W and satisfy the following "base conditions":

BQ1 If f and g are any of the functions n^+, q^+, q^- or n^- and $f \neq g$, then, for every term x, $f(x) \cap g(x) = \emptyset$; and for every x, $n^+(x) \cup q^+(x) \cup q^-(x) \cup n^-(x) = W$
BQ2 (For every x and y) if $^+(x) \subseteq n^-(y)$ then $^+(y) \subseteq n^-(z)$
BQ3 If $^+(y) \subseteq n^+(z)$ and $^+(x) \circ {}^+(y)$ then $n^+(x) \circ n^+(z)$
BQ4 If $^+(y) \subseteq n^-(z)$ and $^+(x) \circ {}^+(y)$ then $n^+(x) \circ n^-(z)$
BQ5 If $^+(z) \subseteq n^+(y)$ and $n^+(x) \circ n^-(y)$ then $n^+(x) \circ n^-(z)$
BQ6 If $^+(y) \subseteq q(z)$ and $^+(x) \subseteq q(y)$ then $^+(x) \subseteq q(z)$
BQ7 If $^+(y) \subseteq q(z)$ and $^+(x) \circ q(y)$ or $q(x) \circ {}^+(y)$ then $^+(x) \circ q(z)$ or $q(z) \circ {}^+(x)$

BQ8 If $^{+}(y) \subseteq +(z)$ and $^{+}(x) \subseteq q(y)$ then $n^{+}(x)$ does not overlap $n^{-}(z)$

Definition 8.14. (valuation relative to a Q-model) $V_{\mathcal{M}}$ is a *valuation relative to a Q-model* $\mathcal{M}$ iff it is is a valuation that satisfies the following "superstructural conditions":

S1 (For every x and y) $V_{\mathcal{M}}(Axy) = t$ iff $^{+}(x) \subseteq {}^{+}(y)$
S2 $V_{\mathcal{M}}(Ixy) = t$ iff $^{+}(x) \circ {}^{+}(y)$.
S3 $V_{\mathcal{M}}(LAxy) = t$ iff $^{+}(x) \subseteq n^{+}(y)$
S4 $V_{\mathcal{M}}(LIxy) = t$ iff $n^{+}(x) \circ n^{+}(y)$
S5 $V_{\mathcal{M}}(L\neg Axy) = t$ iff $n^{+}(x) \circ n^{-}(y)$
S6 $V_{\mathcal{M}}(L\neg Ixy) = t$ iff $^{+}(x) \subseteq n^{-}(y)$
S7 $V_{\mathcal{M}}(QAxy) = t$ iff $^{+}(x) \subseteq q(y)$
S8 $V_{\mathcal{M}}(QIxy) = t$ iff $^{+}(x) \circ q(y)$ or $q(x) \circ {}^{+}(y)$
S9 $V_{\mathcal{M}}(Q\neg Axy) = t$ iff $^{+}(x) \circ q(y)$ or $q(x) \circ {}^{+}(y)$
S10 $V_{\mathcal{M}}(Q\neg Ixy) = t$ iff $^{+}(x) \subseteq q(y)$

Definition 8.15. (Q-valid) $\models_Q \alpha$ (α is *Q-valid*) iff, for every Q-model $\mathcal{M}$, $V_{\mathcal{M}}(\alpha) = t$. α is *Q-invalid* ($\models_Q \alpha$) iff α is not Q-valid.

Theorem 8.16. (soundness) If α is an assertion in QLXM$'$ then $\models_Q \alpha$.

Proof. We need to show that i) if $^{\vdash}\alpha$ is an axiom of QLXM$'$ then $\models_Q \alpha$; and ii) each assertion transformation rule of QLXM$'$ preserves Q-validity. Some examples of the reasoning needed are given. For A1, $\models_Q Aaa$ since, for every Q-model $\mathcal{M}$, $V_{\mathcal{M}}(Aaa) = t$ since $^{+}(a) \subseteq {}^{+}(a)$. For A2, $\models_Q Iaa$ since, for every Q-model $\mathcal{M}$, $V_{\mathcal{M}}(Iaa) = t$ since $^{+}(a) \circ {}^{+}(a)$. For A5, suppose there is a Q-model $\mathcal{M}$ such that $V_{\mathcal{M}}(LAbc) = t$, $V_{\mathcal{M}}(Aab) = t$ and $V_{\mathcal{M}}(LAac) = f$. Then $^{+}(b) \subseteq n^{+}(c)$, $^{+}(a) \subseteq {}^{+}(b)$ and $^{+}(a) \not\subseteq n^{+}(c)$, which is impossible. So $\models_Q (LAbc \to (Aab \to LAac))$. For A15, suppose there is a Q-model $\mathcal{M}$ such that $V_{\mathcal{M}}(QAbc) = t$, $V_{\mathcal{M}}(QAab) = t$ and $V_{\mathcal{M}}(QAac) = f$. Then $^{+}(b) \subseteq q(c)$, $^{+}(a) \subseteq q(b)$ and $^{+}(a) \not\subseteq q(c)$, which is impossible given **BQ6**. So $\models_Q (QAbc \to (QAab \to QAac))$. For AR1 suppose $\models_Q (\ldots x \ldots x \ldots)$ but $\not\models_Q (\ldots y \ldots y \ldots)$, where $(\ldots y \ldots y \ldots)$ is the result of replacing every occurrence of term x in $(\ldots x \ldots x \ldots)$ with term y. Then, for some Q-model $\mathcal{M}$, $V_{\mathcal{M}}(\ldots y \ldots y \ldots) = f$, where $V_{\mathcal{M}}(y)$ is set S. Let $V_{\mathcal{M}}(x) = S$. Then $V_{\mathcal{M}}(\ldots x \ldots x \ldots) = f$. So it is impossible for AR1 not to preserve validity. For AR2 suppose a) $\models_Q (p \to q)$, b) $\models_Q p$ and c) $\not\models_Q q$. Then, for some Q-model $\mathcal{M}$, $V_{\mathcal{M}}(q) = f$, given c). Then, by a), $V_{\mathcal{M}}(p) = f$, which conflicts with b). So AR2 preserves validity. Reasoning for the other axioms and rules is straightforward and is omitted. ■

Given the soundness of QLXM$'$ every asserted sentence in Łukasiewicz's ŁA is Q-valid since ŁA is a fragment of QLXM$'$. All of the L-X-M syllogisms marked as asserted on table 5 are Q-valid since all of them are asserted in QLXM$'$. And, given the following theorem, all of the syllogisms marked as invalid on table 5 are Q-invalid.

Theorem 8.17. Models $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_3$ and $\mathcal{M}_4$ are Q-models.

Proof. By earlier arguments the four models satisfy conditions **BQ1** to **BQ5**. Consider $\mathcal{M}_1$. Suppose $^+(y) \subseteq q(z)$. Then $z = c$. So **BQ6** is trivially satisfied. For all x, $^+(x) \circ q(c)$. So **BQ7** is satisfied. For all z, $n^-(z) = \emptyset$. So **BQ8** is satisfied. Consider $\mathcal{M}_2$. Suppose $^+(y) \subseteq q(z)$. Then $y = a$ or $y = b$, and $z = c$. So **BQ6** is trivially satisfied. For all x, $^+(x) \circ q(c)$. So **BQ7** is satisfied. For all z, if $^+(c) \subseteq^+ (z)$ then $z = c$. Since $n^-(c) = \emptyset$, **BQ8** is satisfied. Consider $\mathcal{M}_3$. Suppose $^+(y) \subseteq q(z)$. Then $y = b$ or $y = c$, and $z = a$. So **BQ6** is trivially satisfied. For all x, $^+(a) \circ q(x)$. So **BQ7** is satisfied. For all z, if $^+(a) \subseteq^+ (z)$ then $z = a$. Since $n^-(a) = \emptyset$, **BQ8** is satisfied. Consider $\mathcal{M}_4$. For all y and z, if $^+(y) \not\subseteq q(z)$. So **BQ6**, **BQ7** and **BQ8** are trivially satisfied. ■

Table 16. Q-model $\mathcal{M}_5$

	n^+	q^+	n^-	q^-
a	1	2	3	
b	3	1		2
c	1,3		2	

Table 16 expresses a model. **BQ1** and **BQ2**, here and below, require no comment. For every y and z, if $+(y) \subseteq n^+(z)$ then $z = c$. For every x, $n^+(x) \circ n^+(c)$. So **BQ3** is satisfied. For every y and z, $^+(y) \not\subseteq n^-(z)$. So **BQ4** is trivially satisfied. If $^+(y) \subseteq n^+(z)$ then $z = c$. For every x, y and z, if $z \subseteq n^+(y)$ then $n^+(x)$ does not overlap $n^-(y)$. So **BQ5** is satisfied. For every x and y, if $x \subseteq q(y)$ then $x = a$ and $y = b$. So **BQ6** is trivially satisfied. For all x, $x \circ q(b)$. So **BQ7** is satisfied. For all z, $^+(a)$ does not overlap $n^-(z)$. So **BQ8** is satisfied.

Given Q-model $\mathcal{M}_5$, $\not\models_Q (LAbc \rightarrow (QAab \rightarrow LAac))$. For, $V_{\mathcal{M}_5}(LAbc) = t$ since $^+(b) \subseteq n^+(c)$. $V_{\mathcal{M}_5}(QAab) = t$ since $^+(a) \subseteq q(b)$. And $V_{\mathcal{M}_5}(QAac) = f$ since $^+(a) \not\subseteq q(c)$. The occurrence of '4ac' in the Barbara/LQQ cell indicates Q-model $\mathcal{M}_5$ is a countermodel for Barbara LQQ, where 'a' is the minor term and 'c' is the major term. This method of listing minor and major terms will be followed below.

Table 17. Model $\mathcal{M}_6$

	n^+	q^+	n^-	q^-
a	1	2	3,4	
b	3	1	4	2
c	4		1,2,3	

Table 17 expresses a model. For every y and z, if $^+(y) \subseteq n^+(z)$ then $y = z = c$. For every x and y, if $^+(x) \circ {}^+(y)$ then $x = c$. Since $n^+(c) \circ n^+(c)$, **BQ3** is satisfied. For every y, if $^+(y) \subseteq n^-(a)$ then $y = b$ or $y = c$. For every x, if $x \circ {}^+(b)$ or $x \circ {}^+(c)$ then $n^+(x) \circ n^-(a)$. For every y, if $^+(y) \subseteq n^-(b)$ then $y = b$. For every x, if $x \circ {}^+(b)$

then $n^+(x) \circ n\ (b)$. For every y, if $^+(y) \subseteq n^-(c)$ then $y = a$ or $y = b$. For every x, if $x \circ\ ^+(a)$ or $x \circ\ ^+(b)$ then $n^+(x) \circ n^-(c)$. So **B4** is satisfied. For every y and z, if $^+(z) \subseteq n^+(y)$ then $n^-(y) \subseteq n^-(z)$ (that is, Thomason's **BT5** is satisfied).[28] So, **BQ5** is satisfied. For every x and y, if $^+(x) \subseteq q(y)$ then $x = a$ and $q = b$. So **BQ6** is trivially satisfied. For all w, if $^+(w) \circ q(a)$ or $q(w) \circ\ ^+(a)$ then $w = a$ or $w = b$. Since $^+(a) \circ q(b)$ and $^+(b) \circ q(b)$, **BQ7** is satisfied. For every z, if $^+(b) \subseteq z$ then $z = b$. Since $^+(a)$ does not overlap $n^-(b)$, **BQ8** is satisfied.

Given Q-model $\mathcal{M}_6$, *not* $\models_Q (LEac \rightarrow (QAab \rightarrow QObc))$. For, $V_{\mathcal{M}_6}(LEac) = t$ since $^+(a) \subseteq n^-(c)$. $V_{\mathcal{M}_6}(QAab) = t$ since $^+(a) \subseteq q(b)$. And $V_{\mathcal{M}_6}(QObc) = f$ since $^+(b)$ does not overlap $q(c)$ and $q(b)$ does not overlap $^+(c)$. The occurrence of '6bc' in the Felapton/LQQ cell indicates that Q-model $\mathcal{M}_6$ is a countermodel for Felapton LQQ, where 'b' is the minor term and 'c' is the major term.

Table 18. Model $\mathcal{M}_7$

	n^+	q^+	n^-	q^-
a	1		3	2
b	2	3		1
c	1		3	2

Table 18 expresses a Q-model. Since Thomason's **BT3** (if $^+(x) \circ\ ^+(y)$ then $^+(x) \circ n^+(y)$) is satisfied, both **BQ3** and **BQ4** are satisfied. Since **BT5** is satisfied **BQ5** is satisfied. If $^+(x) \subseteq q(y)$ then $y = b$ and either $x = a$ or $x = c$. Then **BQ6** is trivially satisfied. If $^+(z) \circ q(a)$ or $q(z) \circ\ ^+(a)$ and if $^+(z) \circ q(c)$ or $q(z) \circ\ ^+(c)$ then $^+(z) \circ\ ^+(b)$. So **BQ7** is satisfied. If $^+(b) \subseteq\ ^+(z)$ then $z = b$. Since $n^-(b) = \emptyset$, **BQ8** is satisfied.

Use Q-model $\mathcal{M}_8$ to show that Barbari LQX and others are invalid.

Table 19. Model $\mathcal{M}_8$

	n^+	q^+	n^-	q^-
a	1	2	3	4
b	4	3		1,2
c	3,4		2	1

Table 19 expresses a Q-model. Suppose $^+(y) \subseteq n^+(z)$. Then $y = b$ or $y = c$, and $z = c$. Since $n^+(b) \circ n^+(c)$ and $n^+(c) \circ n^+(c)$, **BQ3** is satisfied. Since there is no x such that $n^+(x) \circ n^-(c)$, **BQ5** is satisfied. Since, for every x and y, $x \not\subseteq y$, **BQ4** is

[28]As noted above, **BQ5** is a weaker condition than **BT5**. Replacing **BQ5** with **BT5** in the definition of a Q-model, forming a Q′ model, yields this highly unAristotelian result: $\models_{Q'} (QAab \rightarrow Eab)$. For, suppose that for some Q′-model $\mathcal{M}$, $V_{\mathcal{M}}(Eab) = f$. Then $V_{\mathcal{M}}(Iab) = t$. By S2 and **BT5**, $^+(a) \circ n^+(b)$. By S9, $V_{\mathcal{M}}(QAab) = f$.

trivially satisfied. Suppose $^{+}(y) \subseteq q(z)$. Then $y = a$ and $z = b$. Then **BQ6** is trivially satisfied. Since, for every x, $^{+}(x) \circ q(b)$ or $q(x) \circ {}^{+}(b)$, **BQ7** is satisfied. If $^{+}(b) \subseteq {}^{+}(z)$ then $z = b$ or $z = c$. Since, for all x, $n^{+}(x)$ does not overlap $n^{-}(b)$ and $n^{+}(x)$ does not overlap $n^{-}(c)$, **BQ8** is satisfied.

Table 20. Model $\mathcal{M}_9$

	n^+	q^+	n^-	q^-
a	1	2	3,4	
b	3,4	1		2
c	3		1,2,4	

Table 20 expresses a Q-model. If $^{+}(y) \subseteq n^{+}(z)$ then $y = c$ and either $z = c$ or $z = b$. If $^{+}(x) \circ {}^{+}(c)$ then $x = b$ or $x = c$. Since $n^{+}(x) \circ n^{+}(z)$, **BQ3** is satisfied. For every y, if $^{+}(y) \subseteq n^{-}(a)$ then $y = c$. For every x, if $x \circ {}^{+}(c)$ then $n^{+}(x) \circ n^{-}(a)$. There are no y such that $y \subseteq n^{-}(b)$. For every y, if $^{+}(y) \subseteq n^{-}(c)$ then $y = a$. For every x, if $x \circ {}^{+}(a)$ then $n^{+}(x) \circ n^{-}(c)$. So **BQ4** is satisfied. If $^{+}(z) \subseteq n^{+}(y)$ then $z = c$ and either $y = b$ or $y = c$. $n^{+}(x)$ does not overlap $n^{-}(b)$. If $n^{+}(x) \circ n^{-}(c)$ then $x = a$ or $x = b$. Since $n^{+}(a) \circ n^{-}(c)$ and $n^{+}(b) \circ n^{-}(c)$, **BQ5** is satisfied. For every x and y, if $x \subseteq q(y)$ then $x = a$ and $y = b$. So **BQ6** is trivially satisfied. For all z, if $^{+}(z) \circ {}^{+}(a)$ then $z = a$ or $z = b$. Since $^{+}(a) \circ q(b)$ and $^{+}(b) \circ q(b)$, **BQ7** is satisfied. For all z, if $^{+}(b) \subseteq {}^{+}(z)$ then $b = z$. Since $n^{-}(z) = \emptyset$, **BQ8** is satisfied.

Table 21. Model $\mathcal{M}_{10}$

	n^+	q^+	n^-	q^-
a	1	2		3,4
b	3	2	1	4
c	4	2	1,3	

Table 21 expresses a Q-model. For every x and y, $^{+}(x) \not\subseteq n^{+}(y)$. So **BQ3** and **BQ5** are satisfied. For all x and y, $^{+}(x) \not\subseteq n^{-}(y)$. So **BQ4** is satisfied. If $^{+}(x) \subseteq q(y)$ and $^{+}(y) \subseteq q(z)$ then $x = a$ and $z = c$. So **BQ6** is satisfied. For every x and y, $^{+}(x) \circ q(y)$ or $q(x) \circ {}^{+}(y)$. So **BQ7** is satisfied. If $^{+}(x) \subseteq q(y)$ then $x = b$ or $x = c$. For all z, $n^{+}(b)$ does not overlap $n^{-}(z)$ and $n^{+}(c)$ does not overlap $n^{-}(z)$. So **BQ8** is satisfied.

Table 22 expresses a Q-model. Since **BT3** is satisfied, **BQ3** and **BQ4** are satisfied. Since **BT5** is satisfied, **BQ5** is satisfied. For every x and y, $x \not\subseteq y$. So **BQ6**, **BQ7** and **BQ8** are trivially satisfied.

Table 22. Model $\mathcal{M}_{11}$

	n^+	q^+	n^-	q^-
a	1,2			
b	1	2		
c	1,2			

8.2 *Q-valid moods needed for completeness*

Aristotle did not discuss any moods with possiblity, as opposed to contingency, premises (or antecedents). But, given the semantics proposed for QLXM′ we must recognize the Q-validity of some moods in which an M-wff is a premise (or an antecedent). In particular Darii QMQ is Q-valid. So, to move in the direction of obtaining completeness results for QLXM′ we shall amend the system by making Darii QMQ axiom 29 (A29).

Theorem 8.18. (soundness of amended QLXM′) Suppose QLXM′ is amended by making the assertion of Darii QMQ, $\vdash(QAbc \rightarrow (MIab \rightarrow QIac))$ be an axiom. Leave everything else unchanged. Then the resulting system is sound.

Proof. Suppose $\mathcal{M}$ is a Q-model, $V_{\mathcal{M}}(QAbc) = t$ and $V_{\mathcal{M}}(MIab) = t$. Given the definition of a Q-model, at least one of these three conditions is met: i) ${}^+(a) \circ {}^+(b)$, ii) ${}^+(a) \circ q(b)$ or iii) ${}^+(a) \subseteq n^-(b)$. If i)is met then ${}^+(a) \circ q(c)$ and thus $V_{\mathcal{M}}(QIac) = t$. If ii) is met then ${}^+(a) \circ q(c)$ or $q(a) \circ {}^+(c)$ and thus $V_{\mathcal{M}}(QIac) = t$. If iii) is met then $V_{\mathcal{M}}(MIab) = t$ and $V_{\mathcal{M}}(MIab) = f$. Given this absurdity $V_{\mathcal{M}}(QIac) = t$. So $\models_Q (QAbc \rightarrow (MIab \rightarrow QIac))$. ■

Assertions that are Q-valid correspond to unmarked cells on table 23. The marks in cells indicate how countermodels may be found for the Q-invalid syllogisms the table refers to.

For each unmarked cell we shall show how the indicated syllogism is asserted in the system.

Theorem 8.19. (asserted QMQs and MQQs) The non-numbered QMQ and MQQ cells on table 23 correspond to asserted wffs.

Proof.

1. $\vdash(QAbc \rightarrow (MIab \rightarrow QIac))$ (Darii QMQ, A29)
2. $\vdash(QEbc \rightarrow (MIab \rightarrow QOac))$ (Ferio QMQ, from 1 by CC, AS, CW)
3. $\vdash(QAbc \rightarrow (MIba \rightarrow QIac))$ (Datisi QMQ, from 1 by Ap-con, AS)
4. $\vdash(QAbc \rightarrow (MAba \rightarrow QIac))$ (Darapti QMQ, from 3 by Ap-con, AS)
5. $\vdash(QEbc \rightarrow (MIba \rightarrow QOac))$ (Ferison QMQ, from 3 by CC, AS, CW)
6. $\vdash(QEbc \rightarrow (MAba \rightarrow QOac))$ (Felapton QMQ, from 4 by CC, AS, CW)
7. $\vdash(QAbc \rightarrow (MAab \rightarrow QIac))$ (Barbari QMQ, from 1 by Ap-sub-a, AS)

Table 23. Additional Q-syllogisms

		QMQ	MQQ	QMM	MQM	QLL	LQL	QQM
Figure 1	Barbara	12ac	5ac	12ac	15ca	7ab	8ac	
	Celarent	12ac	6ac	13ac	7ac	7ab		
	Darii		5ac		15ca	7ab	14bc	
	Ferio		6ac	14ac	14ac	7ab	15ba	14ac
Figure 2	Cesare	9ca	6ac	7ac	7ac	14ab		7ac
	Camestres	6ca	9ac	7ac	7ac		14ba	7ac
	Festino	9ca	6ac	7ac	7ac	13ab	15ba	7ac
	Baroco	6ca	9ac	7ac	7ac	12ab	14ba	7ac
Figure 3	Darapti					7cb	7bc	
	Felapton		9bc	14ac	14ac	7cb	15bc	14ac
	Disamis	5ca		15ac		7cb	7bc	
	Datisi		5ca		15ca	7cb	14bc	
	Bocardo	5ca	9bc	14ac	8bc	7cb	15ba	14ac
	Ferison		9bc	14ac	14ac	7cb	15ba	14ac
Figure 4	Bramantip	5ca		15ac		8ca	7ba	
	Camenes	6ca	7bc	7ac	13ca		7ba	
	Dimaris	5ca		15ac		7cb	7ba	
	Fresison	5ca	6bc	7ac	7ac	13ab	15ba	7ca
	Fesapo	5ca	6bc	7ac	8bc	16cb	15ba	
Subalterns	Barbari		5ac		15ac	7ab	14bc	
	Celaront		6ac	13ac	7ac	7ab		
	Cesaro	9ca	6ac	7ac	7ac	13ab		7ac
	Camestrop	6ca	9ac	7ac	7ac		14ba	7ac
	Camenop	6ca		7ac	16ac		7ba	

8. $\vdash(QEbc \rightarrow (MAab \rightarrow QOac))$ (Celaront QMQ, from 7 by CC, AS, CW)
9. $\vdash(MIbc \rightarrow (QAba \rightarrow QIac))$ (Disamis MQQ, from 3 by AI, Q-con, CW)
10. $\vdash(MAbc \rightarrow (QAba \rightarrow QIac))$ (Darapti MQQ, from 3 by Ap-sub-a, AS)
11. $\vdash(MIcb \rightarrow (QAba \rightarrow QIac))$ (Dimaris MQQ, from 1 by AI, Q-con, CW, US)
12. $\vdash(MAcb \rightarrow (QAba \rightarrow QIac))$ (Bramantip MQQ, from 11 by Ap-sub-a, AS)
13. $\vdash(MAcb \rightarrow (QEba \rightarrow QOac))$ (Camenop MQQ, from 12 by CC, AS, CW)

■

Theorem 8.20. (asserted QMMs and MQMs) The non-numbered QMM and MQM cells on table 23 correspond to asserted sentences.

Proof. Use theorem 8.19 and Q-sub-o. ■

Theorem 8.21. (asserted QLLs and LQLs) The non-numbered QLL and LQL cells on table 23 correspond to asserted wffs.

Proof. Use theorem 8.20 and RV. ■

Theorem 8.22. (asserted QQMs) The non-numbered QQM cells on table 23 correspond to asserted wffs.

Proof. Use theorem 8.6, Q-sub-o and CW for cells other than Camenes, Fesapo, Celaront and Camenop QQM. For them use the following reasoning.

1. $\vdash(QEbc \to (QAab \to MEac))$ (Celarent QQM)
2. $\vdash(QEbc \to (QAab \to MOac))$ (Celaront QQM, from 1 by Ap-sub-a, CW)
3. $\vdash(QAcb \to (QAba \to MEac))$ (Camenes QQM, from 1 by AI, Ap-con, CW, US)
4. $\vdash(QAcb \to (QAba \to MOac))$ (Camenop QQM, from 3 by Ap-sub-a, CW)
5. $\vdash(QEcb \to (QAba \to MOac))$ (Fesapo QQM, from 4 by CC, AS) ■

Table 24. Model $\mathcal{M}_{12}$

	n^+	q^+	n^-	q^-
a	1	2	3	4
b	3	4	2	1
c	2	3	1	4

Table 24 expresses a Q-model. For every x and y, $^+(x) \not\subseteq n^+(y)$. So **BQ3** and **BQ5** are trivially satisfied. For every x and y, $^+(x) \not\subseteq n^-(y)$. So **BQ4** is trivially satisfied. Suppose $^+(y) \subseteq q(z)$. Then $y = b$ and $z = c$. So **BQ6** is trivially satisfied. For every x, $^+(x) \circ q(b)$ or $q(x) \circ {}^+(b)$. So **BQ7** is satisfied. If $^+(c) \subseteq z$ then $z = c$. Since $n^+(b)$ does not overlap $n^-(c)$, **BQ8** is satisfied.

Table 25. Model $\mathcal{M}_{13}$

	n^+	q^+	n^-	q^-
a	1	2	3	4
b	4	3	2	1
c	1,2	3		4

Table 25 expresses a Q-model. Suppose $^+(y) \subseteq n^+(z)$. Then $y = a$ and $z = c$. If $^+(x) \circ {}^+(a)$ then $x = a$ or $x = c$. Since $n^+(a) \circ n^+(c)$, **BQ3** is satisfied. Since $n^-(c) = \emptyset$, **BQ5**is trivially satisfied. For every x and y, $^+(x) \not\subseteq n^-(y)$. So **BQ4** is trivially satisfied. Suppose $^+(y) \subseteq q(z)$. Then $y = b$ and $z = c$. So **BQ6** is trivially satisfied. For all x, $^+(c) \circ q(x)$. So **BQ7** is satisfied. Since $n^-(z) = \emptyset$, **BQ8** is satisfied.

Table 26 expresses a Q-model. For every x and y, if $^+(x) \subseteq n^+(y)$ then $x = a$ and $y = c$. If $^+(x) \circ {}^+(a)$ then $n^+(x) \circ n^+(c)$. So **BQ3** is satisfied. For every x and y,

Table 26. Model $\mathcal{M}_{14}$

	n^+	q^+	n^-	q^-
a	1	2		3,4
b	3	4	1	2
c	1,2	4		3

$^+(x) \not\subseteq n^-(y)$. So **BQ4** is satisfied. Since $n^-(c) = \emptyset$, **BQ5** is satisfied. For every x and y, if $^+(x) \subseteq q(y)$ then $y = a$ or $y = c$. So **BQ6** is trivially satisfied. For every z, $^z \circ\ q(a)$ or $q(z) \circ\ ^+(a)$. And for every z, $^z \circ\ q(c)$ or $q(z) \circ\ ^+(c)$. So **BQ7** is satisfied. For every z, if $^+(a) \subseteq\ ^+(z)$ then $z = a$. And for every z, if $^+(c) \subseteq\ ^+(z)$ then $z = c$. Since $n^-(a) = \emptyset$ and $n^-(a) = \emptyset$, **BQ8** is satisfied.

Table 27. Model $\mathcal{M}_{15}$

	n^+	q^+	n^-	q^-
a	1		2,3	4
b	4	3		1,2
c	2		1,4	3

Table 27 expresses a Q-model. Since **BT3** is satisfied, both **BQ3** and **BQ4** are satisfied. Since **BT5** is satisfied **BQ5** is satisfied. Suppose $^+(y) \subseteq q(z)$. Then $z = b$. So **BQ6** is trivially satisfied. For all x, $x \circ\ q(b)$. So **BQ7** is satisfied. If $^+(b) \subseteq\ ^+(x)$ then $x = b$. Since $n^-(b) = \emptyset$, **BQ8** is satisfied.

Table 28. Model $\mathcal{M}_{16}$

	n^+	q^+	n^-	q^-
a	1	2		3
b	2	3		1
c	1,2			3

Table 28 expresses a Q-model. Suppose $^+(y) \subseteq n^+(z)$. Then $z = c$. Since, for all x, $n^+(x) \circ\ n^+(c)$, **BQ3** is satisfied. Since $n^-(c) = \emptyset$, **BQ5** is satisfied. Since, for all x and y, $^+(x) \not\subseteq n^-(y)$, **BQ4** is trivially satisfied. Suppose $^+(y) \subseteq q(z)$. Then $y = b$ and $z = a$. So **BQ6** is trivially satisfied. For all x, $x \circ\ q(a)$. So **BQ7** is satisfied. For all x, $n^-(x) = \emptyset$. So **BQ8** is trivially satisfied.

Note that the acceptance of all but two of the QLM and LQM moods is generated from acceptances involving the MLM and LMM moods.

9 THE ARISTOTELICITY OF QLXM′

Of the 154 first, second or third figure syllogisms referred to on table 15 there are exactly thirteen that are Q-valid but invalid for Aristotle. And there are exactly nine that are Q-invalid but are valid for Aristotle. So the Aristotelicity of QLXM′ system is about 86%. Of the twenty-two discrepancies seventeen are due to mistakes involving the use of Reversal. These mistakes are marked on table 14 by using pairs of numbers from 1 to 17. So, for example, on this table both Barbara QXM and Baroco QLX are marked with '1', indicating that by Reversal both should be valid or both should be invalid. But Aristotle regarded only the former as valid. Both Ferison QLM and Camestres LQM are marked with '17', indicating that by Reversal both should be valid or both should be invalid. Aristotle regarded only the former as valid.

So, there are five remaining discrepancies to account for. i) Darapti XQQ: Aristotle could have used Disamis XQQ to show its validity. ii) Darapti LQQ: Aristotle could have used Darapti XQQ to show its validity. iii) Festino QXM: As noted by McCall in [1963, p. 93], Aristotle could have used Festino MXM to show its validity. Given Reversal, Festino MXM is valid in virtue of Disamis XLL. iv) Celarent QLX: Aristotle could have used Reversal and Festino QXM to show it is valid. v) Felapton XQM: Aristotle properly regarded it as valid since he regarded Ferio XQM as valid. Given our interest in developing a formal system that would not have the unAristotelian results, noted in theorem 7.2, which are present in McCall's Q-L-X-M system, we chose to regard Ferio XQM as Q-invalid.

10 TALLY OF THE TWO-PREMISED Q-VALID SYLLOGISMS

The 333 syllogisms marked on Table 13 are the Q-valid apodeictic two-premised syllogisms in which no contingent wff is a premise or a conclusion. Table 15 and table 23 refer to some of the Q-valid 2-premised syllogisms that involve contingent wffs. To count all of them we need to take account of complementary conversions. Note, for example, that AEA QQQ-figure 1 (that is $(QAbc \rightarrow (QEab \rightarrow QAac))$) is Q-valid by complementary conversion since Barbara QQQ is Q-valid.[29] In this section we shall count all of the 2-premised syllogisms that are Q-valid.

When counting the valid moods we shall use '[A]' to mean that the premise or conclusion indicated may be either an A or an E wff. Similarly we shall use '[I]' to mean the premise or the conclusion indicated may be either an I or an O formula. So, by saying that QQQ [A][A][A] in figure 1 is Q-valid, we are claiming the validity of eight figure 1 QQQ syllogisms: QQQ AAA (AAE, AEA, AEE, EAA, EAE, EEA, and EEE). By saying that QXQ [A]I[I] in figure 1 is Q-valid, we are claiming the Q-validity of four figure 1 QXQ syllogisms: QXQ AII (AIO, EII, and EIO).

[29] See Ross's table in [1949, facing p. 286] for references to this as well as several other syllogisms that may be validated by using complementary conversion.

Q-valid QQQs (64):
Figure 1: [A][A][A], [A][I][I], [A][A][I]
Figure 3: [A][A][I], [I][A][I], [A][I][I]
Figure 4: [A][A][I], [I][A][I]

Q-valid QXQs and QLQs (40):
Figure 1: [A]A[A], [A]I[I], [A]A[I],
Figure 3: [A]A[I], [A]I[I]

Q-valid XQQs and LQQs (32):
Figure 3: A[A][I], I[A][I]
Figure 4: A[A][I], I[A][I]

Q-valid QXMs (34):
Figure 1: [A]AA, [A]AE, [A]II, [A]IO, [A]AI, [A]AO
Figure 2: [A]IO, [A]AO
Figure 3: [A]AI, [A]AO, [I]AI, [A]II, [A]IO
Figure 4: [A]AI, [I]AI, [A]IO, [A]AO

Q-valid XQMs (20):
Figure 1: A[A]A, A[I]I, A[A]I
Figure 3: A[A]I, I[A]I, A[I]I,
Figure 4: A[A]I, A[A]E, I[A]I, A[A]O

Q-valid QLXs (24):
Figure 1: [A]AE, [A]AO
Figure 2: [A]AE, [A]EE, [A]IO, [A]OO, [A]AO, [A]EO
Figure 4: [A]EE, [A]IO, [A]AO, [A]EO

Q-valid LQXs (30):
Figure 1: E[A]E, E[I]O, E[A]O
Figure 2: E[A]E, A[A]E, E[I]O, E[A]O, A[A]O
Figure 3: E[A]O, O[A]O, E[I]O
Figure 4: A[A]E, E[I]O, E[A]O, A[A]O

Q-valid QLMs (46):
Figure 1: [A]AA, [A]AE, [A]II, [A]IO, [A]AI, [A]AO
Figure 2: [A]AE, [A]EE, [A]IO, [A]OO, [A]AO, [A]EO
Figure 3: [A]AI, [A]AO, [I]AI, [A]II, [A]IO
Figure 4: [A]AI, [A]EE, [I]AI, [A]IO, [A]AO, [A]EO

Q-valid LQMs (46):
Figure 1: A[A]A, E[A]E, A[I]I, E[I]O, A[A]I, E[A]O
Figure 2: E[A]E, A[A]E, E[I]O, E[A]O, A[A]O

Figure 3: A[A]I, E[A]O, I[A]I, A[I]I, O[A]O, E[I]O
Figure 4: A[A]I, A[A]E, I[A]I, E[I]O, E[A]O, A[A]O

Q-valid QMQs (16):
Figure 1: [A]I[I], [A]A[I]
Figure 3: [A]A[I], [A]I[I]

Q-valid MQQs (16):
Figure 3: A[A][I], I[A][I]
Figure 4: A[A][I], I[A][I]

Q-valid QMMs (8):
Figure 1: [A]II, [A]AI
Figure 3: [A]AI, [A]II

Q-valid MQMs (8):
Figure 3: A[A]I, I[A]I
Figure 4: A[A]I, I[A]I

Q-valid QLLs (8):
Figure 2: [A]EE, [A]EO
Figure 4: [A]EE, [A]EO

Q-valid LQLs (8):
Figure 1: E[A]E, E[A]O
Figure 2: E[A]E, E[A]O

Q-valid QQMs (48):
Figure 1: [A][A]A, [A][A]E, [A][I]I, [A][A]I, [A][A]O
Figure 3: [A][A]I, [I][A]I, [A][I]I
Figure 4: [A][A]I, [A][A]E, [I][A]I, [A][A]O

There are 333 + 64 + 40 + 32 + 34 + 20 + 24 + 30 + 46 + 46 + 16 + 16 + 8 + 8 + 8 + 8 + 48 (that is, 781) Q-valid 2-premised syllogisms found in thirty five "general moods": LLX, LLM, LXX, LXM, XLX, XLM, XXX, XXM, LLL, LXL, XLL, LMM, MLM, MXM, XMM, LMX, MLX, QQQ, QXQ, QLQ, XQQ, LQQ, QXM, XQM, QLX, LQX, QLM, LQM, QMQ, MQQ, QMM, MQM, QLL, LQL, and QQM.

11 EXTENSIONS

The most natural extension of the above work on QLXM′ would be to develop a Smiley-type decision procedure for validity for the n-premised syllogisms, for $n \geq 2$, where these syllogisms meet the chain condition. Though Smiley's decision procedure for the assertoric syllogistic pairs inconsistent sets of wffs with syllogisms construed as inferences,

the pairing could also be between sets of wffs and syllogisms constructed as implications. The decision procedure would list Q-inconsistent sets such as $\{P_1Ax_1x_2, P_2Ax_3x_4, \ldots, P_nAx_{2n-1}x_{2n}, \widetilde{Q}Ax_1x_{2n}\}$, where: i) each P_i, for $1 \leq i < n$, is X, L or Q; ii) P_n is Q; and iii) $\widetilde{Q}$ (the negation of Q) is a new quantifier. Given the decision procedure it would follow that $(QAab \rightarrow (Abc \rightarrow (LAcd \rightarrow (QAde \rightarrow QAae))))$, for example, is Q-valid.

Though it is argued above that QLXM′ is more Aristotelian than McCall's Q-L-X-M there are several other systems that could be developed to bring coherence into Aristotle's discussions of modalities. For example, consider Barbara XQM. McCall points out that Aristotle's defense of its validity is flawed, but McCall chooses to take it as an axiom in his Q-L-X-M. It is also an axiom in QLXM′. Dropping this axiom would mean that the semantics for the weaker system would be simpler.

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INDIAN LOGIC

Jonardon Ganeri

1 ARGUMENTATION WITHIN DIALECTIC AND DEBATE: PRAGMATIC CRITERIA FOR GOOD ARGUMENTATION

1.1 Early dialogues: information-seeking, interrogation and cross-checking

The intellectual climate of ancient India was vibrant, and bristled with controversy. Debates were held on a great variety of matters, philosophical, scientific and theological. Quite soon, the debates became formal affairs, with reputations at stake and matters of importance in the balance. Already in the *Bṛhadāraṇyaka Upaniṣad* (*c.* 7^{th} century BCE), we find the sage Yājñavalkya being quizzed by the king's priestly entourage on tricky theological puzzles:

> Once when Janaka, the king of Videha, was formally seated, Yājñvalkya came up to him. Janaka asked him: 'Yājñvalkya, why have you come? Are you after cows, or discussion about subtle truths?' He replied: '*Both*, your majesty.' (BU 4.1.1).

What followed was a question-answer type dialogue in which Janaka interrogated the sage, not only to solicit information but to test Yājñavalkya's mettle. The sage had earlier granted Janaka a wish, and the wish he chose was the freedom to ask any question at will. Yājñavalkya was not to be released from this wish until he had fully satisfied Janaka's probing inquiry:

> [Janaka] 'Here, sire, I'll give you a thousand cows! But you'll have to tell me more than that to get yourself released!' At this point Yājñvalkya became alarmed, thinking: 'The king is really sharp! He has flushed me out of every cover.' (BU 4.3.33–4).

It is in fact a characteristic of the earliest recorded debates that they take the form of question-answer dialogues. As a form of debate, the goal of a question-answer dialogue is not restricted merely to one party soliciting information from another, for there are, as this dialogue shows, elements too of testing out one's opponent and cross-checking what he says. A particularly important early question-answer dialogue is the *Milinda-pañha*, or *Questions of King Milinda*. It records the encounter between a Buddhist monk Nāgasena and Milinda, also known as

Handbook of the History of Logic. Volume 1
Dov M. Gabbay and John Woods (Editors)

Menander, an Indo-Bactrian king who ruled in the part of India that had fallen under Greek influence at the time of Alexander's Indian campaign. The document dates from around the first century CE, although Milinda's reign was 155–130 BCE. At the outset, Nāgasena insists that their dialogue is conducted as scholarly debate and not merely by royal declaration[1] —

> King Milinda said: Reverend Sir, will you discuss with me again?
>
> Nāgasena: If your Majesty will discuss (*vāda*) as a scholar, well, but if you will discuss as a king, no.
>
> Milinda: How is it then that scholars discuss?
>
> Nāgasena: When scholars talk a matter over one with another, then is there a winding up, an unravelling, one or other is convicted of error, and he then acknowledges his mistake; distinctions are drawn, and contra-distinctions; and yet thereby they are not angered. Thus do scholars, O King, discuss.
>
> Milinda: And how do kings discuss?
>
> Nāgasena: When a king, your Magesty, discusses a matter, and he advances a point, if any one differ from him on that point, he is apt to fine him, saying "Inflict such and such a punishment upon that fellow!" Thus, your Magesty, do kings discuss.
>
> Milinda: Very well. It is as a scholar, not as a king, that I will discuss. (MP 2.1.3).

Vāda, the type of dialogue Nāgasena depicts as that of the scholar, is one in which there are two parties. Each defends a position with regard to the matter in hand; there is an 'unravelling' (*nibbeṭhanam*; an unwinding, an explanation) and a disambiguation of the positions of both — a process of revealing commitments, presumptions and faulty argument; there is also a 'winding up' ending in the censure (*niggaho*; Skt. *nigraha*) of one party, a censure based on reasons he himself will acknowledge (*paṭikamman*; 're-action', rejoinder). This is a species of the persuasion dialogue, a 'conversational exchange where one party is trying to persuade the other part that some particular proposition is true, using arguments that show or prove to the respondent that the thesis is true'[2]. Indeed, it would seem to be the species that has come to be known as the *critical discussion*, a

[1] A similar distinction, in the types of scientific debate held between physicians, will be drawn a little later by Caraka, a medical theorist, and an important source of information about ancient Indian logic. He says, in an echo of the *Meno* 7.5 c–d, that debate (*saṃbhāṣā*) among specialists is of two types — friendly (*sandhāya*) and hostile (*vigṛhya*). See *Caraka-Saṃhitā* 3.8.16–17 and Ernst Prets, 'Theories of Debate, Proof and Counter-Proof in the Early Indian Dialectical Tradition', in Balcerowicz, Piotr & Mejor, Marek eds., *On the Understanding of other cultures: Proceedings of the International Conference on Sanskrit and Related Studies to Commemorate the Centenary of the Birth of Stanislaw Schayer*, Warsaw 1999. (Warsaw: Oriental Institute, Warsaw University, 2000).

[2] Douglas Walton, *The New Dialectic: Conversational Contexts of Argument* (Toronto: University of Toronto Press, 1998), p. 37.

persuasion dialogue in which the conflict is resolved 'only if somebody retracts his doubt because he has been convinced by the other party's argumentation or if he withdraws his standpoint because he has realized that his argumentation cannot stand up to the other party's criticism'[3]. Not every persuasion dialogue need end in one party recognising defeat, for an important function of the general persuasion dialogue is to be maieutic, helping each side to clarify the nature of their commitments and the presuppositions upon which their positions depend.[4] In the to-and-fro of such a dialogue, each party is allowed to retract earlier commitments, as it becomes clear what the consequences of such a commitment would be. This maieutic, clarificatory function of a dialogue is perhaps what Nāgasena intends when he speaks of an 'unravelling', and it seems clearer still in his characterisation of 'investigation' (*vikāra*) as a 'threshing-out':

Milinda: What is the distinguishing characteristic, Nāgasena, of reflection (*vitakka*)?

Nāgasena: The effecting of an aim.

Milinda: Give me an illustration.

Nāgasena: It is like the case of a carpenter, great king, who fixes in a joint a well-fashioned piece of wood. Thus it is that the effecting of an aim is the mark of reflection.

Milinda: What is the distinguishing characteristic, Nāgasena, of investigation (*vikāra*)?

Nāgasena: Threshing out again and again.

Milinda: Give me an illustration.

Nāgasena: It is like the case of the copper vessel, which, when it is beaten into shape, makes a sound again and again as it gradually gathers shape. The beating into shape is to be regarded as reflection and the sounding again and again as investigation. Thus it is, great king, that threshing out again and again is the mark of investigation.

Milinda: Very good, Nāgasena. (MP 2.3.13-14).

So it is through reflection and argumentation that the parties to an investigation together thrash out a position. Nāgasena tells us very little about the sort of argumentation that is appropriate, and we can learn little more about argument within persuasion dialogues from the *Questions of King Milinda* (although Milinda's repeated request to be given an illustration is suggestive of the importance that would later be attached to the citation of illustrative examples in good argumentation; see §1.3 below). And yet there is still something to learn. For

[3]Frans van Eemeren & Rob Grootendorst, *Argumentation, Communication and Fallacies* (Hillsdale: Lawrence Erlbaum Associates,1992), p. 34.
[4]Walton (1988: 48).

the dialogue of the *Questions of King Milinda* is not, contrary to Nāgasena's initial statement, a straighforwardly scholarly debate, but proceeds instead with his being interrogated at the hands of Milinda. Ostensibly Milinda wishes to be informed as to the answer to a range of thorny ethical and metaphysical questions, but his questioning is not so innocent, and at times he seems intent on entrapping Nāgasena in false dichotomies and leading questions. So it is said of him:

> Master of words and sophistry (*vetaṇḍī*), clever and wise
> Milinda tried to test great Nāgasena's skill.
> Leaving him not, again and yet again,
> He questioned and cross-questioned him, until
> His own skill was proved foolishness. (MP 4.1.1).

Milinda here is significantly described as a 'master of sophistry' or *vetaṇḍī*, a practitioner of the dialogue form known as *vitaṇḍā*, a 'refutation-only' type of dialogue in which the opponent defends no thesis of his own but is set only on refuting that of the proponent (see §1.4). The implication here is that such dialogues are essentially eristic. And it is, in particular, the eristic use of questioning that Milinda is a master of. Questions need not be innocent requests for information; they can also be disguised arguments. To reply to the question 'When did you stop cheating on your tax returns?' at all, affirmatively or negatively, is already to commit oneself to the 'premise' of the question, that one has indeed been cheating on one's tax returns. In the intellectual climate of ancient India, when interrogative dialogue was common-place, it was very well known that questions can be used to entrap the unwitting, and counter-strategies were invented to avoid entrapment. The Buddha himself was well aware that replying to a yes-no question can commit one to a proposition, whatever answer one gives, and his solution, famously, was to refuse to answer. Thus when asked a series of ten leading questions — is the soul is eternal? is it non-eternal? etc. — the Buddha declined to offer a reply. For any reply would commit him, against his wish, to the existence of souls. In the *Questions of King Milinda*, we see Nāgasena experimenting with a different technique to avoid entrapment. To some of Milinda's more devious yes-no questions, instead of refusing to reply at all, Nāgasena replies 'Both yes and no'! To others he replies 'Neither yes nor no'! For example:

> Milinda: He who is born, Nāgasena, does he remain the same or become another?
>
> Nāgasena: Neither the same nor another.
>
> Milinda: Give me an illustration.
>
> Nāgasena: Now what do you think, O king? You were once a baby, a tender thing, and small in size, lying flat on your back. Was that the same as you who are now grown up?
>
> Milinda: No. That child was one, I am another.

Nāgasena: If you are not that child, it will follow that you have had neither mother nor father, no! nor teacher. You cannot have been taught either learning, or behaviour, or wisdom. ...Suppose a man, O king, were to light a lamp, would it burn the night through?

Milinda: Yes, it might do so.

Nāgasena: Now, is it the same flame that burns in the first watch of the night, Sir, and in the second?

Milinda: No.

Nāgasena: Or the same that burns in the second watch and the third?

Milinda: No.

Nāgasena: Then there is one lamp in the first watch, and another in the second, and another in the third?

Milinda: No. The light comes from the same lamp all the night through.

Nāgasena: Just so, O king, is the continuity of a person or thing maintained. One comes into being, another passes away; and the rebirth is, as it were, simultaneous. Thus neither as the same nor as another does a man go on to the last phase of his self-consciousness. (MP 2.2.1)

The 'premise' of the question, that to change is to cease to be, is very effectively refuted with a 'neither yes nor no' reply. Nāgasena first makes Milinda acknowledge that, with this as the background premise, answering either 'yes' or 'no' leads to an absurdity. For if he is strictly identical to the child, then he must share that child's properties; and if he is different, then he cannot. Having exposed the false premise, Nāgasena, rejects it in favour of the view that persistence through time requires not strict identity but causal continuity. Here is a different kind of example:

Milinda: Does memory, Nāgasena, always arise subjectively, or is it stirred up by suggestion from outside?

Nāgasena: Both the one and the other.

Milinda: But does not that amount to all memory being subjective in origin, and never artificial?

Nāgasena: If, O king, there were no artificial (imparted) memory, then artisans would have no need of practice, or art, or schooling, and teachers would be useless. But the contrary is the case.

Milinda: Very good, Nāgasena. (MP 3.6.11).

Here the question's hidden premise is that memories are caused either wholly by what goes on in the mind or wholly by factors external to it, and the 'both yes and no' reply makes plain that what ought to be said is that memories are

wholly caused either by what goes on in the mind or by factors external to it, but not caused wholly by one or the other. Again, subsidiary argumentation exposes the absurdity in replying with an unqualified 'yes' or an unqualified 'no'. It was perhaps in recognition of the tactical importance of such 'neither yes nor no' and 'both yest and no' replies that it became a common-place that there are *four possible ways* of responding to any question of the yes-no type, an idea that was systematised in the work of Nāgārjuna (§1.4). What we see very clearly in the *Questions of King Milinda* is a sophisticated early appreciation of the pragmatics of interogative dialogues.

1.2 On balance and fairness in the conduct of dialogue: The Kathāvatthu

The *Kathāvatthu* or *Points of Controversy* (circa third century BCE) is a book about method. It describes, for the benefit of adherents to various Buddhist schisms, the proper method to be followed in conducting a critical discussion into an issue of doctrinal conflict. Recent scholarship has largely focussed on the question of the extent to which there is, in the *Kathāvatthu*, an 'anticipation' of results in propositional logic.[5] For, while it is true that the formulation of arguments there is term logic rather than propositional, and true also that the propositional rules are nowhere formulated in the abstract, the codified argumentation clearly exploits manipulations that trade on the definition of material implication, on contraposition, and on at least one of *modus tollens*, *modus ponens* and *reductio ad absurdum.* The preoccupation with this question of anticipation, assumes, however, a methodology for the interpretation of Indian logic that suffers a number of serious disadvantages. For, first, in presupposing that the only matter of interest is the extent to which a given text displays recognition of principles of formal logic, the methodology fails to ask what it was that the authors themselves were trying to do, and in consequence, is closed to the possibility that these texts contribute to logical studies of a different kind. And second, in supposing that arguments have to be evaluated formally, the important idea that there are *informal* criteria for argument evaluation is neglected. In fact, the *Kathāvatthu* offers a particularly clear example of a text whose richness and interest lies elsewhere than in its anticipation of deductive principles and propositional laws. As a meticulous analysis of the argumentation properly to be used in the course of a dialogue of a specific type, its concern is with the pragmatic account of argument evaluation, the idea that

[5] Aung, S.Z., *Points of Controversy, or, Subjects of Discourse: Being a Translation of the Kathāvatthu from the Abhidhammapiṭaka*, eds. S.Z. Aung and C.A.F. Rhys Davids. Pali Text Society, translation series no.5. London: Luzac & Co. 1915; reprint 1960; Schayer, St., "Altindische Antizipationen der Aussagenlogik", *Bulletin international de l'Academie Polonaise des Sciences et des Lettres*, classe de philologies: 90 96 (1933), translated in Jonardon Ganeri ed., *Indian Logic: A Reader* (London: Curzon, 2001); Bochenski, J. M., "The Indian Variety of Logic", in his *A History of Formal Logic.* Freiburg. Trans. I. Thomas, Notre Dame: University of Notre Dame Press (1961), pp. 416 447., reprinted in Jonardon Ganeri ed., *Indian Logic: A Reader*; Matilal, Bimal Krishna, *The Character of Logic in India.* Albany: State University of New York Press, 1998.

arguments have to be evaluated as good or bad with regard to their contribution towards the goals of the dialogue within which they are embedded. The leading concern of the *Kathāvatthu* is with issues of balance and fairness in the conduct of a dialogue, and it recommends a strategy of argumentation which guarrantees that both parties to a point of controversy have their arguments properly weighed and considered. It is important, in the normative framework of the *Kathāvatthu*, that there is a distinction between the global aim of the dialogue as a whole — here to rehearse in an even-handed manner all the considerations that bear upon an issue of dispute, to clarify what is at stake even if no final resolution is achieved - and the local aim of each participant — to advocate the stance they adopt with regard to that issue by supplying arguments for it and attacking the arguments of the other parties.

A dialogue conducted in accordance with the prescribed method of the *Kathāvatthu* is called a *vādayutti*. The goal of a *vādayutti* is the reasoned examination (*yutti*; Skt. *yukti*) of a controversial point in and through a noneristic dialogue (*vāda*). The dialogue is highly structured, and is to be conducted in accordance with a prescribed format of argumentation. There is a given point at issue, for example, whether 'a person is known in the sense of a real and ultimate fact' (i.e. whether persons are conceived of as metaphysically irreducible), whether there are such things as morally good and bad actions, and so, in general, whether A is B. A dialogue is now divided into eight sub-dialogies or 'openings' (*aṭṭhamukha*). These correspond to eight attitudes it is possible to adopt with regard to the point at issue. So we have:

[1] Is A B?

[2] Is A not B?

[3] Is A B everywhere?

[4] Is A B always?

[5] Is A B in everything?

[6] Is A not B everywhere?

[7] Is A not B always?

[8] Is A not B in everything?

The introduction of an explicit quantification over times, places and objects serves to determine an *attitude* of proponent and respondent to the point of controversy. If the issue in question is, for example, whether lying is immoral, the clarification would be as to whether that proposition is to be maintained or denied, and in either case, whether absolutely, or only as relativised in some way to circumstances, times or agents. So an opening thesis here is by definition a point at issue together with an attitude towards it.

Each such'opening' now proceeds as an independent dialogue, and each is divided into five stages: the way forward (*anuloma*), the way back (*paṭikamma*), the refutation (*niggaha*), the application (*upanayana*), and the conclusion (*niggamana*). In the way forward, the proponent solicits from the respondent their endorsement of a thesis, and then tries to argue against it. In the way back, the respondent turns the tables, soliciting from the proponent their endorsement of the counter-thesis, and then trying argue against it. In the refutation, the respondent, continuing, seeks to refutes the argument that the proponent had advanced against the thesis. The application and conclusion repeat and reaffirm that the proponent's argument against the respondent's thesis is unsound, while the respondent's argument against the proponent's counter-thesis is sound.

It is significant to note that there is here no *pro-argumentation*, either by the respondent for the thesis or by the proponent for the counter-thesis. There is only *contra-argumentation*, and that in two varieties. The respondent, in the 'way back', supplies an argument against the proponent's counter-thesis, and in the refutation stage, against the proponent's alleged argument against the thesis. So we see here a sharp distinction between three types of argumentation — *pro argumention*, argumentation that adduces reasons in support of one's thesis, *counter* argumenation — argumentation that adduces reasons against the counter-thesis, and *defensive* argumentation, argumentation that defends against counter-arguments directed against one's thesis. The respondent, having been 'attacked' in the first phase, 'counter-attacks' in the second phase, 'defends' against the initial attack in the third, and 'consolidates' the counter-attack and the defence in the fourth and fifth. The whole pattern of argumentation, it would seem, is best thought of as *presumptive*, that is, as an attempt to switch a burden of proof that is initially even distributed between the two parties. The respondent tries to put the burden of proof firmly onto the proponent, by arguing against the proponent while countering any argument against herself. The fact that the respondent does not offer any pro argumentation in direct support of the thesis means that the whole pattern of argumentation is technically *ab ignorantium*; that is, argumentation of the form "I am right because not proved wrong". But ab *ignorantium* reasoning is not always fallacious; indeed, it is often of critical importance in swinging the argument in one's favour in the course of a dialogue (see §1.5).

In the first stage, the way forward, the proponent elicits from the respondent an endorsement of a thesis, and then sets out to reason against it. Not any form of reasoning is allowed; indeed the *Kathāvatthu* prescribes a very specific method of counter-argumentation. Thus:

I. The Way Forward

Theravādin: Is the soul (*puggala*) known as a real and ultimate fact?
[1] Puggalavādin : Yes.
Theravādin: Is the soul known in the same way as a real and ultimate fact is known?
[2] Puggalavādin: No, that cannot be truly said.

Theravādin: Acknowledge your refutation (*niggaha*):
[3] If the soul be known as a real and ultimate fact, then indeed, good sir, you should also say, the soul is known in the same way as any other real and ultimate is known.
[4] That which you say here is false, namely, that we should say, "the soul is known as a real and ultimate fact", but we should not say, "the soul is known in the same way as any other real and ultimate fact is known."
[5] If the later statement cannot be admitted, then indeed the former statement should not be admitted either.
[6] In affirming the former, while denying the latter, you are wrong.

The respondent, here a *puggalavādin* or believer in the existence of personal souls, is asked to endorse the thesis. The proponent then attempts to draw out an implication of that thesis, an implication more over to which the *puggalavādin* will not be willing to give his consent. Here the thesis that persons are thought of as metaphysically irreducible elements of the world is held to imply that knowledge of persons is knowledge of the same kind as that of other types of thing. The *puggalavādin*, will perhaps want to draw an epistemological distinction between empirical knowledge of external objects and self-knowledge, and so will not endorse this derived proposition. And now the proponent, in a fresh wave of argumentation, demonstrates that it is inconsistent for the *puggalavādin* to endorse the thesis but not the derived consequence. So a counter-argument has three components: the initial thesis or *thapanā* (Skt. *sthāpanā*), the derived implication or *pāpanā*, and the demonstration of inconsistency or *ropanā*.

It is in the *ropanā* that there seems to be an 'anticipation' of propositional logic. Of the four steps of the *ropanā*, the first, from [3] to [4], looks like an application of the definition of material implication or its term-logical equivalent:

$$(A \text{ is } B \rightarrow (C \text{ is } D) =_{\text{defn}} \neg((A \text{ is } B)\&\neg(C \text{ is } D)).$$

Notice here that an effect of soliciting from the respondent a 'no' in answer to the proponent's second question is that the negation is external and not internal. Thus, we have '$\neg(C \text{ is } D)$' rather than '$(C \text{ is } \neg D)$'. This what one needs in the correct definition of material implication.

The second step, from [4] to [5], looks like a derivation of the contraposed version of the conditional, a derivation that depends on the stated definition of the conditional. From that definition, and assuming that '&' is commutative, it follows that

$$(A \text{ is } B) \rightarrow (C \text{ is } D) \text{ iff } \neg(C \text{ is } D) \rightarrow \neg(A \text{ is } B).$$

The final step now is an application of *modus ponens*. So what we have is:

[1] $(A \text{ is } B)$ premise

[2] $\neg(C \text{ is } D)$ premise

[3] $(A \text{ is } B) \to (C \text{ is } D)$ additional premise?

[4] $\neg((A \text{ is } B) \& \neg(C \text{ is } D))$ 3, defn. of $\to$

[5] $\neg(C \text{ is } D) \to \neg(A \text{ is } B)$ 4, defn. of $\to$

[6] $\neg(A \text{ is } B)$ 2, 5, MP

This is how Matilal[6] reconstructs the *ropanā* stage of argumentation. Earlier, Bochenski[7] recommended a variant in which steps [3] and [4] "together constitute a kind of law of contraposition or rather a *modus tollendo tollens* in a term-logical version". Still another alternative is to see step [3] as a piece of enthymematic reasoning from the premise already given, rather than as the introduction of an additional premise. In other words, the 'if...then' in [3] is to be understood to signify the logical consequence relation rather than material implication. Then step [4] negates the premise in an application of reductio *ad absurdum.* That is:

[1,2] $(A \text{ is } B) \& \neg(C \text{ is } D)$ premise

[3] $(C \text{ is } D)$ $1 + 2$, enthymematic derivation

[4] $\neg((A \text{ is } B) \& \neg(C \text{ is } D))$ $1 + 2, 3$; RAA

[5] $\neg(C \text{ is } D) \to \neg(A \text{ is } B)$ 4, defn. of $\to$

[6] $\neg((A \text{ is } B) \& \neg(C \text{ is } D))$ 5, defn. of $\to$

This reconstuction seems more in keeping with the overall pattern of argumentation — to take the respondent's thesis and derive from it consequences the respondent will not endorse, and thereby to argue against the thesis (and it preserves the repetition of the original). Here again we see that the form of argumentation in the *Kathāvatthu* is better understood if we bear in mind the function it is intended to serve within a dialogue context.

The same dialogue context is normative, in the sense that it gives the grounds for evaluating any actual instance of such argumentation as good or bad. It seems possible to understand the 'way forward' in terms of certain concepts from the theory of argumenation. Hamblin introduced the idea that each participant in a dialogue has a 'commitment store', a set of propositions to which they commit themselves in the course of the dialogue, primarily by asserting them directly.[8] In Hamblin's model, the commitments of each party are on public display, known to every participant in the dialogue. In order to represent the fact that this is very often *not* the case, Walton[9] employs a distinction between open or 'light-side' commitments, and veiled or 'dark-side' commitments. The veiled commitments of

[6] Matilal (1998: 33–37)
[7] Bochenski (1961: 423)
[8] Hamblin, C. L., *Fallacies.* London: Methuen, 1970.
[9] Walton (1998: 50–51).

a participant are not on public view, and might not be known even to that participant themselves: but perhaps the participant trades on them in making certain kinds of dialogue move. Indeed, it is part of what Walton[10] calls the 'maieutic' role of dialogue to make explicit the veiled commitments of the participants, a process of clarification that is valuable even if it does not lead to the issue at stake being decided in favour of one party or the other.[11]

Something of this sort is what is being described in the initial stages of the 'way forward'. Steps [1] and [2] elicit from the respondent an explicit and open commitment to the propositions 'A is B' and '$\neg$ (C is D)'. ¿From the respective assertion and denial, these become parts of her explicit commitment store. But next, though the enthymematic argumentation that constitutes the *pāpanā* or stage [3], it is made clear that the respondent has a veiled commitment to the proposition 'C is D'. For this is shown to follow from propositions in the explicit commitment store of the respondent. Finally, the *ropanā* stage of reasoning reveals this newly explosed commitment to be inconsistent with the respondent's other explicit commitments. The overall effect is to force the respondent into a position where she must retract at least one of the propositions to which she has committed herself. Indeed, we can say that such a retraction is the primary goal of the way forward. The primary aim is not to *disprove* the thesis, but to force a retraction of commitment. So when we evaluate the argumentation used in this part of the dialogue, it is to be evaluated as good or bad with reference to how well it succeeds in forcing such a retraction, and not simply or only or even in terms of its deductive or inductive soundness. The strategic problem here is how to persuade the respondent to accept some proposition that is meant ultimately to be used to force a retraction, and the type of strategy being recommended is the one Walton calls that of "separating", where "two or more propositions are proved separately and then eventually put together in an argument structure that is used to prove one's own thesis or argue against an opponent's".[12] In setting out the reasoning in this way, the intention of the author of the *Kathāvatthu* is not to imply that precisely this sequence of arguments is sound. What is being shown is the *form* that any counter-argument should take. It is a description, in generic terms, of the strategic resources open to the proponent, and serves rather as a blue-print for any actual *vādayutti* dialogue.

At this point in the sub-dialogue that is the first opening, then, the burden of proof seems to lie squarely with the respondent, the *puggalavādin*, who is being pressured into the uncomfortable position of having to retract his stated thesis. The remaining four phases of the first opening are a summary of the strategic resources open to the respondent to recover his position, and indeed to turn the tables against the proponent. First, the way back. This is a phase of counter-

[10]Walton (1998: 58).

[11]The term 'maieutic', from *maieutikos* 'skill in midwifery, is taken from the *Theaetetus*, where Socrates describes himself as a midwife for beautiful boys - helping them to give birth to whatever ideas are in them, and test them for whether they are sound.

[12]Walton (1998: 44).

attack, in which the respondent uses parallel reasoning to force the proponent too into a position of retraction with regard to the counter-thesis.

II. The Way Back

Puggalavādin: Is the soul not known as a real and ultimate fact?
[1] Theravādin: No, it is not known.
Puggalavādin: Is it not known in the same way as any real and ultimate fact is known?
[2] Theravādin: No, that cannot be truly said.
Puggalavādin: Acknowledge the rejoinder (*paṭikamma*):
[3] If the soul is not known as a real and ultimate fact, then indeed, good sir, you should also say: it is not known in the same way as any other real and ultimate fact is known.
[4] That which you say is false, namely, that we should say "the soul is not known as a real and ultimate fact", but we should not say "it is not known in the same way as any other real and ultimate fact is known".
[5] If the latter statement cannot be admitted, then indeed the former statement should not be admitted either.
[6] In affirming the former while denying the latter, you are wrong.

At the end of the 'way back', if the respondent's arguments have gone well, the proponent has been pressed in the direction of retracting his commitment to the counter-thesis. If the respondent were to leave matters here, however, he would have failed in the global aim of the 'opening'. The aim of the opening is to shift the burden of proof decisively onto the proponent. After the second stage in the opening, however, the burden of proof is again symmetrically distributed among the parties to the dialogue — both are in a position of being pressed to retract their respective commitment. So, in the third phase, the respondent seeks, in a defensive move, to diffuse the argument of the proponent that is forcing this retraction. Again, the cited reasoning is schematic, it indicates a general strategy the details of which must be worked out differently in each specific case. The distinction being drawn is the one between *counter-argument*, and *defensive repost*, a distinction that makes sense only within the normative framework of a dialogical exchange.

The first opening in the *vādayutti* has rehearsed the best argumentation that available against someone whose attitude towards the point at issue is one of unqualified affirmation. Remember, however the global aim of a *vādayutti* — to be the form of dialogue most conducive to a balanced examination of the best arguments, both for and against. It is the function now of the second opening to rehearse the best argumentation against someone whose attitude towards the point at issue is one of unqualified denial, and of the subsequent openings to do likewise with respect to attitudes of qualified affirmation and denial. Even at the end of the dialogue, there may be no final resolution, but an important maieutic function has been served — the clarification of the commitments entailed by each position, of their best strategies and forms of argumentation. So, indeed, it is

as a rich account of presumptive reasoning in dialogue, and not so much for its 'anticipations' of formal logic, that the *Kathāvatthu* makes a rewarding object of study.

1.3 Case-based reasoning, extrapolation and inference from sampling: The Nyāyasūtra

It was Henry Colebrooke[13] who first brought Indian logic to the attention of the English philosophical world, announcing in a famous lecture to the Royal Asiatic Society in 1824 his discovery of what he called the 'Hindu Syllogism'. Colebrooke's 'discovery' consisted in fact in a translation of an ancient Indian treatise called the *Nyāyasūtra.* It dates from around the 1^{st} or 2^{nd} century AD, and is said to be the work of Gautama Akṣapāda. Scholars are now inclined to regard it as the amalgamation of two earlier works on philosophical method, one a treatise on the rules and principles of debate, the other a discussion of more general issues in epistemology and metaphysics. In a section on the proper way for a debater to set out their argument, the *Nyāyasūtra* prescribes a five-step schema for well-formed argument, and it is this schema that Colebrooke identified as the Indian syllogism. We now know much much more than Colebrooke about the historical development of Indian logic. He, for instance, was unaware of the informal logic and anticipations of propositional calculus in the *Kathāvatthu* (§1.2), or the theories of the Buddhists Vasubandhu, Diṅnāga and Dharmakīrti on formal criteria for inference (§§2.1–5). And scholars had yet to learn the complexities of the later logical school of Navya-Nyāya (§§4.1–3), with its intriguing treatment of negation, logical consequence and quantification, and even, as Daniel Ingalls has shown in his pioneering book entitled *Materials for the Study of Navya-Nyāya Logic*, the formulation of De Morgan's Laws.[14] Nevertheless, in spite of Colebrooke's lack of acquaintance with the historical context, he and those who followed him were right to see the *Nyāyasūtra* as a treatise of fundamental importance in Indian logical thinking, and I would like to pick up and continue the thread of their discussion. I want to argue that the *Nyāyasutra* begins a transformation in Indian thinking about logic. And this in two inter-related respects: in the beginnings of a shift of interest away from the place of *argumentation* within dialectic and debate and towards a greater concern with the more formal properties of sound *inference*; and in a parallel and correlated shift from *case-based* to *rule-governed* accounts of logical reasoning. I will discuss each of these in turn.

In the *Nyāyasutra*, there is a more systematic discussion of the categories and methods of debate than in earlier debating manuals. Three kinds of debate are

[13] H. T. Colebrooke, "On the Philosophy of the Hindus: Part II - On the Nyāya and Vaiśeshika systems". *Transactions of the Royal Asiatic Society (1824)*, 1: 92–118; reprinted in Jonardon Ganeri ed., *Indian Logic: A Reader*.

[14] D. H. H. Ingalls, *Materials for the Study of Navya-Nyāya Logic* (Cambridge Mass.: Harvard University Press), 1951, pp. 65–67.

distinguished: good or honest debate (*vāda*), tricky or bad debate (*jalpa*) and a refutation-only debate (*vitaṇḍā*):

> Good debate (*vāda*) is one in which there is proof and refutation of thesis and antithesis based on proper evidence (*pramāṇa*) and presumptive argumentation (*tarka*), employing the five-step schema of argumentation, and without contradicting any background or assumed knowledge (*siddhānta*).
>
> Tricky debate (*jalpa*) is one in which, among the features mentioned before, proof and refutation exploit such means as quibbling (*chala*), false rejoinders (*jāti*), and any kind of clincher or defeat situation (*nigrahasthāna*).
>
> Refutation-only debate (*vitaṇḍā*) is one in which no counter-thesis is proven. (NS 1.2.1–3).

Here is our first reference to the Indian five-step inference pattern. It is a schema for proper argumentation among disputants who are engaged in an honest, friendly, noneristic, and balanced debate (*vāda*). In the dialectical context in which such arguments are embedded, a proponent attempts to prove a thesis and to refute the counter-thesis of the opponent, both parties drawing upon a shared body of background knowledge and received belief, and using properly accredited methods for the acquisition and consideration of evidence. The aim of each participant in the dialogue is not victory but a fair assessment of the best arguments for and against the thesis. In Indian logic, *vāda* represents an *ideal* of fair-minded and respectful discourse. By contrast, in a tricky debate (*jalpa*), underhanded debating tactics are allowed, and the aim is to win at all costs and by any means necessary. The third kind of debate, the refutation-only debate (*vitaṇḍā*), is the variety of dialogue preferred by the sceptics — to argue against a thesis without commitment to any counter-thesis. It is not entirely clear whether this is a type of good or tricky debate. We might conjecture, however, that if dialectic is a rough kin of *vāda*, and sophistic of *jalpa*, then the Socratic elenchus could be regarded as a species of *vitaṇḍā*, which is not, therefore, an entirely disreputable method of debate.

The aim, in a good debate between friends, is the assessment of the best arguments for or against the thesis. And that leads to the question: how are arguments to be assessed or evaluated? Are the criteria for assessment formal, to do only with the form of the argument schema itself; or are they informal, pragmatic criteria, according to which arguments have to be evaluated as good or bad with regard to their contribution towards the goals of the dialogue within which they are embedded?

With this question in mind, let us look at the five-step proof pattern. The proper formulation of an argument is said to be in five parts: tentative statement of the thesis to be proved (*pratijñā*); citation of a reason (*hetu*); mention of an example (*udāharaṇa*); application of reason and example to the case in hand (*upanaya*);

final assertion of the thesis (*nigamana*). An unseen fire is inferred to be present on the mountain, on the basis of a plume of smoke; just as the two have been found associated in other places like the kitchen. The terms used here are defined in a series of admittedly rather gnomic utterances (NS 1.1.34–39):

1.1.32 'the parts [of an argument scheme] are thesis, reason, example, application and conclusion' (*pratijñāhetudāharaṇopanayanigamanānyavayavāḥ*).

1.1.33 'the thesis is a statement of that which is to be proved' (*sādhyanirdeśaḥpratijñā*).

1.1.34 'the reason is that which proves what is to be proven in virtue of a similarity with the example' (*udāharaṇasādharmyāt sādhyasādhanaṃ hetuḥ*).

1.1.35 'again, in virtue of a dissimilarity' (*tathā vaidharmyāt*).

1.1.36 'the example is an illustration which, being similar to that which is to be proved, has its character' (*sādhyasādharmyāt taddharmabhāvī dṛṣṭanta udāharaṇam*).

1.1.37 'or else, being opposite to it, is contrary' (*tadviparyayād vā viparītam*).

1.1.38 'the application to that which is to be proved is a drawing in together (*upasaṃhāra*) "this is so" or "this is not so," depending on the example' (*udāharaṇāpekṣas tathety upasaṃharo na tatheti vā sādhyasyopanayaḥ*).

1.1.39 'the conclusion is a restatement of the thesis as following from the statement of the reason' (*hetvapadeśāt pratijñāyāḥ punarvacanaṃ nigamanam*).

The basic idea is that an object is inferred to have one (unobserved) property on the grounds that it has another, observed, one — "there is fire on the mountain because there is smoke there". The most distinctive aspect of the schema, though, is the fundamental importance given to the citation of an example, a single case said either to be similar or else dissimilar to the case in hand. Suppose I want to persuade you that it is about to rain. I might reason as follows: "Look, it is going to rain (thesis). For see that large black cloud (reason). Last time you saw a large black cloud like that one (example), what happened? Well, its the same now (application). It is definitely going to rain (conclusion)."

Let us try to unpick the *Nyāyasutra* definitions. Suppose we let 'F' denote the property that serves as the reason here (*hetu*), 'G' the property whose presence we are seeking to infer (*sādhya*), 'a' the new object about which we are trying to decide if it is G or not (*pakṣa*), and 'b' the cited example (*udaharaṇa*). Then we seem to have a pair of schematic inferences, one based on similarity, the other on dissimilarity:

Five-step proof based on similarity

[thesis] Ga

[reason] Fa proves Ga, because b is similar to a.

[example] b has the 'character of a' because it is similar to a.

[application] a is the same as b with respect to G.

[conclusion] Ga

Five-step proof based on dissimilarity

[thesis] Ga

[reason] Fa proves Ga, because b is dissimilar to a.

[example] b does not have the 'character of a' because it is dissimilar to a.

[application] a is not the same as b with respect to G.

[conclusion] Ga

The counter-proof follows the same pattern, proving the counter-thesis ($\neg Ga$) by means of a different reason and example:

Counter-proof

[thesis] $\neg Ga$

[reason] $F'a$ proves Ga, because b is similar to a.

[example] c has the 'character of a' because it is similar to a.

[application] a is the same as c with respect to G.

[conclusion] $\neg Ga$

The five-step schema was interpreted in a particular way by Vātsyāyana, the first commentator on the *Nyāyasūtra*. His interpretation is largely responsible for shaping the direction Indian logic was later to take. At the same time, it was an interpretation that made the citation of an example essentially otiose. Vātsyāyana was, in effect, to transform Indian logic, away from what it had been earlier, namely a theory of inference from case to case on the basis of resemblance, and into a rule-governed account in which the citation of cases has no significant role.

Let us then consider first Vātsyāyana's interpretation. What Vātsyāyana says is that the similarity between a and b just consists in their sharing the reason property F. The basic pattern of inference is now: a is like b [both are F]; $Gb \therefore Ga$. *Or else*: a is unlike b [one is F and the other isn't]; $\neg Gb \therefore Ga$. Writing it out as before, what we have is:

Proof based on similarity

[thesis]	Ga	
[reason]	Fa	
[example]	Fb	b is similar to a [both are F].
[application]	Gb	
[conclusion]	Ga	

In a counterproof, a is demonstrated to be similar in some other respect to some other example, one that lacks the property G:

Counterproof

[thesis]	$\neg Ga$	
[reason]	$F'a$	
[example]	F'c	c is similar to a [both are F'].
[application]	$\neg Gc$	
[conclusion]	$\neg Ga$	

Thus, for example, a proof might be: the soul is eternal because it is uncreated, like space. And the counterproof might be: the soul is non-eternal because it is perceptible, like a pot.

The proposal is that if a resembles b, and b is G, then a can be inferred to be G too. But there is an obvious difficulty, which is that mere resemblance is an insufficient ground. Admittedly, the soul and space are both uncreated, but why should that give us any grounds for transferring the property of being eternal from one to the other? The *respect* in which the example and the case in hand resemble one another must be relevant to the property whose presence is being inferred. This is where the idea of a 'false proof' or 'false rejoinder' (*jāti*) comes in. Any argument that, while in the form of the five-step schema, fails this relevance requirement is called a 'false proof' and the *Nyāyasūtra* has a whole chapter (chapter 5) classifying and discussing them. A 'false rejoinder' is defined in this way:

> NS 1.2.18 'a *jāti* is an objection by means of similarity and dissimilarity' (*sādharymavaidharmyābhāṃ pratyavasthānaṃ jātiḥ*).

It appears to be admissible to transfer the property 'rain-maker' from one black cloud to another black cloud, but not from a black cloud to a white cloud. It appears to be admissible to transfer the property 'has a dewlap' from one cow to another cow, but not from one four-legged animal (a cow) to another (a horse). It

is clear what now needs to be said. The argument is good if there exists a *general* relationship between the reason F and the property being proved G, such that the latter never occurs without the former.

It is the Buddhist logician Diṅnāga (480–540 CE) who seems to have been the first to make this explicit (see also §2.2). According to him, a reason must satisfy three conditions. Define a 'homologue' (*sapakṣa*) as an object other than a that possesses G, and a 'heterologue' (*vipakṣa*) as an object other than a that does not possess G. Then Diṅnāga's three conditions on a good reason are:

[1] F occurs in a.

[2] F occurs in some homologue.

[3] F occurs in no heterologue.

Condition [3] asserts, in effect, that F never occurs without G, and this, together with [1] that F occurs in a, implies of course that G occurs in a. In effect, the citation of an example in the original *Nyāyasūtra* formula has been transformed into a statement of a general relationship between F and G. There remains only a vestigial role for the example in condition [2], which seems to insist that there be an instance of F other than a which is also G. Diṅnāga is worried about the soundness of inferences based on a reason which is a property unique to the object in hand; for example, the inference "sound is eternal because it is audible". For if this is sound, then why not the counter-argument "sound is non-eternal because it is audible"? And yet there are many inferences like this that are sound, so it seems to be a mistake to exclude them all. In fact condition [2] soon came to be rephrased in a way that made it logically equivalent to [3], namely as saying that F occur only in homologues (*eva* 'only' is used here as a quantifier). In asking for the respect in which the example and the new case must resemble each other, for the presence of G in the example to be a reason for inferring that G is present in the new case, we are led to give the general relationship that any such respect must bear to G, and that in turn makes citation of an example otiose. The five-step schema becomes:

[thesis] Ga

[reason] because F

[example] where there is F, there is G; for example, b.

[application] Fa

[conclusion] Ga

It is the five-step argument pattern so transformed that has suggested to Colebrooke and other writers on Indian logic a comparison with an Aristotelian syllogism in the first figure, *Barbara*. We simply re-write it in this form:

All F are G.
Fa.
Therefore, Ga.

This assimilation seems forced in at least two respects. First, the conclusion of the *Nyāyasūtra* demonstration is a singular proposition. In Aristotle's system, on the other hand, it is always either a universal proposition with 'all' or 'no', or a particular proposition with 'some'. Second, and relatedly, the role of the 'minor term' is quite different: in the Indian schema, it indicates a locus for property-possession, while in Aristotle, the relation is 'belongs to'. Again, in reducing the Indian pattern to an Aristotelian syllogism, the role of the example, admittedly by now rather vestigial, is made to disappear altogether.

A rather better reformulation of the five-step schema is suggested by Stanisław Schayer,[15] who wants to see the Indian 'syllogism' as really a proof exploiting two rules of inference:

[thesis]	Ga	There is fire on a (= on this mountain).
[reason]	Fa	There is smoke on a.
['example']	$(x)(Fx \to Gx)$	For every locus x: if there is smoke in x then there is fire in x.
[application]	$Fa \to Ga$	This rule also applies for $x = a$.
[conclusion]	Ga	Because the rule applies to $x = a$ and the statement Ga is true, the statement Fa is true.

Two inference rules are in play here — a rule of substitution, allowing us to infer from '$(x)\zeta x$' to 'ζa', and a rule of separation, allowing us to infer from '$\theta \to \varphi$' and 'θ' to 'φ'. Schayer thereby identifies the Indian syllogism with a proof in a natural deduction system:

THESIS. Ga because Fa.

Proof.

1	(1)	Fa	Premise
2	(2)	$(x)(Fx \to Gx)$	Premise
3	(3)	$Fa \to Ga$	2, by $\forall$ Elimination
1,2	(4)	Ga	1 & 3, by $\to$ Elimination. ■

We have seen how the *Nyāyasūtra* model of good argumentation came to be transformed and developed by later writers in the Indian tradition in the direction of a formal, rule-governed theory of inference, and how writers in the West have interpreted what they have called the Indian 'syllogism'. I suggested at the beginning that we might try to interpret the Nyāya model along different lines altogether, seeing it an early atttempt at what is now called 'case-based reasoning'. Case-based reasoning begins with one or more prototypical exemplars of a

[15]Schayer, St., "Altindische Antizipationen der Aussagenlogik", *Bulletin international de l'Academie Polonaise des Sciences et des Lettres*, classe de philologies: 90 96 (1933); translated in Jonardon Ganeri ed., *Indian Logic: A Reader*.

category, and reasons that some new object belongs to the same category on the grounds that it resembles in some appropriate and context determined manner one of the exemplars. Models of case-based reasoning have been put forward for medical diagnostics and legal reasoning, and some have been implemented in artificial intelligence models. It has been shown, for example, that training a robot to solve problems by having it retrieve solutions to stored past cases, modifying them to fit the new circumstances, can have great efficiency gains over seeking solutions through the application of first principles. Perhaps something like this underlies a lot of the way we actually reason, and perhaps it was as an attempt to capture *this* type of reasoning that we should see the ancient logic of the *Nyāyasūtra* and indeed of the medical theorist Caraka.[16] In this model, a perceived association between symptoms in one case provides a reason for supposing there to be an analogous association in other, resembling cases. The physician observing a patient A who has, for example, eaten a certain kind of poisonous mushroom, sees a number of associated symptoms displayed, among them F and G, say. He or she now encounters a second patient B displaying a symptom at least superficially resembling F. The physician thinks back over her past case histories in search of cases with similar symptoms. She now seeks to establish if any of those past cases resembles B, and on inquiry into B's medical history, discovers that B too has consumed the same kind of poisonous mushroom. These are her grounds for inferring that B too will develop the symptom G, a symptom that had been found to be associated with F in A. A common etiology in the two cases leads to a common underlying disorder, one that manifests itsself in and explains associations between members of a symptom-cluster.

Can we find such a model of the informal logic of case-based reasoning in the *Nyāyasūtra*? Consider again NS 1.1.34. It said that 'the reason is that which proves what is to be proved in virtue of a similarity with the example.' On our reading, what this says is that a similarity between the symptom F in the new case and a resembling symptom F' in the past-case or example is what grounds the inference. And NS 1.1.36 says that 'the example is something which, being similar to that which is to be proved, has its character'. Our reading is that the old case and the new share something in their circumstances, like having eaten the same kind of poisonous mushroom, in virtue of which they share a 'character', an underlying disorder that expains the clustering of symptoms. So the five-step demonstration is now:

[16] *Caraka-Saṃhitā* 3.8.34: 'what is called "example" is that in which both the ignorant and the wise think the same and that which explicates what is to be explicated. As for instance, "fire is hot," "water is wet," "earth is hard," "the sun illuminates." Just as the sun illuminates, so knowledge of sāṃkhya philosophy illuminates'.

[thesis]	Ga	
[reason]	Fa	F is similar to F' in b.
[example]	b	exhibits the same underlying structure as a, because it resembles a.
[application]		a is the same as b with respect to G.
[conclusion]	Ga	

Let us see if this pattern fits examples of good inference taken from a variety of early Indian logical texts. One pattern of inference, cited in the *Nyāyasūtra*, is causal-predictive: Seeing the ants carrying their eggs, one infers that it will rain; seeing a full and swiftly flowing river, one infers that it has been raining; seeing a cloud of smoke, one infers the existence of an unseen fire. Presumably the idea is that one has seen other ants carrying their eggs on a past occasion, and on that occasion it rained. The inference works if, or to the exent that, we have reasons for thinking that those ants and these share some unkown capacity, a capacity that links detection of the imment arrival of rain with the activity of moving their eggs. The pattern is similar in another kind of inference, inference from sampling: Inferring from the salty taste of one drop of sea water that the whole sea is salty; inferring that all the rice is cooked on tasting one grain. The assumption again is that both the sampled grain of rice and any new grain share some common underlying structure, a structure in virtue of which they exhibit the sydromes associated with being cooked, and a structure whose presence in both is indicated by their being in the same pan, heated for the same amount of time, and so forth.

I will make two final comments about these patterns of case-based reasoning. First, it is clear that background knowledge is essentially involved. As the *Nyāyasutra* stresses in its definition of a good debate, both parties in a debate much be able to draw upon a commonly accepted body of information. Such knowledge gets implicated in judgements about which similarities are indicative of common underlying disorders, and which are not. Second, in such reasoning the example does not seem to be redundant or eliminable in favour of a general rule. For although there always will be a general law relating the underlying disorder with its cluster of symptoms, the whole point of this pattern of reasoning is that the reasoner need not be in a position to *know* what the underlying disorder is, and so what form the general law takes. In conclusion, while the history of logic in India shows a strong tendency towards formalisation, the logic of ancient India tried to model informal patterns of reasoning from cases that are increasingly becoming recognised as widespread and representative of the way much actual reasoning takes place.

1.4 *Refutation-only dialogue: vitaṇḍā*

We have already seen how 'refutation-only' debate is defined in the *Nyāyasutra*:

> Refutation-only debate (*vitaṇḍā*) is one in which no counter-thesis is proven. (NS 1.2.1–3).

For the Naiyāyika, to argue thus is to argue in a purely negative and destructive way. Here one has no goal other than to undermine one's opponent. People who use reason in this way are very like the sceptics and unbelievers of the epics. Vātsyāyana claims indeed that to use reason in this way is virtually self-defeating:

> A *vaitāṇḍika* is one who employs destructive criticism. If when questioned about the purpose [of so doing], he says 'this is my thesis' or 'this is my conclusion,' he surrenders his status as a *vaitāṇḍika*. If he says that he has a purpose, to make known the defects of the opponent, this too will is the same. For if he says that there is one who makes things known or one who knows, or that there is a thing by which things are made known or a thing made known, then he surrenders his status as a *vaitāṇḍika*.[17]

Vitaṇḍā is the sceptic's use of argumentation, and it is a familiar move to attempt to argue that scepticism is self-defeating. In India, it is the Mādhyamika Buddhist Nāgārjuna (circa first century CE) who is most closely associated with the theoretical elaboration of refutation-only argumentation, through the method of 'four-limbed refutation' (*catuṣkoṭi*) and the allied technique of presumptive reasoning (*prasaṅga; tarka*). In the next section, I will show how this latter technique became a device for shifting the burden of proof onto one's opponent. First, I will examine the method of 'four-limbed' refutation in the context of Nāgārjuna's wider philosophical project.

Reasoning, for Nāgārjuna, is the means by which one 'steps back' from common sense ways of understanding to a more objective view of the world. Reason is a mode of critical evaluation of one's conceptual scheme. A more objective understanding is one in which one understands that things are not necessarily as they appear. It is a view from which one can see how and where one's earlier conceptions are misleading. One learns not to trust one's perceptions when a large object far away looks small, or a stick half submerged in water looks bent, and in learning this one exercises a mode of self-critical reason. So too a rational person learns not to trust their conceptions when they presuppose the existence of independent, self-standing objects. From the vantage point of an objective view, it is easy to see that one's old conceptions had false presuppositions. The real trick, however, is to be able to expose those presuppositions while still 'within' the old conception, and so to lever oneself up to a more objective view. This levering–up from–within requires a new way of reasoning: Nāgārjuna's celebrated *prasaṅga-type* rationality. It is a self-critical rationality which exposes as false the existential presuppositions on which one's present conceptions are based. The same method can equally well be used to expose the false presuppositions on which one's dialectical opponents' views are based, and for this reason the whole technique is strongly maieutic, in the sense defined earlier.

A simple example will illustrate the kind of reasoning Nāgārjuna thinks is needed if one is to expose the presuppositions of one's conceptual scheme from within. A

[17] *Nyāyabhāṣya* 3, 15 20.

non-compound monadic concept 'F' has the following application-condition: it applies only to things which are F. It is therefore a concept whose application presupposes that there is a condition which divides the domain into two. For our purposes, the condition can be thought of either as 'belonging to the class of Fs' or 'possessing the property being-F'. Now take an arbitrary object, a, from some antecedently specified domain. There are apparently two possibilities for a: either it falls under the concept, or else it is not. That is, the two options are:

(I) F applies to a

(II) F does not apply to a.

Suppose that one can disprove *both* of these options. How one would try to do this will vary from case to case depending on the individual concept under scrutiny. But if one is able to disprove (I) and to disprove (II), then the concept in question can have no application-condition. The presupposition for the application of the concept, that there is a condition (class, property) effecting a division within the domain, fails. A later Mādhyamika master[18] expresses the idea exactly:

> When neither existence nor nonexistence presents itself before the mind, then, being without objective support (*nirālambana*) because there is no other way, [the mind] is still.

Sentences are used to make statements, but if the statement so made is neither true nor false, then, because there is no third truth-value, the statement must be judged to lack content.[19]

Nāgārjuna's developed strategy involves a generalization. A generalization is needed because many if not most of the concepts under scrutiny are *relational* rather than *monadic*; centrally: causes, sees, moves, desires. When a concept is relational, there are four rather than two ways for its application-condition to be satisfied (see Figure 1, page 332):

(I) R relates a only to itself

(II) R relates a only to things other than itself

(III) R relates a both to itself and to things other than itself

(IV) R relates a to nothing.

As an illustration of the four options, take R to the square-root relation $\sqrt{}$, and the domain of objects to be the set of real numbers. Then the four possibilities are exemplified by the numbers 0, 4, 1 and −1 respectively. For $\sqrt{0} = 0$, $\sqrt{4} = 2$ and also −2, $\sqrt{1} = 1$ and also −1, while finally −1 does not have a defined square root among the real numbers. The list of four options is what is called in Madhyamaka a *catuṣkoṭi*.

[18] Śāntideva, *Bodhicaryāvatāra* 9.34.

[19] On presupposition and truth-value gaps, see P. F. Strawson, *Introduction to Logical Theory* (London: Methuen, 1952).

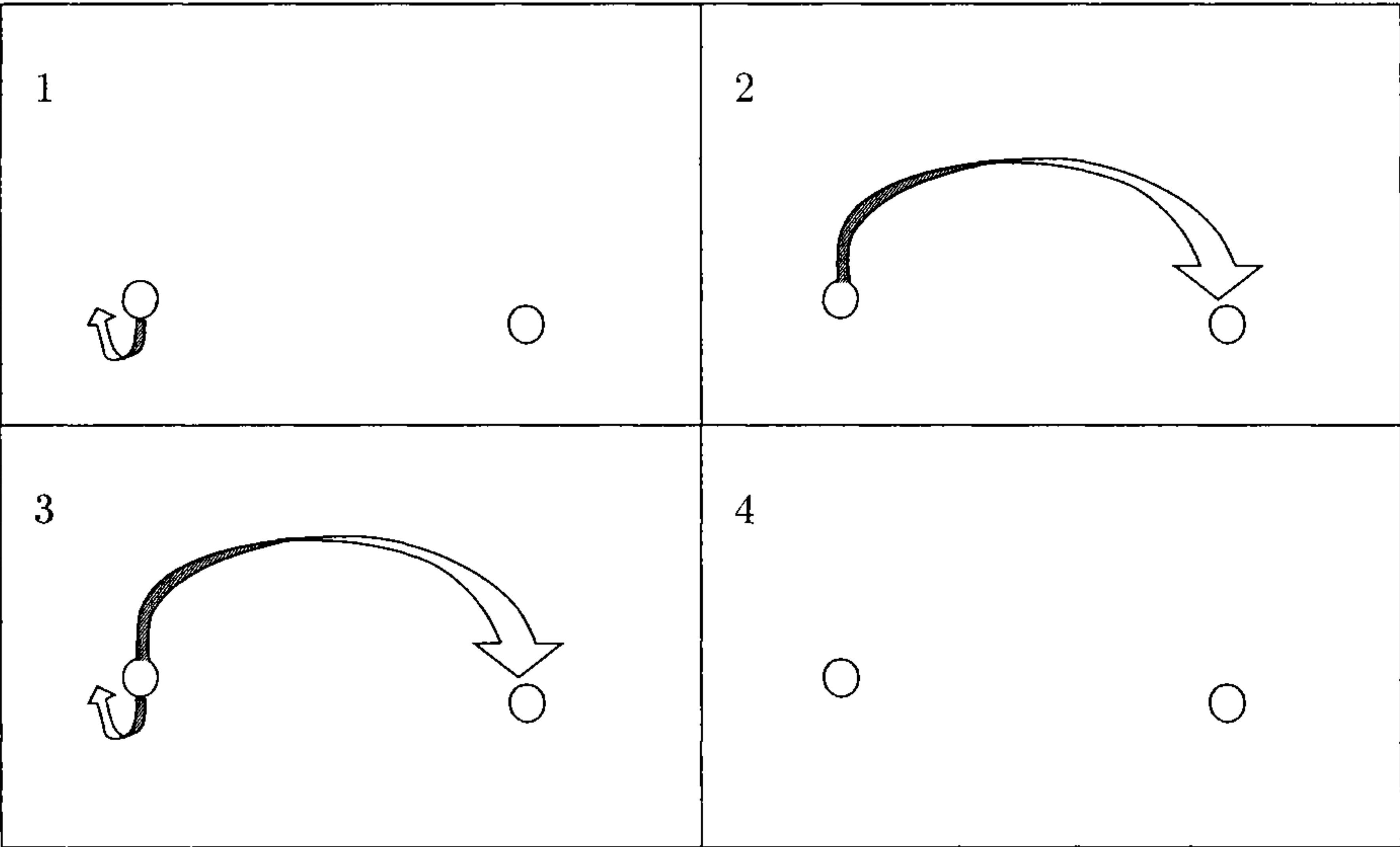


Figure 1. The Four Options

> Everything is thus, not thus, both thus and not thus, or neither thus nor not thus. That is the Buddha's [provisional] instruction. [*Mūlamadhyanaka-ka=arikā*, MK 18.8]

> Some say that suffering (*duḥkha*) is self-produced, or produced from another, or produced from both, or produced without a cause. [MK 12.1]

> Since every factor in existence (*dharma*) are empty, what is finite and what is infinite? What is both finite and infinite? What is neither finite nor infinite? [MK 25.22]

It is easy to see that the four options are mutually exclusive and jointly exhaustive. For the class of objects to which R relates a is either (IV) the empty set $\emptyset$ or, if not, then either (I) it is identical to $\{a\}$, or (II) it excludes $\{a\}$, or (III) it includes $\{a\}$. Not every relation exhibits all four options. (I) not exhibited if R is anti-reflexive. (II) is not exhibited if R is reflexive and bijective. (IV) is not exhibited if R is defined on every point in the domain. Note in particular that if R is the identity relation, then neither (III) nor (IV) are exhibited, not (III) because identity is transitive, and not (IV) because identity is reflexive. Indeed, options (III) and (IV) are not exhibited whenever R is an equivalence (transitive, symmetric, and reflexive) relation.

The next step in the 'refutation-only' strategy is to construct subsidiary 'disproofs', one for each of the four options. Although there is no pre-determined procedure for constructing such disproofs, by far the most commonly used method is to show that the option in question has some unacceptable consequence (*prasaṅga*). I will examine this method in detail in §1.5. A major dispute for later Mādhyamikas was over what sort of reasoning is permissible in the four subsidiary disproofs, the proofs that lead to the rejection of each of the four options. It is a difficult question to answer, so difficult indeed that it led, at around 500 AD, to a fission within the school of Madhyamaka. The principal group (Prāsaṅgika, headed by Buddhapālita) insisted that only *prasaṅga*-type, 'presupposition-negating' reasoning is admissible. This faction is the more conservative and mainstream, in the sense that their teaching seems to be in keeping with Nāgārjuna's own method of reasoning. The important later Mādhyamika masters Candrakīrti and Śāntideva defended this view. A splinter faction, however, (Svātantrika, headed by Bhāvaviveka) allowed 'independent' inference or inductive demonstration into the disproofs. Perhaps this was done so that the inferential methods developed by Diṅnāga (§2.2) could be deployed in establishing the Mādhyamika's doctrinal position. Clearly, the fewer restrictions one places on the type of reasoning one permits oneself to use, the greater are the prospects of successfully finding arguments to negate each of the four options. On the other hand, we have seen that the citation of paradigmatic examples is essential to this type of reasoning (§1.3), and it is hard to see how one could be entitled to cite examples in support of one's argument, when the very conception of those examples is in question.

The effect of the four subsidiary disproofs is to establish that none of the four options obtains:[20]

> Neither from itself nor from another, nor from both, nor without a cause, does anything whatever anywhere arise. [MK 1.1]
>
> One may not say that there is emptiness, nor that there is non-emptiness. Nor that both, nor that neither exists; the purpose for so saying is only one of provisional understanding. [MK 22.11]

The emptiness of the concept in question is now deduced as the final step in the process. For it is a presupposition of one of the four options obtaining that the concept does have an application-condition (a class of classes or relational property). If all four are disproved, then the presupposition itself cannot be true. When successful, the procedure proves that the concept in question is empty, null, *śūnya*. This is Nāgārjuna's celebrated and controversial '*prasaṅga*-type' rational inquiry, a sophisticated use of rationality to annul a conceptual scheme.

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[20] Further examples: MK 25.17, 25.18, 27.15–18. Interesting is the suggestion of Richard Robinson that the method of reasoning from the four options has two distinct functions, a positive therapeutic role as exhibited by the unnegated forms, and a destructive dialectical role, exhibited by the negated forms. Richard H. Robinson, *Early Mādhyamika in India and China*, (Madison, Milwauke and London: University of Winsconsin Press, 1967), pp. 39 58, esp. pp. 55-56.

A statement is truth-apt if it is capable of being evaluated as either true or false. When Nāgārjuna rejects each of the four options, he is rejecting the claim that a statement of the form '*aRb*' is truth-apt, since the four options exhaust the possible ways in which it might be evaluated as true. But if the statements belonging to a certain discourse are not truth-apt, then the discourse cannot be part of an objective description of the world (a joke is either funny or unfunny, but it cannot be evaluated as true or false.) The *prasaṅga* negates a presupposition for truth-aptness and so for objective reference.

Nāgārjuna applies the procedure in an attempt to annul each of the concepts that are the basic ingredients of our common-sense scheme. In each case, his method is to identify a relation and prove that none of the four options can obtain. On closer inspection, it turns out that his argumentation falls into two basic patterns.[21] One pattern is applied to any concept involving the idea of an ordering or sequence, especially the concept of a causal relation, of a temporal relation and of a proof relation. The paradigm for this argument is Nāgārjuna's presentation of a paradox of origin (chapter 1), which serves as model for his analysis of causation (chapter 8), the finitude of the past and future (chapter 11), and suffering (chapter 12). The argument seeks to establish that a cause can be neither identical to, nor different from, the effect. If nothing within the domain is uncaused, then the four options for the realization of a causal relation are foreclosed.

The other pattern of argumentation in Nāgārjuna is essentially grammatical. When a relational concept is expressed by a transitive verb, the sentence has an Agent and a Patient (the relata of the relation). For example, "He sees the tree," "He goes to the market," "He builds a house." The idea of the grammatical argument is that one can exploit features of the deep case structure of such sentences in order to prove that the Patient can be neither identical to the Agent, nor include it, nor exclude it, and that there must be a Patient. Nāgārjuna uses this pattern of argumentation in constructing a paradox of motion (MK, chapter 2), and this chapter serves as a model for his analysis of perception (chapter 3), composition (chapter 7), fire (chapter 10), and of bondage and release (chapter 16). Indeed, the same pattern of argument seems to be applicable whenever one has a concept which involves a notion of a single process extended in time. What exactly these arguments show and how well they succeed is a matter of debate, but what we have seen is the elaboration of a sophisticated sceptical strategy of argumentation, based on the idea of 'refutation-only' dialogue.

1.5 *Presumptive argumentation (*tarka*) and burden-of-proof shifting*

Indian logicians developed a theory of what they call 'suppositional' or 'presumptive' argumentation (*tarka*). It is a theory about the burden of proof and the role of presumption, about the conditions under which even inconclusive evidence is sufficient for warranted belief. As we have already seen, it is a style of reasoning that is regarded as permissible within a well-conducted dialogue (*vāda*; see

[21] On other patterns in Nāgārjuna's argumentation, see Robinson (1967: 48).

§1.3). In the canonical and early literature, *tarka* is virtually synonymous with reasoned thinking in general. The free-thinkers so derided in the epics were called *tārkikas* or 'followers of reason'. Even later on, when the fashion was to adorn introductory surveys of philosophy with such glorious names as *The Language of Reason* (*Tarkabhāṣā*, Mokṣākaragupta), *Immortal Reason* (*Tarkāmṛta*, Jagadīśa), *Reason's Moonlight* (*Tarkakaumudī*, Laugākṣi Bhāskara), it was usual to confer on a graduate of the medieval curriculum an honorific title like Master or Ford of Reason (*tarkavāgīśa, tarkatīrtha*). Such a person is a master in the art of evidence and the management of doubt, knowing when to accept the burden of proof and also when and how to deflect it.

Extrapolative inference (*anumāna*, see §1.3) rests on the knowledge of universal generalisations, and it is the possibility of such knowledge that the most troubling forms of scepticism call into question. How can one be entitled to believe that something is true of every member of a domain without inspecting each member individually? How does one cope with the ineliminable possibility that an unperceived counterexample exists in some distant corner of the domain? The difficulty here is with the epistemology of negative existentials. The Buddhist Diṅnāga formulates the extrapolation relation as a 'no counterexample' relation. For him, x extrapolates y just in case there is no x without y (y-*avina* x-*abhāva*). The Navya-Nyāya logicians prefer a different negative existential condition, one derived from the reflexivity and transitivity of the extrapolation relation. Given transitivity, if x extrapolates y then, for any z, if y extrapolates z, so does x. The converse of this conditional holds too, given that the extrapolation relation is reflexive (proof: let $z = y$). So let us define an 'associate condition' (*upādhi*) as a property which is extrapolated by y but not x. Then x extrapolates y just in case there is no associate condition.[22] One can infer fire from smoke but not smoke from fire, for there is an associate condition, dampness-of-fuel, present wherever smoke is but not wherever fire is. Tinkering with the definition, though, does not affect the epistemological problem; it remains the one of proving a nonexistence claim.

Presumptive argumentation, *tarka*, is a device for appropriating a presumptive right – the right to presume that one's own position is correct even without conclusive evidence in its support. One is, let us imagine, in a state of doubt as to which of two hypotheses A and B is true. A and B are exclusive (at most one is true) but not necessarily contradictory (both might be false). Technically, they are in a state of 'opposition' (*virodha*).[23] The doubt would be expressed by an exclusive disjunction in the interrogative – Is it that A or that B? Uncertainty initiates inquiry, and at the beginning of any inquiry the burden of proof is symmetrically distributed among the alternative hypotheses. A piece of presumptive

[22] For a survey of the literature on this theory, see Karl Potter ed., "Indian Epistemology and Metaphysics", *Encyclopedia of Indian Philosophies*, Volume 2 (Princeton: Princeton University Press, 1977), pp. 203 206; Karl Potter and Sibajiban Bhattacharyya eds., "Indian Analytical Philosophy: Gaṅgeśa to Raghunātha", *Encyclopedia of Indian Philosophies*, Volume 6 (Princeton: Princeton University Press, 1993), pp. 187 192.

[23] Nandita Bandyopadhyay, "The Concept of Contradiction in Indian Logic and Epistemology," *Journal of Indian Philosophy* 16.3 (1988), pp. 225 246, fn. 1.

argumentation shifts the burden of proof by adducing a *prima facie* counterfactual argument against one side. The form of the argument is the same in all cases. It is that one alternative, supposed as true, would have a consequence in conflict with some set of broadly defined constraints on rational acceptability. The existence of such an argument gives one the right to presume that the other alternative is true, even though one has no conclusive proof of its truth, and even though the logical possibility of its being false remains open. In the psychologized language of the Nyāya logician, a suppositional argument is a 'blocker' (*bādhaka*) to belief in the supposed alternative, and an 'eliminator' (*nirvartaka*) of doubt. The Naiyāyika Vācaspati (9th century) comments:[24]

> Even if, following a doubt, there is a desire to know [the truth], the doubt still remains after the desire to know [has come about]. This is the situation intended for the application of presumptive argumentation. Of two theses, one should be admitted as known when the other is rejected by the reasoning called 'suppositional.' Thus doubt is suppressed by the application of presumptive argumentation to its subject matter... A means of knowing is engaged to decide a question, but when there is a doubt involving its opposite, the means of knowing fails [in fact] to engage. But the doubt concerning the opposite is not removed as such by the undesired consequence. What makes possible its removal is the means of knowing.

Vācaspati stresses that a thesis is not itself proved by a suppositional demonstration that the opposite has undesired consequences; one still needs evidence corroborating the thesis. But there is now a presumption in its favour, and the burden of proof lies squarely with the opponent. Presumptive argumentation 'supports' one's means of acquiring evidence but it not itself a source of evidence. It role is to change the standard of evidence required for proof in the specific context.

A radical sceptical hypothesis is a proposition inconsistent with ordinary belief but consistent with all available evidence for it. The aim of the radical sceptic is to undermine our confidence that our beliefs are justified, to introduce doubt. The Nyāya logicians' response to scepticism is not to deny that there is a gap between evidence and belief, or to deny the logical possibility of the sceptical hypothesis. It is to draw a distinction between two kinds of doubt, the *reasonable* and the *reasonless*. A doubt is reasonable only when both alternatives are consistent with all the evidence *and the burden of proof is symmetrically distributed* between them. One paradigmatic example is the case of seeing in the distance something that might be a person or might be a tree-stump. Udayana gives the epistemology of such a case: it is a case in which one has knowledge of common aspects but not of specific distinguishing features. What we can now see is that the example gets its force only on the assumption that there is a level epistemic playing field, with both hypotheses carrying the same *prima facie* plausibility. Presumptive

[24] *Nyāyavārttikatātparyaṭīkā*, below NS 1.1.40.

argumentation has the potential to break the impasse — imagine, for example, that the unidentified lump is just one of ten in an orderly row not there an hour ago. The perceptual evidence remains the same, but the burden of proof is on anyone who wants to maintain in this situation that the lump is a stump.

The other paradigm is knowledge of extrapolation relations. The problem here is that the thesis is one of such high generality that the burden of proof is already heavily against it! How can a few observations of smoke with fire ground a belief that there is fire *whenever* there is smoke? Suppositional argument has a different supportive role here. Its function is to square the scales, to neutralise the presumption *against* the belief in generality. It does so by finding *prima facie* undesirable consequences in the supposition that an associate condition or counterexample exists. Then sampling (observation only of confirmatory instances in the course of a suitably extensive search for counterexamples), though still weak evidence, can tilt the scale in its favour.

A presumptive argument moves from conjecture to unacceptable consequence. Modern writers often identify it with the medieval technique of *reductio ad absurdum*, but in fact its scope is wider. The 'unacceptable consequence' can be an out-and-out contradiction but need not be so. For we are not trying to prove that the supposition is false, but only to shift the burden of proof onto anyone who would maintain it. And for this it is enough simply to demonstrate that the supposition comes into conflict with some well-attested norm on rationality. Udayana, the first to offer any systematic discussion, does not even mention contradiction as a species of unacceptable consequence. He says[25] that presumptive argumentation is of five types –

1. self-dependence (*ātmāśraya*)
2. mutual dependence (*itaretarāśraya*)
3. cyclical dependence (*cakraka*)
4. lack of foundation (*anavasthā*)
5. undesirable consequence (*aniṣṭaprasaṅga*)

The last of these is really just the generic case, what distinguishes presumptive argumentation in general. The first four form a tight logical group. If the supposition is the proposition A, then the four types of unacceptable consequence are (1) proving A from A, (2) proving A from B, and B from A, (3) proving A from B, B from C, and C from A – or any higher number of intermediate proof steps eventually leading back to A, and (4) proving A from B, B from C, C from $D, \ldots$, without end. So what presumptive argumentation must show is that the supposition is ungrounded, its proof being either regressive or question-begging.

Two points are noteworthy about Udayana's list. First, rational unacceptability bears upon the proof adduced for the supposition, not the supposition itself. The

[25] *Ātmatattvaviveka*, p. 863.

underlying implication is that one has the right to presume that one's thesis is correct if one can find fault with the opponent's proof of the antithesis. Principles of this sort are familiar from discussion of the informal logic of arguments from ignorance in which one claims entitlement to assert A on the grounds that it is not known (or proved) that $\neg A$.[26] In general such a claim must be unfounded – it amounts to the universal appropriation of a presumptive right in all circumstances.

The second point to notice about Udayana's list, however, is that it is very narrow. Udayana places strict constraints on what will count as an unacceptable consequence, constraints which are more formal than broadly rational. Conflict with other well-attested belief is not mentioned, for instance. Udayana severely limits the scope of presumptive argumentation. His motive, perhaps, is to disarm the sceptic. For presumptive argumentation is the favoured kind of reasoning of the sceptic-dialecticians (and indeed the term Udayana uses is *prasaṅga*, the same term Nāgārjuna had used for his dialectical method). Sceptics typically will want to loosen the conditions on what constitutes an unacceptable consequence of a supposition, so that the scope for refutation is expanded. So what Udayana seems to be saying is that one does indeed have the right to presume that one's thesis is correct when the argument for the counter-thesis commits a fallacy of a particularly gross type — not mere conflict with other beliefs but formal lack of foundation. If the best argument for the antithesis is *that* bad, then one has a *prima facie* entitlement to one's thesis.

Śrīharṣa (c. AD 1140) is an Advaita dialectician, a poet and a sceptic.[27] He expands the notion of unacceptable consequence, noticing several additional types unmentioned by Udayana.[28] One is 'self-contradiction' (*vyāghāta*). It was Udayana himself[29] who analysed the notion of opposition as noncompossibility, and cited as examples the statements "My mother is childless," "I am unable to speak", and "I do not know this jar to be a jar." In the first instance, the noncompossibility is in what the assertion states, in the second it is in the speech-act itself, while in the third the propositional attitude self-ascription is self-refuting (a case akin to the Cartesian impossibility of thinking that one is not thinking).

Another refutation-exacting circumstance is the one called 'recrimination' (*pratibandī*). This is a situation in which one's opponent accuses one of advancing a faulty proof, when his own proof suffers exactly the same fault! There is a disagreement about what this state of equifallaciousness does to the burden of proof. The

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[26] Douglas Walton, *Arguments from Ignorance* (University Park, PA: The Pennsylvania State University Press, 1996).

[27] On Śrīharṣa: Phyllis Granoff, *Philosophy and Argument in Late Vedānta: Śrīharṣa's Khaṇḍanakhaṇḍakhādya* (Dordrecht: Reidel Publishing Company, 1978); Stephen Phillips, *Classical Indian Metaphysics* (La Salle: Open Court, 1995), chapter 3.

[28] *Khaṇḍanakhaṇḍakhādya* IV, 19 (*aprasaṅgātmakatarkanirūpaṇa*, pp. 777 788, 1979 edition; section numbering follows this edition). Śrīharṛṣa the negative dialectician wants to criticise even the varieties of presumptive argumentation, although his own method depends upon it. So he says: "By us indeed were presumptive argumentations installed in place, and so we do not reject them with [such] counter-arguments. As it is said – 'it is wrong to cut down even a poisonous tree, having cultivated it oneself"' (p. 787).

[29] *Ātmatattvaviveka*, p. 533.

practice of Naiyāyikas is to take the circumstance as tilting the balance against the opponent the opponent discredits himself in pressing an accusation without seeing that it can be applied with equal force to his own argument. But Śrīharṣa quotes with approval Kumārila's assertion that "all things being equal, where the same fault afflicts both positions one should not be censured [and not the other]".[30]

Śrīharṣa, the sceptic, would like to see both parties refuted by this circumstance. The same point underlies his mention as an unacceptable consequence the circumstance of 'lack of differential evidence' (*vinigamanāviraha*), when thesis and antithesis are in the same evidential situation. Again, what we see is a jostling with the burden of proof. Here Śrīharṣa is saying that absence of differential evidence puts a burden of proof on both thesis and antithesis — doubt itself refutes. It is the sceptic's strategy always to seek to maximise the burden of proof, and so to deny that anyone ever has the right to presume their position to be correct. That is, as Stanisław Schayer observed a long time ago, a difference between the *tarka* of the Naiyāyika and the *prasaṅga* of a sceptic like Śrīharṣa or Nāgārjuna.[31] For the latter, the demonstration that a thesis has an allegedly false consequence does not commit the refuter to an endorsement of the antithesis. Nāgārjuna wants to maintain instead that thesis and antithesis share a false existential precommitment.

Simplicity (*laghutva*) is, Śrīharṣa considers and the Naiyāyikas agree, a *ceteris paribus* preference-condition. Of two evidentially equivalent and otherwise rationally acceptable theses, the simpler one is to be preferred. The burden of proof lies with someone who wishes to defend a more complex hypothesis when a simpler one is at hand. The Nyāya cosmological argument appeals to simplicity when it infers from the world as product to a *single* producer rather than a multiplicity of producers. Here too the role of the simplicity consideration is to affect the burden of proof, not itself to prove. Cohen and Nagel[32] make a related point when they diagnose as the 'fallacy of simplism' the mistake of thinking that "of any two hypotheses, the simpler is the *true* one." In any case, simplicity can be a product not of the content of a hypothesis but only of its mode of presentation — the distinction is made by the Naiyayikas themselves.[33] And it is hard to see how it can be rational to prefer one hypothesis to another only because it is simpler in form.

We have assumed that the rival hypotheses are both empirically adequate, that is to say, they are both consistent with all known facts. Śrīharṣa mentions an unacceptable consequence involving empirical evidence (*utsarga*). It is an objec-

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[30] *Khaṇḍanakhaṇḍakhādya* II, 2 (*pratibandīlakṣaṇakhaṇḍana*, pp. 571 572). The full quotation is given in his commentary by Śaṃkara Miśra.

[31] Stanisław Schayer, "Studies on Indian Logic, Part II: Ancient Indian Anticipations of Propositional Logic," [1933], translated into English by Joerg Tuske in Jonardon Ganeri ed., *Indian Logic: A Reader*.

[32] Morris R. Cohen and Ernest Nagel, *An Introduction to Logic and Scientific Method* (London: Routledge & Kegan Paul, 1934), p. 384.

[33] Bhimacarya Jhalakikar, *Nyāyakośa or Dictionary of Technical Terms of Indian Philosophy* (Poona: Bhandarkar Oriental Research Institute, 1928), s.v. *laghutvam.*

tion to the usual idea that if there is empirical evidence supporting one hypothesis but not the other, then the first is confirmed. Śrīharṣa's sceptical claim is that a hypothesis must be considered refuted unless it is conclusively proved; nonconclusive empirical evidence does nothing to affect this burden of proof. Likewise, he says, a hypothesis must be considered refuted if it is incapable of being proved or disproved — this at least seems to be the import of the unacceptable consequence he calls 'impertinence' (*anucitya*) or 'impudence' (*vaiyātya*).

Other varieties of suppositional refutation have been suggested along lines similar to the ones we have reviewed. Different authors propose different sets of criteria for rational nonacceptance. What we have seen is that there is, in the background, a jostling over the weight and place of the burden of proof. The sceptic presses in the direction of one extreme — that a thesis can be considered refuted unless definitively proven. The constructive epistemologist tries to press in the direction of the opposite extreme — that a thesis can be considered proved unless definitively disproved. The truth lies somewhere in between, and it is the role of presumptive argumentation to locate it.

2 BUDDHIST CONTRIBUTIONS IN INDIAN LOGIC: FORMAL CRITERIA FOR GOOD ARGUMENTATION

2.1 The doctrine of the triple condition (trairūpya)

The Buddhist logician Diṅnāga (c. 480–540 AD) recommends a fundamental restructuring of the early Nyāya analysis of reasoned extrapolation and inference. Recall that analysis. It is an inference from likeness and unalikeness. In the one case, some object is inferred to have the target property on the grounds that it is 'like' a paradigmatic example. The untasted grain of rice is inferred to be cooked on the grounds that it is in the same pan as a test grain which is found to be cooked. In the other case, the object is inferred to have the target property on the grounds that it is 'unlike' an example lacking the target property. Likeness and unalikeness are matters of sharing or not sharing some property, the reason-property or evidence grounding the inference. Examples are either 'positive' — having both the reason and the target property, or 'negative' — lacking both. Extrapolation is the process of extrapolating a property from one object to another on the basis of a likeness or unalikeness between them.

The difficulty is that not every such extrapolation is rational or warranted. The extrapolation of a property from one object to another is warranted only when the two objects are *relevantly* alike or *relevantly* unalike. That two objects are both blue does not warrant an extrapolation of solidity from one to the other; neither can we infer that they are different in respect to solidity because they are of different colours. What one needs, then, is a theory of relevant likeness or unalikeness, a theory, in other words, of the type of property (the reason property) two objects must share if one is to be licensed to extrapolate another property (the target property) from one to the other.

This is exactly what Diṅnāga gives in his celebrated theory of the 'reason with three characteristics' (*trairūpya*). Diṅnāga's thesis is that relevant likeness is an exclusion relation. Two objects are relevantly alike with respect to the extrapolation of a property S just in case they share a property excluded from what is other than S. In other words, a reason property H for the extrapolation of a target property S is a property no wider in extension than S (assuming that 'non' is such that $H \cap \text{non}S = \emptyset$ iff $H \subseteq S$). Here is the crucial passage in the *Pramāṇa-samuccaya*, or *Collection on Knowing*:

> The phrase [from II 1b] "through a reason that has three characteristics" must be explained.
>
> > [A proper reason must be] present in the site of inference and in what is like it and absent in what is not [II 5cd].
>
> The object of inference is a property-bearer qualified by a property. After observing [the reason] there, either through perception or through inference, one also establishes in a general manner [its] presence in some or all of the same class. Why is that? Because the restriction is such that [the reason] is present *only* in what is alike, there is no restriction that it is *only* present. But in that case nothing is accomplished by saying that [the reason] is "absent in what is not". This statement is made in order to determine that [the reason], absent in what is not [like the site of inference], is not in what is other than or incompatible with the object of inference. Here then is the reason with three characteristics from which we discern the reason-bearer.

Diṅnāga's important innovation is to take the notions of likeness and unalikeness in extrapolation to be relative to the *target* property rather than the reason property. Two objects are 'alike' if they both have, or both lack, the target property. Two objects are 'unalike' if one has and the other lacks the target property. We want to know if our object — the 'site' of the inference — has the target property or not. What we do know is that our object has some other property, the reason property. So what is the formal feature of that reason property, in virtue of which its presence in our object determines the presence or absence of the target property? The formal feature, Diṅnāga claims, is that the reason property is present only in what is alike and absent in whatever is unalike our object.

This can happen in one of two ways. It happens if the reason property is absent from everything not possessing the target property and present only in things possessing the target property. Then we can infer that our object too possesses the target property. It can also happen if the reason property is absent from everything possessing the target property and present only in things not possessing the target property. Then we can infer that our object does not possess the target property.

Call the class of objects which are like the site of the inference the 'likeness class', and the class of objects unlike the site the 'unlikeness class' (Diṅnāga's terms are

sapakṣa and *vipakṣa*). Interpreters have traditionally taken the likeness class to be the class of objects which possess the target property, and the unlikeness class to be the class of objects which do not possess the target property. I read Diṅnāga differently. I take his use of the terms 'likeness' and 'unlikeness' here at face-value, and identify the likeness class with the class of things *in the same state* vis-à-vis the target property as the site of the inference. We do not know in advance what that state is, but neither do we need to. The pattern of distribution of the reason property tells us what we can infer – that the site has the target property, that it lacks it, or that we can infer nothing. My approach has several virtues, chief among which is that it preserves the central idea of likeness as a *relation* between objects rather than, as with the traditional interpretation, referring to a *property* of objects. I think it also avoids many of the exegetical problems that have arisen in the contemporary literature with regard to Diṅnāga's theory.

One of the traditional problems is whether the site of the inference is included in the likeness class or not.[34] If the likeness class is the class of objects possessing the target property, then to include it seems to beg the question the inference is trying to resolve: does the site have that property or not. But to exclude it implies that the union of the likeness and unlikeness classes does not exhaust the universe (the site cannot, for obvious reasons, be unlike itself). So one is left with two disjoint domains, and an apparently insuperable problem of induction – how can correlations between the reason property and the target property in one domain be *any* guide to their correlation in another, entirely disjoint, domain?[35]

If we take Diṅnāga's appeal to the idea of likeness at face-value, however, the problem simply does not arise. The site of the inference is in the likeness class on the assumption that likeness is a reflexive relation — but that begs no question, for we do not yet know whether the likeness class is the class of things which possess the target property, or the class of things which do not possess it. It is the class of things which are in the same state vis-à-vis the target property as the inferential site itself. We can, if needs be, refer to objects 'like the site but not identical to it;' or we can take likeness to be nonreflexive, and refer instead, if needs be, to 'the site *and* objects like it' — but this is a matter only of labelling, with no philosophical interest.

Another of the traditional problems with Diṅnāga's account is an alleged logical equivalence between the second and third conditions.[36] The second condition states that the reason property be present *only* in what is alike.[37] The third condition states that it be absent in what is not. But if it is present *only* in what

[34] Tom F. Tillemans, "On sapakṣa," *Journal of Indian Philosophy* 18 (1990), pp. 53–80.

[35] Hans H. Herzberger, "Three Systems of Buddhist Logic," in B. K. Matilal and R. D. Evans eds., *Buddhist Logic and Epistemology: Studies in the Buddhist Analysis of Inference and Language* (Dordrecht: Reidel Publishing Company, 1982), pp. 59–76.

[36] Bimal Matilal, "Buddhist Logic and Epistemology," in Matilal and Evans (1982: 1–30); reprinted in Matilal, *The Character of Logic in India* (Albany: State University Of New York Press, 1998), chapter 4.

[37] There is some debate among scholars over whether it was Diṅnāga himself or his commenator Dharmakīrti who first inserts the particle *only* into the clauses.

is alike, it must be absent in what is not; and if it is absent in what is not alike, it must be present *only* in what is. Now it is clear that Diṅnāga's reason for inserting the particle *only* into his formula is to prevent a possible misunderstanding. The misunderstanding would be that of taking the second condition to assert that the reason property must be present in all like objects. That would be too strong a condition, ruling out any warranted inferences in which the reason property is strictly narrower than the target. On account of the meaning of the particle *only*, we can see that it is also one of the two readings of the statement:

> In what is alike, there is only the presence [of the reason]

where the particle *only* is inserted into the predicate position. Diṅnāga eliminates this unwanted reading of the second condition, but he does so in a disastrous way. He eliminates it by inserting the particle into the subject position:

> Only in what is alike, there is the presence [of the reason].

The reason this is disastrous is that it makes the second condition logically equivalent to the third. Notice, however, that when *only* is in predicate position, there are still two readings. The reading one needs to isolate is the second of these two readings:

> In what is alike, there is *indeed* the presence [of the reason]

That is, the reason is present in *some* of what is alike.

Accordingly, the theory is this. The extrapolation of a property S to an object is grounded by the presence in that object of any property X such that X excludes non$S$ but not S. A reason property for S is any member of the class

$$\{X : X \cap S \neq \varnothing \ \&\ X \cap \text{ non}S = \varnothing\}.$$

The clause 'but not S' (the second of Diṅnāga's three conditions) has a clear function now. It is there to rule out properties which exclude *both* non$S$ *and* S. Such properties are properties 'unique' to the particular object which is the site of the inference, and Diṅnāga does not accept as warranted any extrapolation based on them. I will look at his motives in the next section.

Reason properties are nonempty subsets of the properties whose extrapolation they ground. If two objects are 'alike' in sharing a property, and one has a second property of wider extension than the first, then so does the second. Inductive extrapolation, in effect, is grounded in the contraposed universal generalisation "where the reason, so the target." A difficult problem of induction remains - how can one come to know, or justifiably believe, that two properties stand in such a relation without surveying all their instances? Diṅnāga has no adequate answer to this problem (but see [Tuske, 1998; Peckhaus, 2001]). Dharmakīrti, Diṅnāga's brilliant reinterpreter, does. His answer is that when the relation between the two properties is one of causal or metaphysical necessity, the observation of a few

instances is sufficient to warrant our belief that it obtains (§2.3). Diṅnāga, however, is not interested in such questions. For him, the hard philosophical question is that of discovering the conditions for rational extrapolation. It is another issue whether those conditions can ever be known to obtain.

To sum up, Diṅnāga's three conditions on the reason are:

1.	Attachment	Presence in the site a attachment	(*pakṣadharmatā*)
2.	Association	Presence (only) in what is like	(*anvaya*)
3.	Dissociation	Absence in what is unlike	(*vyatireka*)

If we take these conditions to be independent, it follows that there are exactly seven kinds of extrapolative inferential fallacy — three ways for one of the conditions to fail, three ways for two conditions to fail, and one way for all three conditions to fail. So the new theory puts the concept of a fallacy on a more formal footing. A fallacy is no longer an interesting but essentially *ad hoc* maxim on reasoned argument. It is now a formal failing of the putative reason to stand in the correct extrapolation-grounding relation. One way for the reason to fail is by not attaching to the site at all, thereby failing to ground any extrapolation of other properties to it. This is a failure of the first condition. Another way for the reason to fail is by 'straying' onto unlike objects, thereby falsifying the third condition. The presence of one property cannot prove the presence of another if it is sometimes present where the other one is not. (It can, however, prove the *absence* of the other if it is *only* present where the other is not — and then the absence of the first property is a proof of the *presence* of the second.) We might then think of the third condition as a 'no counter-example' condition, a counter-example to the extrapolation-warranting relation of subsumption being an object where the allegedly subsumed property is present along with the absence of its alleged subsumer. An extrapolation is grounded just as long as there are no counterexamples.

2.2 Diṅnāga's 'wheel of reasons' *(hetucakra)*

In addition to his *Pramāṇa-samuccaya*, Diṅnāga wrote another, very brief text on logic, the *Wheel of Reasons*, or *Hetucakranirṇaya*. Diṅnāga's aim here is to classify all the different types of argument which fit into the general schema ⟨p has s because it has h⟩, and to give an example of each. It is here that he applies his theory of a triple-conditioned sign to show when an inference is sound or unsound, and the kinds of defect an inferential sign can suffer from. Hence, it leads to a classification of fallacious and non-fallacious inferences.

The 'wheel' or 'cycle' is in fact a 3 by 3 square, giving nine inference types. Diṅnāga derives the square as follows. A 'homologue' (*sapakṣa*) is defined as any object (excluding the locus of the inference) which is possesses the inferrable property, s. Now, a putative inferential sign, h, might be either (i) present in every homologue, (ii) present in only some of the homologues but not in others, or (iii) present in no homologue. Suppose we let 'sp' stand for the class of homologues.

Then we can represent these three possibilities as 'sp+', 'sp±', and 'sp−' respectively. The same three possibilities are also available with respect to the class of heterologues (objects, excluding the locus, which do not possess the inferred property, s). We can denote these by 'vp+', 'vp±', and 'vp−' respectively. Thus, 'vp+' means that every member of vp (every heterologue) possesses the sign property, h, etc. Now since any putative inferential sign must either be present in all, some or no homologue, and also in either all, some or no heterologue, there are just nine possibilities (Figure 2):

		vp		
		+	-	±
sp	+	1 deviating	2 goodK	3 deviating
	-	4 contradictory	5 uniquely deviating	6 contradictory
	±	7 deviating	8 good	9 deviating

Figure 2.

Why does Diṅnāga say that only 2 and 8 are cases of a good inferential sign? Recall the three conditions on a good sign. The first is that the inferential sign must be present in the locus of inference. This is taken for granted in the wheel. The second states that the inferential sign should be present in some (at least one) homologous case. In other words, a good sign is one for which either 'sp+' or 'sp±'. Thus the second condition rules out 4, 5 and 6. Similarly, the third condition states that the inferential sign should be absent from any heterologous case, i.e. that 'vp−'. This rules out 1, 4, 7 and 3, 6, 9. So only 2 and 8 represent inferential signs which meet all three conditions and generate good inferences. Note here that the third condition alone is sufficient to rule out every fallacious case except 5. Hence, seeing why Diṅnāga considers 'type-5' inferences to be unsound will reveal why he considered the second of the three conditions to be necessary (see below).

Diṅnāga next gives an illustration of each of the nine possibilities. They can be tabulated, as in Figure 3.

In each case, the locus of the inference is sound. Note that wherever possible, Diṅnāga cites both a 'positive confirming example', i.e. an object where both

	s	h	positive example	negative example	counter-example
1	eternal	knowable	space		a pot
2	transitory	created	a pot	space	—
3	manmade	tansitory	a pot	space	lightning
4	eternal	crated		—	a pot
5	eternal	audible	—	a pot	—
6	eternal	manmade	—	lightning	a pot
7	natural	transitory	lightning	—	a pot
8	transitory	manmade	a pot	space	—
9	eternal	incorporeal	space	a pot	action

Figure 3.

h and s are present, as well as a negative confirming example', i.e. an object where neither h nor s is present. Both support the inference. He also cites, where relevant, a 'counter-example', i.e. a case where h is present but s is absent. The existence of a counter-example undermines the inference. Let us look at four representative cases.

Case 2: A warranted inference. This inference reads: Sound is transitory, because it is created, e.g. a pot; space. Intuitively, this inference is sound, because the reason-property, createdness, is present only in places where the inferred property, transitoriness, is also present. Hence createdness is a good sign of transitoriness. The inference is supported first by an example where both are present, a pot, and second by an example where neither are present, space.

Case 3: 'deviating' (*asiddha*). This inference reads: Sound is manmade, because it is transitory, e.g. a pot; space. Intuitively, this inference is unsound, because the reason-property, transitoriness, is present in places where the inferred property, manmade, is absent. The counterexample cited is lightning transitory but not manmade. Because we can find such a counter-example, the inferential sign is said to 'deviate' from the inferred property. Deviating inferences are ones which satisfy the second condition but fail the third.

Case 6: 'contradictory' (*viruddha*). The inference reads: Sound is eternal, because it is manmade, e.g. lightning. The sign here fails both conditions 2 and 3 — there is no case of a thing which is eternal and manmade, but there is a counter-example, for instance, a pot, which is manmade but non-eternal. Such an inference is called 'contradictory' because we can in fact infer to the contrary conclusion, namely that sound is non-eternal because it is manmade. We can do this because in the contrary inference, the homologous and heterologous domains are switched round.

Case 5: 'specific' (*asādhāraṇa*). Sound is eternal, because it is audible, e.g. a pot. The first point to notice is that there are no counter-examples to this

inference, for there are no examples, outside the 'locus' domain of sounds, of an audible thing which is non-eternal. This is because there are no audible things other than sounds! Hence the third condition seems to be satisfied trivially. The characteristic of type-5 inferences is that the reason-property is 'unique' to the locus. According to Diṅnāga, such inferences are unsound, and the reason is that they fail the second condition - there is no homologue, i.e. an eternal thing other than sound, which is also audible.

But this just restates the characteristic feature of such inferences, it doesn't explain *why* they are unsound. Some modern authors argue that the significance of the second condition is more epistemological, than logical: the second condition implies that there must be a positive supporting example, and without such an example the inference, even if sound, carries no conviction. Diṅnāga might, however, have had a more formal or logical reason for rejecting type-5 inferences. The universal rule here is "Whatever is audible, apart from sound, is eternal". Now if a universal rule of the form '$(\forall x)(Fx \rightarrow Gx)$' is made true by there being no Fs, then so is the rule '$(\forall x)(Fx \rightarrow \text{not-}Gx)$'. Hence, we could equally infer that sound is non-eternal because it is audible! This resembles the fault which the Nyāya called '*prakaraṇasama*' or 'indecisive'. Diṅnāga, it seems, wants to avoid this by saying that '$(\forall x)(Fx \rightarrow Gx)$' is true only if there is at least one F, which leads to the second condition.

Let us consider the argument from specifics further. I have said that an extrapolation-grounding property is a nonempty subproperty a property narrower in extension than the property being extrapolated, and resident at least in the object to which that property is being extrapolated. The sweet smell of a lotus is a ground for extrapolating that it has a fragrance; its being a blue lotus is a ground for extrapolating its being a lotus. Extrapolation is a move from the specific to the general, from species to genus, from conjunction to conjunct. Extrapolation is a move upwards in the hierarchy of kinds. This model of extrapolation works well in most cases, but what happens at the extremes? The extreme in one direction is a most general property of all, a property possessed by everything. Existence or 'reality', if it is a property, is a property like this, and the theory entails that existence is always extrapolatable — the inference 'a is, because a is F' is always warranted. Diṅnāga's theory faces a minor technical difficulty here. Since everything exists, then everything is 'like' the site of the inference (in the same state as the site with respect to existence), and the unlikeness class is empty. So Diṅnāga has to be able to maintain that his third condition - absence of the reason property in every unlike object is satisfied when there are no unlike objects. The universal quantifier must have no existential import. His innovative distinction between inference 'for oneself' (*svārthānumāna*) and inference 'for others' (*parārthānumāna*) is a help here. It is the distinction between the logical preconditions for warranted extrapolation and the debate-theoretic exigencies of persuasion. While it might be useful, even necessary, to be able to cite a supporting negative example if one's argument is to carry conviction and meet the public norms on believable inference,

there is no corresponding requirement that the unlikeness class be nonempty if an extrapolation is to be warranted.

What happens at the other extreme? Extrapolation is a move from the more specific to the less specific, and the limit is the case when the reason property is entirely specific to the site of the inference. There is no doubt but that Diṅnāga thinks that extrapolation breaks down at this limit. He calls such reason properties 'specific indeterminate' (*asādhāraṇānaikāntika*), and classifies them as bogus-reasons. Indeed it is the entire function of his second condition to rule out such properties. That is why the second condition insists that the reason property must be present in an object like the site. This condition is an addition to the first, that the reason property be present in the site — it demands that the reason be present in some other object like but not identical to the site. Diṅnāga's example in the *Collection on Knowing* [II 7d] is:

[Thesis] Sound is noneternal.

[Reason] Because it is audible.

In the *Wheel of Reasons* [5cd -7a], he gives another example:

[Thesis] Sound is eternal.

[Reason] Because it is audible.

What is the difference? In fact, the difference between these two examples holds the key to what Diṅnāga thinks is wrong. The property audibility, something specific to sound, does not determine whether sound is eternal or noneternal. In either case, audibility is absent from what is unlike sound (because it is unique to sound) but also from what is like sound (except for sound itself). This symmetry in the distribution of the reason property undermines its capacity to discriminate between truth and falsity. To put it another way, if we take the universal quantifier to range over everything except the site of the inference, sound, then it is true both that everything audible is eternal and that everything audible is noneternal — both are true only because there are no audibles in the range of the quantifier.

This seems to be Diṅnāga's point, but it is not very satisfactory. Sound *is* either eternal or noneternal, and so audibility *is* a subproperty of one or the other. One and only one of the above universal quantifications is true when the quantifier is *unrestricted*. In any case, just why is it that we should not reason from the specific properties of a thing? We do it all the time. Historical explanations are notoriously singular — unrepeated historical events are explained by specific features of their context. Diṅnāga, it seems, is like the follower of the deductive-nomological model in insisting on repeatability as a criterion of explanation. What about mundane cases like this one: the radio has stopped because I have unplugged it? Being unplugged by me is a property specific to the radio, and yet the form of the explanation seems unapproachable. Perhaps, however, what one should say is that the explanatory property is 'being unplugged', and not 'being unplugged by me',

and the explanation rests on the generalisation 'whenever a radio is unplugged, it stops.' So then the restriction is not to any property specific to the site, but only to those which are not merely tokens of some more general explanatory property. And yet there are still intuitively rational but specific inferences — that salt is soluble because it has a certain molecular structure, that helium is inert because it has a certain atomic number, flying creatures fly because they have wings. Why shouldn't the specific properties of a thing be implicated in inferences of its other properties?

What we see here is Diṅnāga's adherence to a strictly inductivist model of extrapolation. The specific property audibility does not ground an extrapolation of eternality or noneternality because there can be no inductive evidence for the extrapolation. Inductive evidence takes the form of objects in the likeness and unlikeness classes known to have or not to have the reason. One might think that one does have at least 'negative' evidence, for one knows that audibility is absent from any object in the unlikeness class. So why can one not infer from the fact that audibility is absent in unlike objects that it must be present in like objects? The answer is that one can indeed make that inference, but it does not get one very far. For we must recall again the way these classes are defined – as classes of objects like or unlike the site with respect to eternality. We do not know whether the site is eternal or noneternal, and in consequence we do not know whether unlike things are things which are noneternal or eternal. So while we have plenty of examples of eternal inaudibles and noneternal inaudibles, we still do not know which are the 'alike' ones and which the 'unalike'.

The explanation of salt's solubility by its specific molecular structure exemplifies a quite different model of explanation. It is a theoretical explanation resting on the postulates of physical chemistry. It is from theory, not from observation, that one infers that having an NaCl lattice structure is a subproperty of being soluble. Similarly, within the context of suitable theories about the nature of sound and secondary qualities, one might well be able to infer from sound's being audible to its being noneternal. Diṅnāga, in spite of his brilliance and originality, could not quite free himself from the old model of inference from sampling. His inclusion of the second condition was a concession to this old tradition. He should have dropped it. Later Buddhists, beginning with Dharmakīrti, did just that – they effectively dropped the second condition by adopting the reading of it that makes it logically equivalent to the third.

Diṅnāga's insistence that any acceptible inference should be accompanied by both positive and negative supporting examples provoked the Naiyāyika Uddyotakara to criticise and expand the Wheel. Uddyotakara points out that there are sound patterns of inference in which either the class of homologues or the class of heterologues is empty. These he calls the 'universally negative' (*kevala-vyatikekin*) and 'universally positive' (*kevalānvayin*) inferences. We now have a wheel with sixteen possible cases (Figure 4):

Here, 'o' means that the class (sp or vp) is empty. An example of a sound 'universally positive' inference might be: "This exists because I can see it". There

		vp +	−	±	o
sp	+		good		good
	-				
	±		good		good
	0		good		

Figure 4.

are no heterologues, because there are no things which do not exist, and so there are no negatively supporting examples. Nevertheless, we should recognise the acceptibility such an inference. Examples of 'universally negative' inferences are more difficult to find. The later Nyāya link such inferences with their theory of definition, considering such examples as "Cows are distinct from non-cows, because they have dewlap'. There are no objects which are distinct from non-cows except for cows, and hence no homologues. But the inference might have significance, for it tells us that the property of having dewlap serves to distinguish cows from non-cows, and hence can be used as a definition of cowhood[38].

2.3 Arguments from effect, essence and non-observation

Dharmakīrti (AD 600-660) offers a substantive account of the conditions under which the observation of a sample warrants extrapolation. His claim is that this is so if the reason property is one of three types: an 'effect' reason (*kārya-hetu*), a natural reason (*svabhāva-hetu*), or a reason based on nonobservation (*anupalabdhi-hetu*).[39]

In each case, the presence of the reason in some sense *necessitates* the presence of the target. An effect-reason is a property whose presence is *causally* necessitated by the presence of the target property – for example, inferring that the mountain has fire on it, because of smoke above it. The reason-target relation is a causal relation. Clearly one can, and later philosophers[40] indeed did, extend this to cover other species of causal inference, such as cases when reason and target are both effects of a common cause. The generalisation 'night follows day' is true,

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[38]For further discussion, see B. K. Matilal, "Introducing Indian Logic", in Matilal (1998), reprinted in Jonardon Ganeri ed., (*Indian Logic: A Reader*

[39]Dharmakīrti, *Nyāyabindu* II 11 12.

[40]See Mokṣākaragupta's eleventh century *Tarkabhāṣā* or *Language of Reason.* Yuichi Kajiyama, *An Introduction to Buddhist Philosophy: An Annotated Translation of the Tarkabhāṣā of Mokṣākaragupta*, Memoirs of the Faculty of Letters (Kyoto) 10 (1966), pp. 74–76.

not because day causes night but because both day and night are caused by the rotation of the earth. An example often cited is the inference of lemon-colour from lemon-taste, when both are products of the same cause, viz. the lemon itself. Still another example is the inference of ashes from smoke: ashes and smoke are both effects of fire. Such an inference has two steps. First, fire is inferred from smoke; second, ash is inferred from fire. The second step, in which we infer an effect from its cause, is possible only because ash is a necessary effect of fire.

A natural reason is one whose presence *metaphysically* necessitates that of the target property, for example the inference that something is a tree because it is a *śiṃśapā* (a species of tree). Dharmakīrti appears to regard the law "all *śiṃśapā*s are trees" as necessarily true, even if its truth has to be discovered by observation, and thus to anticipate the idea that there are *a posteriori* necessities.[41] He states, surprisingly, that the reason-target relation in such inferences is the relation of identity. Why? Perhaps his idea is that the two properties being-a-*śiṃśapā* and being-a-tree are token-identical, for the particular tree does not have two distinct properties, being-a-*śiṃśapā* and a separate property being-a-tree, any more than something which weighs one kilogramme has two properties, having-weight and having-weight-one-kilogramme. The properties as types are distinct, but their tokens in individual objects are identical. Trope-theoretically, the point can easily be understood. The very same trope is a member of two properties, one wider in extension than the other, just as the class of blue tropes is a subset of the class of colour tropes. But a blue object does not have two tropes – one from the class of blue tropes and one from the class of colour tropes. It is the self-same trope.

Is absence of evidence evidence of absence? According to Dharmakīrti, nonobservation sometimes proves absence: my failure to see an object, when all the conditions for its perception are met, is grounds for an inference that it is not here. The pattern of argument such inferences exemplify was known to the medievals as *argumentum ad ignorantiam*, or an 'argument from ignorance.' The pattern occurs whenever one infers that p on the grounds that there is no evidence that p is false. Dharmakīrti states that the argument depends on the object's being perceptible, i.e. that all the conditions for its perception (other than its actual presence) are met in the given situation. Douglas Walton, in a major study of arguments from ignorance,[42] claims that they depend for their validity on an implicit conditional premise — if p were false, p would be known to be false. The characteristics of an argument from ignorance are then a 'lack-of-knowledge' premise — it is not known that not-p, and a 'search' premise — if p were false, it would be known that not-p. The underlying hidden premise mentioned by Dharmakīrti seems to be exactly the one Walton gives: if the object were here, one would see it. The necessity here is subjunctive. The argument has a presumptive status - one has a right to presume the conclusion to be true to the extent that one has searched for and failed to find counter-evidence. It is this idea that is strikingly absent in Diṅnāga. Warranted extrapolation depends not on the mere nonobservation of counterexamples, but

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[41] *Pramāṇavārttika* I, 39–42.
[42] Walton (1996).

on one's failing to find them in the course of a suitably extensive search.

In each of the three cases, the universal relation between reason and target is a relation not of coincidence but of necessity - causal, metaphysical or subjunctive. Dharmakīrti's solution to the problem of induction, then, is to claim that observation supports a generalisation only when that generalisation is lawlike or necessary. In this, I think he anticipates the idea that the distinction between lawlike and accidental generalisations is that only the former support the *counterfactual* 'if the reason property were instantiated here, so would be the target property'. In such a context, let us note, the observation of even a single positive example might sometimes be sufficient to warrant the extrapolation: I infer that any mango is sweet having tasted a single mango; I infer that any fire will burn having once been burnt.

Extrapolation is warranted when the reason-target is lawlike, but it does not follow that the extrapolator must know that it is lawlike. What Dharmakīrti has succeeded in doing is to describe the conditions under which extrapolation *works* — the conditions under which one's actions, were they to be in accordance with the extrapolation, would meet with success. It is a description of the type of circumstance in which extrapolation is rewarded (i.e. *true* — if, as it seems, Dharmakīrti has a pragmatic theory of truth[43]). As to how, when or whether one can know that one is in such a circumstance, that is another problem altogether and not one that Dharmakīrti has necessarily to address. For a general theory of rationality issues in conditions of the form 'in circumstances C, it is rational to do ϕ' or 'in circumstances C, it is rational to believe p'. And this is precisely the form Dharmakīrti's conditions take.

2.4 The Jaina reformulation of the triple condition

Diṅnāga had argued that there are three marks individually necessary and jointly sufficient for the warranted extrapolation from reason to target (§2.2). They are (1) that the reason be present in the site of the extrapolation, (2) that the reason be present (only) in what is similar to the target, and (3) that the reason be absent in what is dissimilar to the target. The second of these conditions is, arguably, equivalent to the third, which asserts that the reason property is absent when the target property is absent. That was supposed to capture the idea of a 'no counterexample' condition, according to which an extrapolation is warranted just in case there is nothing in which the reason is present but not the target. What happens to this account if one allows, as the Jaina logicians do, that a property and its absence be compossible in a single object?[44] What happens is that the three marks cease to be sufficient for warranted extrapolation. In particular, the third mark no longer captures the idea behind the 'no counterexample' condition.

[43]Shoryu Katsura, "Dharmakīrti's Concept of Truth," *Journal of Indian Philosophy* 12 (1984), pp. 213–235. Georges B. J. Dreyfus, *Recognizing Reality: Dharmakīrti's Philosophy and its Tibetan Interpretations* (Albany: State University of New York Press, 1997), chapter 17.

[44]See B. K. Matilal, *The Central Philosophy of Jainism* (Ahmedabad: L. D. Institute of Indology, 1981).

For now the absence of the reason property in a place where the target is absent does not preclude its *presence* there too! So the third mark can be satisfied and yet there still be counterexamples — cases of the presence of the reason together with the absence of the target.

The Jainas indeed claim that the three marks are neither necessary nor sufficient for warranted extrapolation. Their response is to substitute for the three marks a new, single, mark. It is clear that if the presence and absence of a property are compossible, then a distinction needs to be drawn between absence and nonpresence. The first is consistent with the presence of the property; the second is not. Early post-Diṅnāga Jainas like Akalaṅka and Siddhasena described the new mark in quasi-Buddhistic terms, as 'no presence without' (*a-vinā-bhāva*) — i.e. no presence of the reason without the target. Thus Akalaṅka:[45]

> An extrapolation is a cognition of what is signified from a sign known to have the single mark of no presence without the target (*sādhyāvinābhāva*). Its result is blocking and other cognitions.
>
> The relata of the causality and identity relations cannot be cognised without the suppositional knowledge (*tarka*) of their being impossible otherwise, [which is] the proof that this is the single mark even without those relations. Nor is a tree the own-nature (*svabhāva*) or the effect (*kārya*) of such things as shade. And there is no disagreement here.

There is an obvious reference to and criticism of Dharmakīrti here,[46] and also a mention of the important idea, which we have already discussed, that presumptive argumentation (*tarka*) is what gives us knowledge of the universal generalisations grounding extrapolations. The crucial difference from the Buddhists is in the meaning of 'no presence'. For the Jainas, it has to stand for nonpresence and not for absence. That led them to reformulate the reason-target relation as a relation of necessitation. Siddhasena:

The mark of a reason is 'being impossible otherwise' (*anyathānupannatva*) [Nyāyāvat 22].

Vādideva Sūri gives the developed Jaina formulation:

> A reason has a single mark, 'determined as impossible otherwise'. It does not have three marks, for fallacies are then still possible [Pramāṇanayatattālokālaṃkāra 3.11-12].

The idea is that the reason *cannot* be present if the target is not. It is impossible for the reason to be present otherwise than if the target is present. The presence of the reason necessitates the presence of the target.

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[45] *Laghīyastraya*, verse 12.

[46] On Akalaṇka on Dharmakīrti: Nagin J. Shah, Akalaṇka's Criticism of Dharmakīrti's Philosophy (Ahmedabad: L. D. Institute, 1967), pp. 267–270.

I said that Diṅnāga's three marks are, for the Jainas, neither necessary nor sufficient. They are not sufficient because they permit extrapolation when the reason is both present and absent, and the target nonpresent. On what grounds are they thought not to be necessary? The theory of extrapolation as developed first by the early Naiyāyikas and then by Diṅnāga has a built-in simplifying assumption. The assumption is that extrapolation is always a matter of inferring from the presence of one property in an object to the presence of a second property *in that same object*. But that assumption excludes many intuitively warranted extrapolations. The main examples considered by the Jainas are: (i) the *śakaṭa* star-group will rise because *kṛttikā* star-group has risen; (ii) the sun is above the horizon because the earth is in light; (iii) there is a moon in the sky because there is a moon in the water.

These examples are said to prove that the first of Diṅnāga's three marks, that the reason property is present in the site, is not a necessary condition on warranted extrapolation. And yet, while it is certainly desirable to broaden the reach of the theory to cover new patterns of extrapolative inference, it is not very clear what these examples show. What is the underlying generalisation? What are the similar and dissimilar examples? In the first case, the extrapolation seems to be grounded in the universal generalisation 'whenever the *kṛttikā* arises, so too does the *śakaṭa*.' But then there is indeed a single site of extrapolation — the present time. The inference is: the *śakaṭa* will rise now because *kṛttikā* has now risen. A similar point could be made about the second example. There seems indeed to be an implicit temporal reference in both of the first two cases, an extrapolation grounded in a universal generalisation over times.

The third case is more convincing, yet here too one might try to discern a common site. For the true form of the extrapolation is: the moon is in the sky because *it* is reflected in the water, an extrapolation grounded in a universal generalisation of the form 'objects cause their own reflections'. Certainly, however, there are patterns of extrapolation for which the 'single site' condition does not hold. If, for example, one can find a universal generalisation of the form '$\forall x \exists y (Fx \rightarrow Gy)$', then from '$\exists x Fx$' one can infer '$\exists x Gx$'. Perhaps *this* is the pattern of extrapolation the Jainas intend to exemplify with their example of a sky-moon and a water-moon. If so, it is represents an important criticism of a simplifying, but in the end also restricting, assumption in the classical theory of extrapolation.

3 JAINA CONTRIBUTIONS IN INDIAN LOGIC: THE LOGIC OF ASSERTION

3.1 *Rationality and Consistency*

What is the rational response when confronted with a set of propositions each of which we have some reason to accept, and yet which taken together form an inconsistent class? This was, in a nutshell, the problem addressed by the Jaina logicians of classical India, and the solution they gave is, I think, of great interest, both for

what it tells us about the relationship between rationality and consistency, and for what we can learn about the logical basis of philosophical pluralism. The Jainas claim that we can continue to reason in spite of the presence of inconsistencies, and indeed construct a many-valued logical system tailored to the purpose. My aim in this chapter is to offer an interpretation of that system and to try to draw out some of its philosophical implications.

There was in classical India a great deal of philosophical activity. Over the years, certain questions came to be seen as fundamental, and were hotly contested. Are there universals? Do objects endure or perdure? Are there souls, and, if so, are they eternal or non-eternal entities? Do there exist wholes over and above collections of parts? Different groups of philosophers offered different answers to these and many other such questions, and each, moreover, was able to supply plausible arguments in favour of their position, or to offer a world-view from which their particular answers seemed true. The body of philosophical discourse collectively contained therefore, a mass of assertions and contradictory counter-assertions, behind each of which there lay a battery of plausible arguments. Such a situation is by no means unique to philosophical discourse. Consider, for instance, the current status of physical theory, which comprises two sub-theories, relativity and quantum mechanics, each of which is extremely well supported, and yet which are mutually inconsistent. The same problem is met with in computer science, where a central notion, that of putting a query to a data-base, runs into trouble when the data-base contains data which is inconsistent because it is coming in from many different sources. For another example of the general phenomenon under discussion, consider the situation faced by an investigator using multiple-choice questionnaires, when the answers supplied in one context are in conflict with those supplied in another. Has the interrogee said 'yes' or 'no' to a given question, when they said 'yes' under one set of conditions but 'no' under another? Do their answers have any value at all, or should we simply discard the whole lot on account of its inconsistency? Perhaps the most apposite example of all is the case of a jury being presented with the evidence from a series of witnesses. Each witness, we might suppose, tells a consistent story, but the total evidence presented to the jury might itself well be inconsistent.

The situation the Jainas have in mind is one in which a globally inconsistent set of propositions, the totality of philosophical discourse, is divided into subsets, each of which is internally consistent. Any proposition might be supported by others from within the same subset. At the same time, the negation of that proposition might occur in a distinct, though possibly overlapping subset, and be supported by other propositions within it. Each such consistent sub-set of a globally inconsistent discourse, is what the Jainas call a "standpoint" (*naya*). A standpoint corresponds to a particular philosophical perspective.

Let us say that a proposition is *arguable* if it is assertible within some standpoint, i.e. if it is a member of a mutually supporting consistent set of propositions. The original problem posed was this: what is the rational reaction to a class of propositions, each of which is, in this sense, arguable, yet which is globally

inconsistent? It seems that there are three broad types of response. The first, which I will dub *doctrinalism*, is to say that it will always be possible, in principle, to discover which of two inconsistent propositions is true, and which is false. Hence our reaction should be to reduce the inconsistent set to a consistent subset, by rejecting propositions which, on close examination, we find to be unwarranted. This is, of course, the ideal in philosophical debate, but it is a situation we are rarely if ever in. The problem was stipulated to be one such that we cannot decide, as impartial observers, which of the available standpoints, if any, is correct. If doctrinalism were the only option, then we would have no choice but to come down in favour of one or other of the standpoints, basing our selection, perhaps on historical, cultural, or sociological considerations, but not on logical ones.

A second response is that of *scepticism*. Here the idea is that the existence both of a reason to assert and a reason to reject a proposition itself constitutes a reason to deny that we can justifiably either assert or deny the proposition. A justification of a proposition can be defeated by an equally plausible justification of its negation. This sceptical reaction is at the same time a natural and philosophically interesting one, and indeed has been adopted by some philosophers, notably *Nāgārjuna* in India and the Pyrrhonic sceptics as reported by Sextus Empiricus. Sextus, indeed states as the first of five arguments for scepticism, that philosophers have never been able to agree with one another, not even about the criteria we should use to settle controversies.

The third response is that of *pluralism*, and this is the response favoured by the Jainas. The pluralist finds some way conditionally to assent to each of the propositions, and she does so by recognising that the justification of a proposition is internal to a standpoint. In this way, the Jainas try "to establish a rapprochement between seemingly disagreeing philosophical schools"[47], thereby avoiding the dogmatism or "one-sidedness" from which such disagreements flow. Hence another name for their theory was *anekāntavāda*, the doctrine of "non-one-sidedness".[48]

In spite of appearances to the contrary, the sceptic and the pluralist have much in common. For although the sceptic rejects all the propositions while the pluralist endorses all of them, they both deny that we can solve the problem by privileging just one position, i.e. by adopting the position of the doctrinalist. (It seems, indeed, that scepticism and pluralism developed in tandem in India, both as critical reactions to the system-based philosophical institutions.) Note too that both are under pressure to revise classical logic. For the sceptic, the problem is with the law of excluded middle, the principle that for all p, either p or $\neg p$. The reason this is a problem for the sceptic is that she wishes to reject each proposition p without being forced to assent to its negation $\neg p$. The pluralist, on the other hand, has trouble with a different classical law, the law of non-contradiction, that for all p, it is not the case both that p and that $\neg p$, for she wishes to assent both

[47] B.K. Matilal, *The Central Philosophy of Jainism*, Calcutta University Press, Calcutta 1977:61.

[48] For a good outline of these aspects of Jaina philosophical theory, see B.K. Matilal, *The Central Philosophy of Jainism*, and P. Dundas, *The Jains*, Routledge Press, London 1992.

to the proposition *p* and to its negation. While a comparative study of the two responses, sceptical and pluralist, would be of interest, I will here confine myself to developing the version of pluralism developed by the Jainas, and discussing the extent to which their system becomes paraconsistent. It is very often claimed that the Jainas 'embrace' inconsistency, but I will be arguing that this is not so, that we can understand their system by giving it a less strongly paraconsistent reading.

3.2 Jaina seven-valued logic

The Jaina philosophers support their pluralism by constructing a logic in which there are seven distinct semantic predicates (*bhaṅgī*), which, since they attach to sentences, we *might* think of as truth-values (for a slightly different interpretation, see Ganeri 2001, chapter 5). I will first set out the system following the mode of description employed by the Jainas themselves, before attempting to reconstruct it in a modern idiom. I will follow here the twelfth century author Vādideva Sūri (1086 1169 A.D.), but similar descriptions are given by many others, including Prabhācandra, Malliṣena and Samantabhadra. This is what Vādideva Sūri says (*Pramāṇa-naya-tattvālokālaṅkāraḥ*, chapter 4, verses 15–21):[49]

> The seven predicate theory consists in the use of seven claims about sentences, each preceded by "arguably" or "conditionally" (*syāt*), [all] concerning a single object and its particular properties, composed of assertions and denials, either simultaneously or successively, and without contradiction. They are as follows:
>
> 1. Arguably, it (i.e. some object) exists (*syād asty eva*). The first predicate pertains to an assertion.
> 2. Arguably, it does not exist (*syān nāsty eva*). The second predicate pertains to a denial.
> 3. Arguably, it exists; arguably, it doesn't exist (*syād asty eva syān nāsty eva*). The third predicate pertains to successive assertion and denial.
> 4. Arguably, it is 'non-assertible' (*syād avaktavyam eva*). The fourth predicate pertains to a simultaneous assertion and denial.
> 5. Arguably, it exists; arguably it is non-assertible (*syād asty eva syād avaktavyam eva*). The fifth predicate pertains to an assertion and a simultaneous assertion and denial.
> 6. Arguably, it doesn't exist; arguably it is non-assertible (*syān nāsty eva syād avaktavyam eva*). The sixth predicate pertains to a denial and a simultaneous assertion and denial.

[49] Vādideva Sūri: 1967, *Pramāṇa-naya-tattvālokālaṃkāra*, ed. and transl. H. S. Battacharya, Jain Sahitya Vikas Mandal, Bombay.

7. Arguably, it exists; arguably it doesn't exist; arguably it is non-assertible (*syād asty eva syān nāsty eva syād avaktavyam eva*). The seventh predicate pertains to a successive assertion and denial and a simultaneous assertion and denial.

The structure here is simple enough. There are three basic truth-values, true (t), false (f), and non-assertible (u). There is also some means of combining basic truth-values, to form four further compound values, which we can designate tf, tu, fu and tfu. There is a hint too that the third basic value is itself somehow a product of the first two, although by some other means of combination - hence the talk of simultaneous and successive assertion and denial. Thus, in Jaina seven valued logic, all the truth-values are thought to be combinations in some way or another of the two classical values.

There is, however, a clear risk that the seven values in this system will collapse trivially into three. For if the fifth value, tu, means simply "true and true-and-false", how is it distinct from the fourth value, u, "true-and-false"? No reconstruction of the Jaina system can be correct if it does not show how each of the seven values is distinct. The way forward is to pay due attention to the role of the conditionalising operator "arguably" (*syāt*). The literal meaning of "*syāt*" is "perhaps it is", the optative form of the verb "to be". The Jaina logicians do not, however, use it in quite its literal sense, which would imply that no assertion is not made categorically, but only as a possibility-claims. Instead, they use it to mean "from a certain standpoint" or "within a particular philosophical perspective". This is the Jaina pluralism: assertions are made categorically, but only from within a particular framework of supporting assertions. If we let the symbol "∇" represent "*syāt*", then the Jaina logic is a logic of sentences of the form "∇p", a logic of conditionally justified assertions. As we will see, it resembles other logics of assertion, especially the ones developed by Jaśkowski[50] and Rescher[51]

The first three of the seven predications now read as follows:

1. $|p| = t$ iff ∇p.

In other words, p is true iff it is arguable that p. We are to interpret this as saying that there is some standpoint within which p is justifiably asserted. We can thus write it as

1. $|p| = t$ iff $\exists\sigma\ \sigma : p$,

where "$\sigma : p$" means that p is arguable from the standpoint σ. For the second value we may similarly write,

2. $|p| = f$ iff $\nabla\ \ \neg p$.

[50] Jaśkowski, S.: 1948, "Propositional calculus for contradictory deductive systems"; English translation in *Studia Logica* **24**: 143 - 157 (1969).

[51] Rescher, N.: 1968, *Topics in Philosophical Logic*, Reidel, Dordrecht.

That is,

$$|p| = f \text{ iff } \exists\sigma\ \sigma : \neg p.$$

The third value is taken by those propositions whose status is controversial, in the sense that they can be asserted from some standpoints but their negations from others. These are the propositions which the Jainas are most concerned to accommodate. Thus

3. $|p| = tf$ iff $|p| = t$ & $|p| = f$.

I.e.

$$|p| = tf \text{ iff } \nabla p \ \&\ \nabla\neg p,$$

or again

$$|p| = tf \text{ iff } \exists\sigma\ \sigma : p \ \&\ \exists\sigma\ \sigma : \neg p.$$

This way of introducing a new truth-value, by combining two others, may seem a little odd. I think, however, that we can see the idea behind it if we approach matters from another direction. Let us suppose that every standpoint is such that for any given proposition, either the proposition or its negation is assertible from within that standpoint. Later, I will argue that the Jainas did not want to make this assumption, and that this is what lies behind their introduction of the new truth-value "non-assertible". But for the moment let us make the assumption, which is tantamount to supposing that every standpoint is '*optimal*', in the sense that for any arbitrary proposition, it either supplies grounds for accepting it, or else grounds for denying it. There are no propositions about which an optimal standpoint is simply indifferent. Now, with respect to the totality of actual optimal standpoints, a proposition can be in just one of three states: either it is a member of every optimal standpoint, or its negation is a member of every such standpoint, or else it is a member of some, and its negation of the rest. If we number these three states, 1, 2 and 3, and call the totality of all actual standpoints, Σ, then the value of any proposition with respect to Σ is either 1, 2 or 3. The values 1, 2 and 3 are in fact the values of a three-valued logic, which we can designate **M3**. There is a correspondence between this logic and the system introduced by the Jainas (**J3**, say). The idea, roughly is that a proposition has the value 'true' iff it either has the value 1 or 3, it has the value 'false' iff it either has the value 2 or 3, and it has the value 'tf' iff it has the value 3. Hence the three values introduced by the Jainas represent, albeit indirectly, the three possible values a proposition may take with respect to the totality of optimal standpoints.

Before elaborating this point further, we must find an interpretation for the Jainas' fourth value "non-assertible". Bharucha and Kamat offer the following analysis of the fourth value:

> The fourth predication consists of affirmative and negative statements made simultaneously. Since an object X is incapable of being expressed in terms of existence and non-existence at the same time, even allowing for Syād, it is termed 'indescribable'. Hence we assign to the

> fourth predication ... the indeterminate truth-value I and denote the statement corresponding to the fourth predication as $(p\&\neg p)$.[52]

Bharucha and Kamat's interpretation is equivalent to

4. $|p| = u$ iff $\nabla(p\&\neg p)$,

that is

$$|p| = u \text{ iff } \exists\sigma\ \sigma : (p\&\neg p).$$

Thus, for Bharucha and Kamat, the Jaina system is paraconsistent because it allows for standpoints in which contradictions are justifiably assertible. This seems to me to identify the paraconsistent element in the Jaina theory in quite the wrong place. For while there may be certain sentences, such as the Liar, which can justifiably be both asserted and denied, this cannot be the case for the wide variety of sentences which the Jainas have in mind, sentences like "There exist universals" and so on. Even aside from such worries, the current proposal has a technical defect. For what now is the fifth truth-value, tu? If Bharucha and Kamat are right then it means that there is some standpoint from which 'p' can be asserted, and some from which '$p\&\neg p$' can be asserted. But this is logically equivalent to u itself. The Bharucha and Kamat formulation fails to show how we get to a seven-valued logic.

Another proposed interpretation is due to Matilal. Taking at face-value the Jainas' elaboration of the fourth value as meaning "simultaneously both true and false", he says

> the direct and unequivocal challenge to the notion of contradiction in standard logic comes when it is claimed that the same proposition is both true and false at the same time in the same sense. This is exactly accomplished by the introduction of the [fourth] value - "Inexpressible", which can also be rendered as paradoxical.[53]

Matilal's intended interpretation seems thus to be

4. $|p| = u$ iff $\nabla(p, \neg p)$,

$$\text{i.e. } |p| = u \text{ iff } \exists\sigma(\sigma : p\&\sigma : \neg p).$$

Matilal's interpretation is a little weaker than Bharucha and Kamat, for he does not explicitly state that the conjunction '$p\&\neg p$' is asserted, only that both conjuncts are. Admittedly, the difference between Matilal and Bharucha and Kamat is very slight, and indeed only exists if we can somehow make out the claim that

[52] Bharucha, F. and Kamat, R. V.: 1984, "*Syādvāda* theory of Jainism in terms of deviant logic", *Indian Philosophical Quarterly*, **9**: 181 - 187; 183.

[53] Matilal, B. K.: 1991, "*Anekānta*: both yes and no?", *Journal of Indian Council of Philosophical Research*, **8**: 1 12; 10.

both a proposition and its negation are assertible without it being the case that their conjunction is. For example, we might think that the standpoint of physical theory can be consistently extended by including the assertion that gods exists, and also by including the assertion that gods do not exist. It would not follow that one could from any standpoint assert the conjunction of these claims. Yet whether there is such a difference between Matilal's position and that of Bharucha and Kamat is rather immaterial, since Matilal's proposal clearly suffers from the precisely the same technical defect as theirs, namely the lack of distinctness between the fourth and fifth values.

Tere is another interpretation, one which gives an intuitive sense to the truth-value "non-assertible", sustains the distinctness of each of the seven values, but does not require us to abandon the assumption that standpoints are internally consistent. Recall that we earlier introduced the idea of an optimal standpoint, by means of the assumption that for every proposition, either it or its negation is justifiably assertible from within the standpoint. Suppose we now retract that assumption, and allow for the existence of standpoints which are just neutral about the truth or falsity of some propositions. We can then introduce a new value as follows:

4. $|p| = u \iff \exists\sigma(\neg(\sigma : p)\&\neg(\sigma : \neg p))$.

Neither the proposition nor its negation is assertible from the standpoint. For example, neither the proposition that happiness is a virtue nor its negation receives any justification from the standpoint of physical theory. We have, in effect, rejected a commutativity rule, that if it not the case that 'p' is assertible from a standpoint σ then '$\neg p$' is assertible from σ and vice versa $[\neg(\sigma : p) \iff (\sigma : \neg p)]$. Our new truth-value, u, is quite naturally called "non-assertible", and it is clear that the fifth value, tu, the conjunction of t with u, is not equivalent simply with u. The degree to which the Jaina system is paraconsistent is, on this interpretation, restricted to the sense in which a proposition can be tf, i.e. both true and false because assertible from one standpoint but deniable from another. It does not follow that there are standpoints from which contradictions can be asserted.

Why have so many writers on Jaina logic have felt that Jaina logic is paraconsistent in the much stronger sense. The reason for this belief is the account which some of the Jainas themselves give of the meaning of their third basic truth-value, "non-assertible". As we saw in the passage from Vādideva Sūri, some of them say that a proposition is non-assertible iff it is arguably both true and false *simultaneously*, as distinct from the truth value tf, which is *successively* arguably true and arguably false. We are interpreting the Jaina distinction between successive and simultaneous combination of truth-values in terms of a scope distinction with the operator "arguably". One reads "arguably ($t\&f$)", the other "(arguably t) & (arguably f)". If this were the correct analysis of the fourth truth-value, then Jaina logic would indeed be strongly paraconsistent, for it would be committed to the assumption that there are philosophical positions in which contradictions are rationally assertible. Yet while such an interpretation is, on the face of it,

the most natural way of reading Vādideva Sūri's elaboration of the distinction between the third and fourth values, it if far from clear that the Jaina pluralism really commits them to paraconsistency in this strong form. Their goal is, to be sure, to reconcile or synthesise mutually opposing philosophical positions, but they have no reason to suppose that a single philosophical standpoint can itself be inconsistent. Internal consistency was, in classical India, the essential attribute of a philosophical theory, and a universally acknowledged way to undermine the position of one's philosophical opponent was to show that their theory contradicted itself. The Jainas were as sensitive as anyone else to allegations that they were inconsistent, and strenuously denied such allegations when made. I have shown that it is possible to reconstruct Jaina seven-valued logic in a way which does not commit them to a strongly paraconsistent position.

The interpretation I give to the value "non-assertible" is quite intuitive, although it does not mean "both true and false *simultaneously*". My interpretation, moreover, is supported by at least one Jaina logician, Prabhācandra. Prabhācandra, who belongs to the first part of the ninth century C.E., is one of the few Jainas directly to address the question of why there should be just seven values. What he has to say is very interesting:

> (Opponent:) Just as the values 'true' and 'false', taken successively, form a new truth-value 'true-false', so do the values 'true' and 'true-false'. Therefore, the claim that there are seven truth-values is wrong.
>
> (Reply:) No: the successive combination of 'true' and 'true-false' does not form a new truth-value, because it is impossible to have 'true' twice. ... In the same way, the successive combination of 'false' and 'true-false' does not form a new truth-value.
>
> (Opponent:) How then does the combination of the first and the fourth, or the second and the fourth, or the third and the fourth, form a new value?
>
> (Reply:) It is because, in the fourth value "non-assertible", there is no grasp of truth or falsity. In fact, the word "non-assertible" does not denote the simultaneous combination of truth and falsity. What then? What is meant by the truth-value "non-assertible" is that it is impossible to say which of 'true' and 'false' it is.[54]

This passage seems to support the interpretation offered above. When talking about the "law of non-contradiction" in a deductive system, we must distinguish between two quite different theses: (a) the thesis that "$\neg(p \& \neg p)$" is a theorem in the system, and (b) the thesis that it is not the case that both 'p' and '$\neg p$' are theorems. The Jainas are committed to the first of these theses, but reject the second. This is the sense in which it is correct to say that the Jainas reject the "law of non-contradiction".

[54] Prabhācandra: 1941, *Prameyakamalamārtaṇḍa*, ed. M. K. Shastri, Nirnayasagar Press, Bombay; p. 683 line 7 ff.

I showed earlier that when we restrict ourselves to optimal standpoints, the total discourse falls into just one of three possible states with respect to each system. The Jainas have a *seven*-valued logic because, if we allow for the existence of non-optimal standpoints, standpoints which are just neutral with respect to some propositions, then, for each proposition, p say, the total discourse has exactly seven possible states. They are as follows:

1. p is a member of every standpoint in Σ.
2. $\neg$p is a member of every standpoint in Σ.
3. p is a member of some standpoints, and $\neg p$ is a member of the rest.
4. p is a member of some standpoints, the rest being neutral.
5. $\neg p$ is a member of some standpoints, the rest being neutral.
6. p is neutral with respect to every standpoint.
7. p is a member of some standpoints, $\neg p$ is a member of some other standpoints, and the rest are neutral.

Although Jainas do not define the states in this way, but rather via the possible combinations of the three primitive values, t, f and u, it is not difficult to see that the two sets map onto one another, just as they did before. Thus $t = (1, 3, 4, 7)$, $f = (2, 3\ 5, 7)$, $tf = (3, 7)$, and so on.

Using many-valued logics in this way, it should be noted, does not involve any radical departure from classical logic. The Jainas stress their commitment to bivalence, when they try to show, as Vādideva Sūri did above, that the seven values in their system are all products of combining two basic values. This reflects, I think, a commitment to bivalence concerning the truth-values of propositions themselves. The underlying logic *within* each standpoint is classical, and it is further assumed that each standpoint or participant is internally consistent. The sometimes-made suggestion[55] that sense can be made of many-valued logics if we interpret the assignment of non-classical values to propositions via the assignment of classical values to related items is reflected here in the fact that the truth-value of any proposition p (i.e. $|p|$) has two values, the status of p with respect to standpoint σ ('$|p|_\sigma$') derivatively has three values, and the status of p with respect to a discourse Σ ('$|p|_\Sigma$'), as we have just seen, has seven.

Consider again the earlier example of a jury faced with conflicting evidence from a variety of witnesses. The Jainas wouldn't here tell us 'who dun it', for they don't tell us the truth-value of any given proposition. What they give us is the means to discover patterns in the evidence, and how to reason from them. For example, if one proposition is agreed on by all the witnesses, and another is agreed on by some but not others, use of the Jaina system will assign different values to the two

[55] Haack, S.: 1974, *Deviant Logic*, Cambridge University Press, Cambridge; 64.

propositions. The Jainas, as pluralists, do not try to judge which of the witnesses is lying and which is telling the truth; their role is more like that of the court recorder, to present the totality of evidence in a maximally perspicuous form, one which still permits deduction from the totality of evidence.

So far so good. But there is another worry now, one which strikes at the very idea of using a many-valued logic as the basis for a logic of discourse. For, when we come to try and construct truth-tables for the logical constants in such a logic, we discover that the logic is not *truth-functional.* That is to say, the truth-value of a complex proposition such as '$p\&q$', is not a function solely of the truth-values of the constituent propositions 'p' and 'q'. To see this, and to begin to find a solution, I shall need briefly to describe the work of the Polish logician, Jaśkowski, who was the founder of discursive logics in the West, and whose work, in motivation at least, provides the nearest contemporary parallel to the Jaina theory.

3.3 Jaśkowski and the Jainas

Philosophical discourse is globally inconsistent, since there are many propositions to which some philosophers assent while others dissent. The Jainas therefore develop a logic of assertions-made-from-within-a-particular-standpoint, and note that an assertion can be both arguably true, i.e. justified by being a member of a consistent philosophical position, and at the same time be arguably false, if its negation is a member of some other consistent philosophical standpoint. This move is quite similar to that of the founder of inconsistent logics, Jaśkowski, who developed a "discussive logic" in which a proposition is said to be 'discussively true' iff it is asserted by some member of the discourse.

Jaśkowski motivates his paper "Propositional Calculus for Contradictory Deductive Systems" with two observations. The first is that

> any vagueness of the term a can result in a contradiction of sentences, because with reference to the same object X we may say that "X is a" and also "X is not a", according to the meanings of the term a adopted for the moment,

the second is that

> the evolution of the empirical sciences is marked by periods in which the theorists are unable to explain the results of experiments by a homogeneous and consistent theory, but use different hypotheses, which are not always consistent with one another, to explain the various groups of phenomena.[56]

He then introduces an important distinction between two properties of deductive systems. A deductive system is said to be *contradictory* if it includes pairs of theorems A and $\neg A$ which contradict each other. It is *over-complete*, on the

[56] Jaśkowski, S.: 1948, "Propositional calculus for contradictory deductive systems"; English translation in *Studia Logica*, **24**: 143 - 157 (1969); 144.

other hand, if every well-formed formula is a theorem of the system. In classical logic, these two properties are conflated; hence the slogan "anything follows from a contradiction". The problem to which Jaśkowski addresses himself, therefore, is that of constructing a non-classical system which is contradictory but not over-complete. In classical logic, given two contradictory theses A, $\neg A$, we may deduce first that $A \& \neg A$, using the &-introduction or Adjunction Rule, $A, B \rightarrow A \& B$. Then, since $A \& \neg A$ iff $B \& \neg B$ for any arbitrary A and B, and since $B \& \neg B \rightarrow B$ from &-elimination or Simplification, $A \& B \rightarrow A$, it follows that B. More clearly:

1.	$A, \neg A$	
2.	$A \& \neg A$,	from 1 by Adjunction.
3.	$A \& \neg A$ iff $B \& \neg B$,	for any arbitrary A and B.
4.	$B \& \neg B \rightarrow B$,	by Simplification.
5.	$A \& \neg A \rightarrow B$,	from 3 and 4.
6.	B,	from 2 and 5 by Modus Ponens. ■

To get an inconsistent (contradictory but not over-complete) system, at least one step in this sequence must be broken. In Jaśkowski's new system, 'discursive logic', it is the Adjunction Rule which no longer holds. Jaśkowski considers the system in which many different participants makes assertions, each thereby contributing information to a single discourse. The best example, perhaps, is one already given, the evidence presented to a jury by witnesses at a trial. Jaśkowski then introduces the notion of discursive assertion, such that a sentence is discursively asserted if it is asserted by one of the participants in the discourse, and he notes that the operator "it is asserted by someone that..." is a modal operator for the semantics of which it should be possible to use an existing modal logic. Thus

A is a theorem of **D2** iff $\Diamond A$,

where **D2** is Jaśkowski's two-valued discursive logic, and "$\Diamond$" is the operator "someone asserts that...". For some reason, Jaśkowski chooses a strong modal system, **S5**, to give the semantics of this operator, but this is surely a mistake. The reason is that the **S5** modal principle '$A \rightarrow \Diamond A$' does not seem to hold for a discursive system, since there will be truths which no-one asserts. It would not be difficult, however, to use a weaker modal system than **S5**, for example $\mathbf{S2}^0$ or $\mathbf{S3}^0$, which lack the above principle, as the basis for **D2**. (The characteristic axiom of $\mathbf{S4}^0$, '$\Diamond\Diamond A \rightarrow \Diamond A$', does not seem to hold in a discursive system: it can be assertible from some standpoint that there is another standpoint in which p is assertible without there being such a standpoint). The point to note is that, in most modal systems, the Adjunction Rule fails, since it does not follow that the conjunction $A \& B$ is possible, even if A is possible and B is separately possible. And this too, is what we would expect from the discursive operator, for one participant may assert A, and another B, without there being anyone who asserts the conjunction. Jaśkowski therefore arrives at a system which is contradictory, since both A and ¬A can be theses, but, because it is non-adjunctive, is not over-complete.

3.4 The Logical Structure of the Jaina System

The parallels in motivation between Jaśkowski's discursive logic, and the Jaina system are unmistakable. There is, however, an important difference, to which I alluded earlier. Modal logics are not truth-functional; one cannot, for example, deduce the truth-value of '$\Diamond(A\&B)$' from the truth-values of '$\Diamond A$' and '$\Diamond B$'. And it seems for the same reason that a discursive logic cannot be truth-functional either. Suppose, for example, that we have two propositions A, and B, both of which are assertible from (possibly distinct) standpoints, and hence both true in the Jaina system. What is the truth-value of $A\&B$? It seems that this proposition could be either true, false, or both.

It is possible to offer a defence of the Jaina position here. For simplicity, let us restrict ourselves to the Jaina system with only optimal standpoints and just three truth-values. If my suggested defence works here, its extension to the full Jaina system **J7**, would not be especially problematic. Consider again the three-valued logic, **M3**, whose values were defined as follows:

$$|p| = 1 \text{ iff } \forall\sigma\ \sigma : p.$$
$$|p| = 2 \text{ iff } \forall\sigma\ \sigma : \neg p$$
$$|p| = 3 \text{ iff } \exists\sigma\ \sigma : p\ \&\ \exists\sigma\ \sigma : \neg p.$$

These correspond to the three possible states of a totality of optimal standpoints. When we try to construct the truth-table for conjunction in such a system, we find that it is non-truth-functional. Thus, consider the truth-value of '$p\&q$', when $|p| = |q| = 3$. Here, $|p\&q|$ might itself be 3, but it might also be 2. Thus, the truth-value of the conjunction is not uniquely determined by those of its conjuncts. What is uniquely determined, however, is that the truth-value belongs to the class (2, 3). To proceed, we can appeal to an idea first introduced by N. Rescher in his paper "Quasi-truth-functional systems of propositional logic".[57] A quasi-truth-functional logic is defined there as one in which "some connectives are governed by many-valued functions of the truth-values of their variables". The entries in the truth-table of such a logic are typically not single truth-values but sets of values. It is clear that the system set up just now is, in this, sense, quasi-truth-functional. Now, as Rescher himself points out, a quasi-truth-functional logic will always be equivalent to a multi-valued strictly truth-functional system. The idea, roughly, is that we can treat a class of truth-values as constituting a new truth-value. Typically, if the quasi-truth-functional system has n truth-values, its strictly truth-functional equivalent will have 2^n - 1 values (Rescher notes that "in the case of a three-valued (T, F, I) quasi-truth-functional system we would need seven truth-values, to represent: T, F, I, (T, F), (T, I), (F, I), (T, F, I)" but argues that there are special reasons entailing that for a two-valued quasi-truth-functional system we need four rather than three values.). The seven-valued system which results in this way from the three-valued logic sketched above has, in fact, been

[57] Rescher, N.: 1962, "Quasi-truth-functional systems of propositional logic", *Journal of Symbolic Logic*, **27**: 1 - 10.

studied notably by Moffat[58]. I will therefore call it **M7**. An initially tempting idea is to identify the Jaina system **J7** with **M7**. This, however, will only work if the fourth value, u, is defined thus:

$$|p| = u \text{ iff } \forall\sigma\ \sigma : p \vee \forall\sigma\ \sigma : \neg p.$$

For then '*tu*' in the Jaina system will be identical with '1' in the Moffat system, etc. This is, however, not an interpretation which receives any textual support.

Instead, let us observe that there is a close connection between **M7** and the restricted Jaina system, **J3**. For note that the value (1, 3) in **M7** is such that

$$\begin{aligned} |p| = (1,3) \quad & \text{iff} \quad |p| = 1 \vee |p| = 3 \\ & \text{iff} \quad \forall\sigma\ \sigma : p \vee (\exists\sigma\ \sigma : p \ \&\ \exists\sigma\ \sigma : \neg p) \\ & \text{iff} \quad \exists\sigma\ \sigma : p. \end{aligned}$$

Thus (1, 3) in **M7** is just the value 'true' in **J3**. Similarly, (1, 2) in **M7** is just the value 'false' in **J3**. Thus, *although **J3** is not strictly truth-functional, its truth-tables are embedded in those of the Moffat logic, **M7**.*

It is presumably possible to find a quasi-truth-functional system whose truth-tables embed those of **J7**, the full Jaina system, in an entirely analogous way. Thus, although the loss of Adjunction means that the Jaina logic **J7**, is not truth-functional, its truth-table is embedded in a suitable quasi-functional system. The lack of truth-functionality is not, after all, a fatal flaw in the Jaina approach.

3.5 Axiomatisation of the Jaina System

We have shown that it is possible to use many-valued truth-tables to formalise the Jaina system. This was, in effect, the approach of the Jaina logicians themselves. Yet it would surely be much better to proceed by axiomatising the modal standpoint operator, ∇. Once again we look to Rescher[59]. His work on what he calls "assertion logics" is an extension of the work of Jaśkowski. Rescher introduces a system **A1**, with the following axiomatic basis:

(A1)	$(\exists p)\sigma : p$	[Nonvacuousness]
(A2)	$(\sigma : p \ \&\ \sigma : q) \supset \sigma : (p \ \&\ q)$	[Conjunction]
(A3)	$\neg\sigma : (p \ \&\ \neg p)$	[Consistency]
(R)	If $p \vdash q$, then $\sigma : p \vdash \sigma : q$	[Commitment]

Note that one effect of the rule (R) is to ensure that the notion captured is not merely explicit assertion but 'commitment to assert', for (R) states that from a standpoint one may assert anything entailed by another of the assertions. I believe that the Jainas would accept each of the axioms (A1) to (A3). Bharucha and

[58] Moffat, D. C. and Ritchie, G. D.: 1990, "Modal queries about partially-ordered plans", *J. Expt. Theor. Artif. Intell.*, **2**: 341 - 368. See also Priest, G.: 1984, "Hypercontradictions," *Logique et Analyse*, **107**: 237 -43.

[59] Rescher, N.: 1968, *Topics in Philosophical Logic*, Reidel, Dordrecht, chapter xiv.

Kamat, it may be noted, would reject (A3), while Matilal, as I have represented him, would reject (A2). I have already argued that these claims are mistaken. In particular, with regard to (A2), although it is true that the Jainas reject Adjunction, what this means is that assertions made from within different standpoints cannot be conjoined, not that assertions made within the same standpoint cannot be conjoined.

We now introduce the modal standpoint operator, ∇ "arguably", via the definition:

$$\nabla p \text{ iff } (\exists\sigma)\sigma : p,$$

and add the axioms of $\mathbf{S3^0}$ or some other suitable modal system.

Rescher defines some further systems by adding further axioms, none of which, I think, the Jainas would accept. For example, he defines **A2** by adding to **A1** the axiom that anything asserted by everyone is true $[(\forall\sigma)\sigma : p \supset p]$. There is no reason to suppose the Jainas commit themselves to this. The system **J3,** however, is distinguished by the new axiom (A4):

(A4) $\neg(\exists\sigma)(\neg\sigma : p\&\neg\sigma : \neg p)$ [Optimality]

Rescher too proposes a "three-valued approach" to assertion logic, via the notion of 'the truth status of the assertion p with respect to an assertor', written '$|p|\sigma$', and the definitions:

$$\begin{aligned} |p|\sigma \quad &= T \text{ iff } \sigma : p, \\ &= F \text{ iff } \sigma : (\neg p), \text{ and} \\ &= I \text{ iff } \neg(\sigma : p)\&\neg(\sigma : \neg p), \end{aligned}$$

and he shows that using the axioms of **A1**, we can derive a quasi-truth-functional logic for this system. These are not quite the Jaina values, as introduced earlier, for they do not quantify over standpoints or assertors. It is clear, however, that the Jaina system is of the same type as a modalised Rescher assertion logic. Their innovation is to introduce three truth-values via the definitions given before ($|p|_\Sigma = t$ iff $(\exists\sigma)(\sigma : p)$; $|p|_\Sigma = f$ iff $(\exists\sigma)(\sigma : \neg p)$; and $|p|_\Sigma = u$ iff $(\exists\sigma)(\neg(\sigma : p)\&\neg(\sigma : \neg p))$, where '$|p|_\Sigma$' stands for 'the status of the assertion p with respect to the total discourse Σ'). It is this attempt to take a many-valued approach to the modalised, rather than the unmodalised, version of assertion logic which generates the extra complexity of the Jaina system. I have already noted that, since the axiom "$p \supset \nabla p$" is lacking, the modal structure of the system will be no stronger than that of $\mathbf{S3^0}$. Yet in principle there seems no reason to think that the Jaina system cannot in this way be given an axiomatic basis.

3.6 Pluralism, Syncretism, and the Many-faceted View of Reality

The Jainas avoid dogmatism and a one-sided view of the world simply by noting that assertions are only justified in the background of certain presuppositions or conditions. It is perfectly possible for an assertion to be justified given one set of presuppositions, and for its negation to be justified given another different set. The

Jainas' ingenuity lies in the skill with which they developed a logic of discourse to make more precise this natural idea. However, they also went beyond this, for they added that every standpoint reveals a facet of reality, and that, to get a full description of the world, what we need to do is to synthesise the various standpoints. As Matilal puts it, "The Jainas contend that one should try to understand the particular point of view of each disputing party if one wishes to grasp completely the truth of the situation. The total truth ... may be derived from the integration of all different viewpoints".[60] But is this further step, the step from pluralism to syncretism, a coherent step to take? In particular, how is it possible to integrate inconsistent points of view? The point is made by Priest and Routley, who, commenting on the Jaina theory, state that "...such a theory risks trivialization unless some (cogent) restrictions are imposed on the parties admitted as having obtained partial truth – restrictions of a type that might well be applied to block amalgamations leading to violations of Non-Contradiction".[61]

Perhaps we can understand the Jaina position as follows. The so-called 'integration' of two points of view, σ_1 and σ_2, does not mean the creation of some new standpoint, which is the combination of the first two. For this would lead to the formation of inconsistent standpoints unless implausible constraints were placed on what can constitute a standpoint. Instead, what it means is that, if p is assertible from some standpoint σ_1, then this fact, that p is assertible from σ_1, can itself be asserted from σ_2 and every other standpoint. In this way, each disputant can recognise the element of truth in the other standpoints, by making explicit the presuppositions or conditions under which any given assertion is made.

If correct, this idea has an interesting consequence. In moving from pluralism to syncretism, the Jainas commit themselves to the claim that we are led to a *complete* account of reality by integrating of all the different points of view . It follows from this that every true proposition must be asserted within some standpoint, i.e. "$p \supset (\exists\sigma)(\sigma : p)$ or "$p \supset \nabla p$". Hence the move from pluralism to syncretism is a move from a logic of assertibility based on **S3**^{0} or weaker to one based on **S3** or stronger.

To conclude, we have seen how the Jainas developed a plausible and interesting logic of philosophical discourse, how they did not (or need not) commit themselves to the strongly paraconsistent position normally attributed to them, and how, as they strengthened their position from one of pluralism to one of syncretism, they had also to strengthen correspondingly the modal logic underlying the operator "*syāt*".

[60]Matilal, B. K.: 1977, *The Central Philosophy of Jainism*, Calcutta University Press, Calcutta.

[61]Priest, G., Routley, R. Norman, J. eds.: 1989, *Paraconsistent Logic: Essays on the Inconsistent*, Philosophia Verlag, Munchen, p.17.

4 LOGIC IN NAVYA-NYĀYA: THE METAPHYSICAL BASIS OF LOGIC

4.1 The use of graphs in interpreting Vaiśeṣika Ontology

Let us turn now to the Navya-Nyāya school, a school and a set of thinkers predisposed towards the study of the metaphysical structure of the natural world, and to the logical theory that is integral to this ontology. Three revisionary Nyāya thinkers - Bhāsarvajña (c. AD 950), Udayana (c. AD 1050), and Raghunātha (c. AD 1500) - saw in effect that there is a graph-theoretic basis to the classical Vaiśeṣika notion of a category. I will show how the graph-theoretic interpretation of their ideas lends itself to a distinctive treatment of negation, logical consequence and number.

Classical Vaiśeṣika lists six *kinds* of thing: substance, quality, motion, universal, individuator, inherence. Later Vaiśeṣika adds a seventh: absence. The basic stuff of the cosmos in the Vaiśeṣika world-view is atomic. Atoms are uncreatable, indestructible, non-compound substances. Atoms can coalesce into composite substances and can move. Indeed, the only changes in this cosmos are changes in the arrangement, properties and positions of the atoms. Creation is a matter of coalescing, destruction of breaking (and even God does not *create* the cosmos *ab nihilo*, but only 'shapes' it, as a potter shapes clay into a pot). A compound substance is a whole, composed out of, and inhering simultaneously in each of, its parts. These substances are individuated by the type and organisation of their parts. A 'quality' in classical Vaiśeṣika is a property-particular – for example, a particular shade of blue colour or a distinct flavour (what one would now call a 'thin' property). Qualities inhere in substances and in nothing other than substances. A 'motion' is another sort of particular; it too inheres in a substance and in nothing but a substance. Universals inhere in substances, qualities and motions. A universal inheres simultaneously in more than one, but has nothing inhering in it. Lastly, the 'individuator' (*viśeṣa*) is a distinctive and eponymous component in classical Vaiśeṣika ontology. An individuator inheres in and is unique to a particular atom: it is that by which the atomic, partless substances are individuated.[62]

Two principles lie at the heart of the Vaiśeṣika system: a principle of identity and a principle of change. The Vaiśeṣika principle of change is this: a becomes b iff the parts of a rearrange (perhaps with loss or gain) into the parts of b. 'Motions' are that in virtue of which the parts rearrange or stay together. There are basic or partless parts, the atoms, which, precisely because they have no parts, are incapable of becoming anything else. They move about but are eternal and indestructible. The Vaiśeṣika principle of identity is this: $a = b$ iff the parts of a are numerically identical to and in the same arrangement as the parts of b. 'Qualities' are that in virtue of which the parts are numerically identical or different. Atoms, precisely because they are partless, require a different principle of identity: atoms are distinct iff they have distinct individuators. Universals are limits on the degree

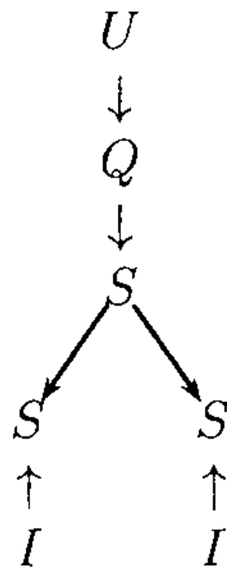
[62]An excellent review of the details of Vaiśeṣika ontology is Karl Potter ed., "Indian Metaphysics and Epistemology – The Tradition of Nyāya-Vaiśeṣika up to Gaṅgeśa", in *The Encyclopedia of Indian Philosophies*, vol. 2 (Delhi: Motilal Banarsidass, 1977), introduction.

of possible difference and change. One thing cannot change into another thing of an entirely different *sort* (a mouse into a mustard seed). One thing a can become another thing b iff the same universal resides in both a and b, that is, if a and b are of the same sort (as Udayana puts it, universals *regulate* causality).

This is the motivation for there being six 'types' of thing (substances, qualities, motions, universals, individuators, inherence). The problem is to find a proper philosophical basis for the notion of a 'type' of thing thus appealed to. In his *Lakṣaṇāvalī*, Udayana reconstructs the categories in a new way, a way which I shall claim explicates the notion of a type *graph-theoretically*. A graph is a simple sort of algebraic structure, consisting of set of nodes or vertices, and a set of edges (an edge being defined as a pair of nodes). A graph is 'directed' if the edges have a direction. Graphs, like many other mathematical structures, are realised in natural phenomena. A striking example is molecular structure: it is because the structure of a molecule is a graph that one can use a graph to depict one:



The implicit structure of the Vaiśeṣika ontology is that of a directed graph. The inherence relation connects things in the ontology in inheror-inheree pairings. So the substances, qualities, motions, universals and individuators are represented as the nodes of a graph whose set of edges represent the inherence relation. A fragment of the graph might look like this:



This graph represents the following state of affairs: a universal U inheres in a quality Q which inheres in a substance S. That substance is a dyad composed of two atoms in which it inheres, and each of which has inhering in it an individuator I. The structure of the world is a directed graph.

The nodes in a graph can be classified according to the number of edges terminating in them, and the number of edges starting from them: so the *valency* of a node in a directed graph is an ordered pair of integers (n, m). What Udayana saw in the *Lakṣaṇāvalī* is that things of different types in the Vaiśeṣika ontology correspond to nodes of different valencies. His brilliant idea is to use the idea of valency to *define* the categories of substance, quality, motion, universal, and individuator. He begins with a classification of the categories into the four valency-groups $(+,+), (+,0), (0,+)$ and $(0,0)$:[63]

[63]Numbering of the verses in the *Lakṣaṇavalī* follows Musashi Tachikawa, *The Structure of*

5. Noneternal [= compound] substance, quality, motion, universal, and individuator inhere.

6. Eternal [i.e. atomic] substance, inherence, and absence lack the property of inhering.

7. Substance, quality, and motion are inhered in.

8. Universal, individuator, inherence, and absence have nothing inhering in them.

In particular then, atoms have valency $(+, 0)$, universals and individuators have valency $(0, +)$, while compound substances, qualities and motions have valency $(+, +)$.

Notice that Udayana says that the inherence relation itself has a valency — $(0, 0)$. We should not take this to mean that the inherence relation is to be represented by a node disconnected from the rest of the graph, but rather that it does not correspond to any *node* in the graph at all. The first and most fundamental graph-theoretic type distinction is the distinction between a node and an edge, and the inherence relation is represented in a graph by the set of edges, not by any node. The set of edges represents the extension of the inherence relation.

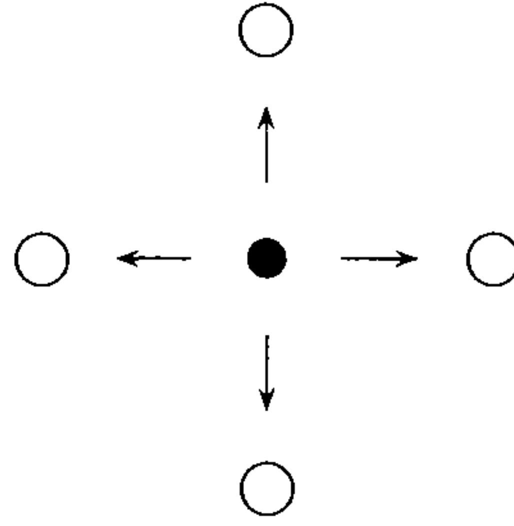
If the categories are to be distinguished from one another according to the valency of the nodes in that graph which is isomorphic to the world of things, then further specification is needed. The distinction between universals and individuators is simple: an individuator has valency $(0, 1)$ while a universal has valency $(0, m)$, with $m > 1$:

202. A universal has nothing inhering in it, inheres, and is co-located with every difference.

203. Individuators lack the property of being inhered in, inhere, and lack the property of inhering by being co-located with every difference.

Udayana's phrase 'co-located with every difference' is a technical device for expressing the idea that a universal inheres in more than one. For if an inheror inheres in exactly one thing x, then all other things are loci of difference–from x, and the inheror is not co-located with difference–from–x. However, if the inheror inheres in two things x and y, then difference–from–x is located in y and difference–from–y is located in x, and the inheror is co-located with both differences. So something co-located with every difference from each of the things in which it inheres is necessarily located in more than one thing. Notice that in classical Vaiśeṣika, individuators are said to have no universals inhering in them precisely because they are fundamental units of individuation, having nothing in common with one another.

UNIVERSAL



INDIVIDUATOR



Any node with valency $(0, m)$ with $m > 1$ is now to be called a 'universal', and any node with valency $(0, 1)$ is to be called an 'individuator':

The valency of atoms is different from that or qualities or motions, but we still need a general definition of substance, covering both atomic and compound substances. For compound substances, like universals but unlike atoms, inhere in other things (their parts). Udayana in fact offers four definitions, of which the first three repeat older definitions. The fourth definition, however, is completely original:

9. A substance is not a substratum of absence of quality.

10. Or, it belongs to such a kind as inheres in what is incorporeal, inheres in what is not incorporeal and does not inhere in what inheres in what is not corporeal.

11. Or, it belongs to such a kind as inheres in space and in a lotus but not in smell.

the World in Udayana's Realism: A Study of the Lakṣaṇāvalī and the Kiraṇāvalī (Dordrecht: Reidel Publishing Company, 1981).

12. Or, it is that in which inheres that in which inheres that which inheres.

The first of these definitions is the classical one in Vaiśeṣika[64] — a substance is that which possesses qualities. Udayana returns to this definition in his famous but conservative commentary, the *Kiraṇ āvalī.* He thinks of replacing it in the more experimental *Lakṣaṇāvalī* with a definition that makes no reference to any other category and indeed is phrased entirely in terms of the notion of inherence: a substance is 'that in which inheres that in which inheres that which inheres'. In other words, a substance is to be represented by a node like this:



The point of the definition is that a substance possesses qualities, and qualities possess universals, and nothing else in the ontology possesses something which possesses something. For universals and individuators possess nothing, while qualities and motions possess universals and nothing else.

Let us define a 'path' between one node and another in the obvious way: there is a path from node **x** to node **w** if there is a sequence of nodes {**x**, **y**, ..., **w**} such that there is an edge from **x** to **y**, an edge from **y** to **z**, ... , an edge from **v** to **w**.[65] Define the 'length' of a path as the number of edges between the first and the last node. Udayana's definition of a substance is now: a node is a substance iff there is a path at least of length 2 leading to it. Substances inhere in their parts; so the definition entails that every part of a substance is a substance.

The classical conception of qualities and motions makes them almost identical: they both inhere only in substances, and they both are inhered in only by universals.[66] Praśastapāda's remark[67] that the qualities other than contact, breaking, number and separateness 'inhere in one thing at a time' should not be construed as implying that they inhere in only one thing, but only that this group of qualities are *monadic* (non-relational) properties. These features are enough to distinguish qualities and motions from all else: from universals and individuators (which do

[64] *Vaiśeṣikasūtra* 1.1.14: "The characteristic of a substance is to possess actions, qualities and to be [their] inherence cause."

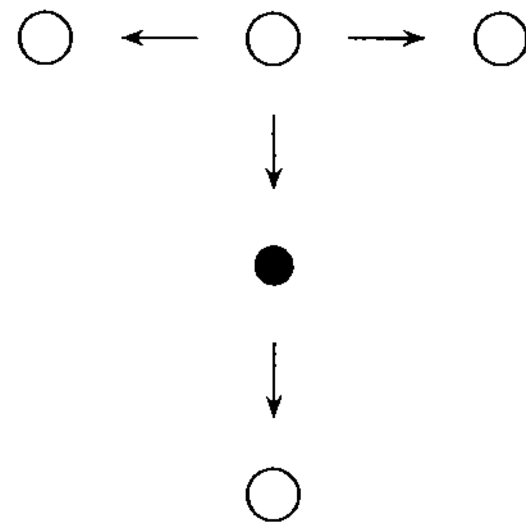
[65] In what follows, bold roman letters denote nodes in the graph, and italic letters denote the entities those nodes represent.

[66] *Vaiśeṣikasūtra* 1.1.15–6. *Padārthadharmasaṃgraha* 18. Section numbering in the *Padārthadharmasaṃgraha* follows Karl Potter ed., "Indian Metaphysics and Epistemology – The Tradition of Nyāya-Vaiśeṣika up to Gaṅgeśa", in *The Encyclopedia of Indian Philosophies*, vol. 2 (Delhi: Motilal Banarsidass, 1977), pp. 282–303.

[67] *Padārthadharmasaṃgraha* 50–51.

not have anything inhering in them), and from substances (which are inhered in by things that are themselves inhered in). It explains too why qualities cannot inhere in qualities — if they did then they would be equivalent graph-theoretically to substances.

QUALITY or MOTION



What is difficult is to find any principled way to distinguish between qualities and motions. There was indeed a persistent revisionary pressure to assimilate these two categories. Bhāsarvajña[68] heads the revisionary move, stating unequivocally that motions should be treated as qualities because, like qualities, they reside in substances and possess universals. From a graph-theoretic perspective, this revision is well motivated: qualities and motions are represented by nodes of the same valency, and so are things of the same *type*. Udayana chooses the harder way, and tries to formulate definitions that will accommodate the distinction. The classical Vaiśeṣika idea[69] that motions are what cause substances to come into contact with one another is reflected in his definitions:

126. A quality belongs to such a kind as inheres in both contact and non-contact, and does not inhere in the non-inherent cause of that sort of contact which does not result from contact.

190. A motion belongs to such a kind as inheres in the non-inherent cause of contact and does not inhere in contact.

These definitions introduce two new relations, contact and causation, neither of which are explicable in terms of inherence nor belongs to the graph-theoretic interpretation of the categories. The very success of that interpretation gives a rationale to the revisionary pressure. Finding a pattern into which all but a few items of some phenomenon fit grounds a presumption that those items are in some way discrepant. This is a general principle of scientific and rational inquiry, and we can see it been used by Bhāsarvajña to motivate revisions in the classical Vaiśeṣika theory. Rationality appears here in the form of *principled* revision.[70]

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[68] *Nyāyabhūṣaṇa*, p. 158.
[69] *Vaiśeṣikasūtra* 1.1.16.
[70] For later comment on Bhāsarvajña's revision: Karl Potter and Sibajiban Bhattacharyya eds.

Let us define a *Vaiśeṣika graph* as a connected directed graph each of whose nodes is a substance, quality, motion, universal or individuator, where:

> A *substance* is a node terminating a directed path of length 2.
>
> A *quality* or *motion* is a node v with valency $(+,+)$, such that the initial node of any edge terminating in v has zero invalency [i.e. such that qualities are not substances].
>
> A *universal* is a node with valency $(0,n)$ with $n > 1$.
>
> An *individuator* is a node with valency $(0,1)$.

Let us say further that node x inheres in a node y iff xy is an edge in a Vaiśeṣika graph. Then we can easily prove some results well-known to the Nyāya-Vaiśeṣika logicians:

LEMMA 1. *No quality inheres in a quality.*

Proof. The invalency of a quality x is nonzero, so any node in which x inheres terminates a path of length 2. ■

LEMMA 2. *Substances inhere only in substances.*

Proof. Any node x in which a substance inheres terminates a path of length 2. ■

THEOREM 3. *A Vaiseṣika graph has no directed cycles.*

Proof. The elements of a directed cycle must have valency $(+,+)$. So no universal or individuator can be a member of such a cycle, because neither has nonzero invalency. No quality or motion can be a member of a cycle, because only universals and individuators inhere in qualities and motions, and there are no universals or individuators in a cycle. No atomic substance can be a member of a cycle, because atoms have zero outvalency. That leaves only cycles of compound substances. But there can be cycles of substances only if a substance can have as a part something of which it is a part and so (if the part-of relation is transitive) be a part of itself. ■

DEFINITION 4. The *level* of a node in a Vaiśeṣika graph is the length of the longest directed path leading to it. (Note — this is well-defined because there are no directed cycles.)

THEOREM 5.

(i) *All and only universals and individuators belong to level 0;*

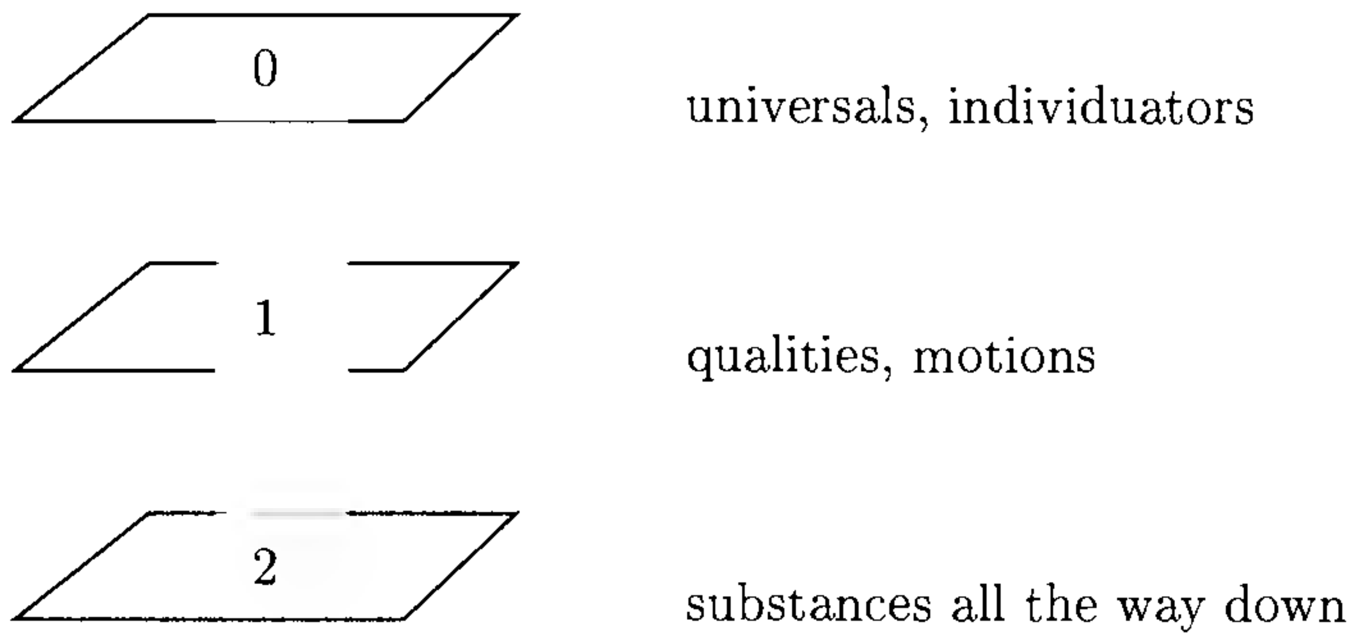
"Indian Philosophical Analysis — Nyāya-Vaiśeṣika from Gaṅgeśa to Raghunātha Śiromaṇi", in *The Encyclopedia of Indian Philosophies*, vol. 6 (Delhi: Motilal Banarsidass, 1993), pp. 323, 525-528.

(ii) All and only qualities and motions belong to level 1;

(iii) All and only substances belong to levels 2 and below.

Proof. (i) Only universals and individuators have zero invalency. (ii) By Lemmas 1 and 2, qualities and motions are inhered in only by universals and individuators, so belong to level 1. Substances do not belong to level 1 by definition. (iii). By definition, any node in level 2 is a substance, and by Lemma 2, any node in level $n > 2$ is a substance. ■

So the structure of the Vaiśeṣika graph is like this:



To what extent are we justified in adopting the graph-theoretical interpretation of Navya-Nyāya? I propose the following *Methodological Test*: The graph-theoretic interpretation is confirmed to the extent that it explains or predicts revisions made to the classical Vaiśeṣika system. Revisions include the introduction of a seventh category absence, the assimilation of qualities and motions, the elimination of individuators, the identification of co-extensive universals, the new account of number. Let us say that a node x is *redundant* in G if its deletion, together with the deletion of any edge incident to it, preserves all paths in G not containing x. The resulting graph $G*$ is a *conservative contraction* of G. Then we have, in effect, the following revisions being proposed by Nyāya authors — (I) All individuators are redundant in a Vaiśeṣika graph (Raghunātha); (II) Two universals are co-extensive only if at least one is redundant (Udayana); and (III) Qualities and motions are entities of the same type (Bhāsarvajña).

4.2 Negation as absence

'Absence' in Navya-Nyāya is not the same as nonexistence. Fictional characters, dream-objects and hallucinations are nonexistent: they do not exist as it were *by nature*. It would be an absurdity to go in search of Hamlet in order to find out

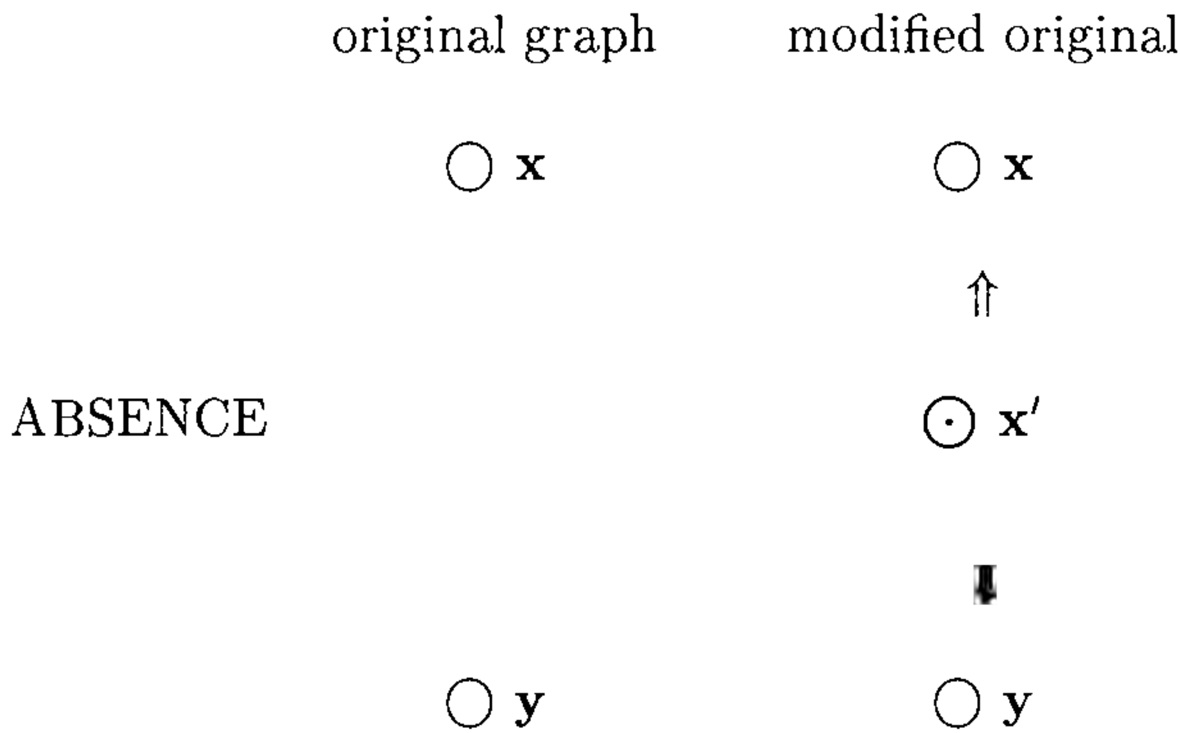
whether he really exists or not — his nonexistence is not a merely contingent lack in the world of things. The absence of water on the moon, on the other hand, is a contingent and concrete fact; so too is the absence of colour in my cheeks. Notice the role of the phrases 'of water', 'of colour' here: an absence has an *absentee* — that which the absence is an absence of. It also has a location (e.g. the moon, my cheeks), and a time. So the proposal is to reparse the sentence 'x does not occur in y at time t' as 'an absence-of-x occurs in y at t.' For it is often the case that the absence of something somewhere is more salient than any fact about what is present there.

There is one relatively straightforward way to interpret the idea of absence graph-theoretically. If x does not inhere in y, then there is no edge $(\mathbf{x}, \mathbf{y})$ in the graph. Now for every graph, there is a dual. The dual has the same nodes as the original graph, but has an edge between two nodes just in case the original does not. So the dual graph does have an edge $(\mathbf{x}, \mathbf{y})$. Following this idea, one would be led to say that absences are things of a different type to any presence because they are edges in the dual graph, rather than edges or nodes in the original.

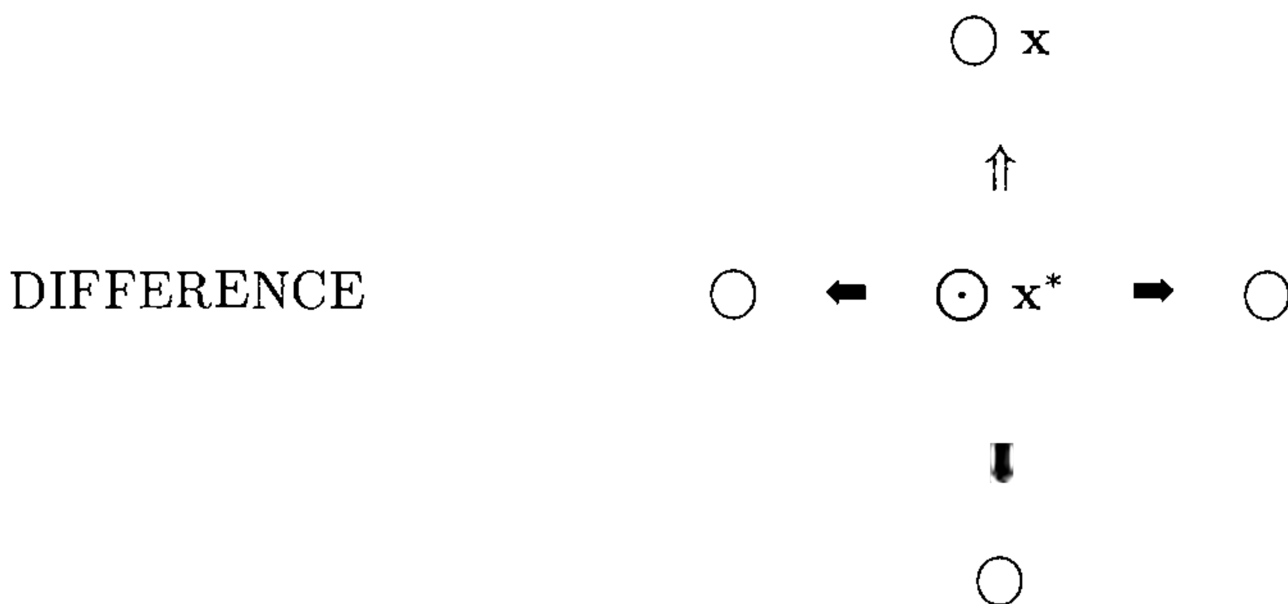
For various reasons, the Vaiśeṣika do not consider this to be an adequate explanation of the category. One problem is that it makes absences more like relations than 'things', and this does not keep to the spirit of the Vaiśeṣika idea that absences are entities. In fact, absences do display much relation-like behaviour after all, absence is always the absence of x in y. Another objection, however, is if absence is a new category, its introduction should result in an extension of the original graph, and not in the introduction of a new graph, let alone a graph completely disconnected from the original. For the connected world of things ought not be represented by a pair of disconnected graphs. A third problem arises if we admit something called 'unpervaded' occurrence, as we will see.

The Vaiśeṣika idea is represent absences as *nodes*, related in new ways to the nodes of the original graph. Here is how to do it. For each unconnected pair of nodes $(\mathbf{x}, \mathbf{y})$, create a new node $\mathbf{x}'$ in the original graph. This new node will have edges to $\mathbf{x}$ and to $\mathbf{y}$, but they will be edges of two new types. The edge $(\mathbf{x}', \mathbf{x})$ is an edge belonging to the extension of the absentee-absence (*pratiyogitā*) relation, which I shall signify as '$\Rightarrow$.' This represents the relation between an absence and what the absence is of. The edge $(\mathbf{x}', \mathbf{y})$ is an edge belonging to the extension of the 'absential special relation' (*abhāvīya-svarūpa-sambandha*), signified here by '➡'. This represents the occurrence relation between an absence and its location. The relation between an absence and its location is clearly not the same as the relation between a presence and its location (inherence, contact), for it is clear that when a person is is absent from a room, their absence is not in the room in the same sense that the other things in the room are.

These new nodes belong in a domain outside the system of levels, for they inhere in nothing and nothing inheres in them (inherence, and the whole system of levels, is a structure on presences). The modified graph is instead a concatenation of the original graph of nodes and edges with a new structure of 'absential nodes' and 'absential edges.'



Navya-Nyāya theory of absence draws a type distinction between *simple absence* (*atyantābhāva*) and *difference* (*anyonyābhāva*). Difference is the absence of a relation of identity between two things. Here '$x \neq y$' is paraphrased as 'a difference-from y occurs in x'. Graph-theoretically, the distinction between absence and difference is a distinction between a negation on edges and a negation on nodes in the original graph. For, trivially, every node is such that it is *different* from every other node. One way to represent this would be to introduce a new kind of 'nonidentity' edge into the graph, an edge which connects every node with every other node. The Naiyāyika, however, wants to the category of absence to correspond to a domain of things rather than relations; so in the graph-theoretic representation, differences have to be represented as nodes rather than edges. So let us say that for every node **x** in the original graph, there is a new node **x***. Call it an 'antinode'. **x*** is connected to *every* node in the graph. It is connected to **x** by an edge of the absentee-absence type, and to every node other than **x** by an absential location edge. There is a one-one correspondence between the new domain of antinodes and the domain of original nodes.



The leading idea behind the graph-theoretic interpretation of the categories is that a type of thing is a type of node, and node-types are determined by patterns of possible valencies in the graph. It was for this reason that we did not need earlier to the *label* the nodes. With the introduction of the category of absence,

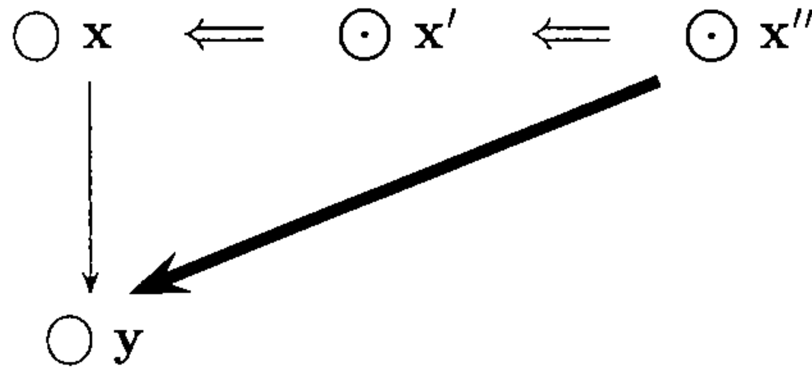
we have two higher-order type distinctions: the distinction between positive and negative nodes, and the distinction among the negative nodes between absential nodes and antinodes. Do these distinctions have a graph-theoretic explanation, or must we allow ineliminable node-labels to demarcate presence nodes, absential nodes and antinodes? What we do have now are three different types of edge — corresponding to the relations of inherence, absence-absentee, and absential location. So we might hope to distinguish between positive and negative nodes as those which are not and those which are at the end of an absential edge. That is, we make it a requirement that no positive node absentially qualify any other node. Clearly, the suggestion will work only if the absence of an absence is not identical to a presence. We will see in the next section that the graph-theoretically oriented Raghunātha indeed denies that this is so. So as not to beg the question at this point, and for the sake of pictorial clarity, I will continue to *mark* positive nodes ○ and negative nodes ⊙ differently.

What about the distinction between absential and antinodes? The traditional way of making the distinction is to say that simple absence is the denial of inherence (or some other nonidentity relation) and difference is the denial of identity. Graph-theoretically, the distinctive feature of an antinode $\mathbf{x}^*$ is that it absentially qualifies every node other than $\mathbf{x}$, while an absential node $\mathbf{x}'$ does not. Does this difference fail when x is something which inheres in nothing (an atom, an individuator)? No, because such things do not inhere in themselves — so $\mathbf{x}'$ unlike $\mathbf{x}^*$ absentially qualifies $\mathbf{x}$. Indeed, this second contrast is itself sufficient to discriminate absential nodes and antinodes.

The above treatment of absence is in effect a procedure for introducing new nodes into the original graph. One set of new nodes fills the 'gaps' in that graph: whenever there is no edge between two nodes, an absential node is introduced between, and linked to, them. Another set of new nodes exactly mirrors the original graph: for each node in the original, there is one and only one antinode, linked to everything the original node is not. But now, having supplemented the original graph with two sets of new nodes, nothing is to stop us from repeating the procedure again — generating new sets of *second-order* absence nodes — and to do this again and again. It seems that we have introduced a procedure for the indefinite recursive expansion of the graph. Fortunately this does not in fact happen. As we will now see, no subsequent recursion of the procedure after the first produces any new nodes.

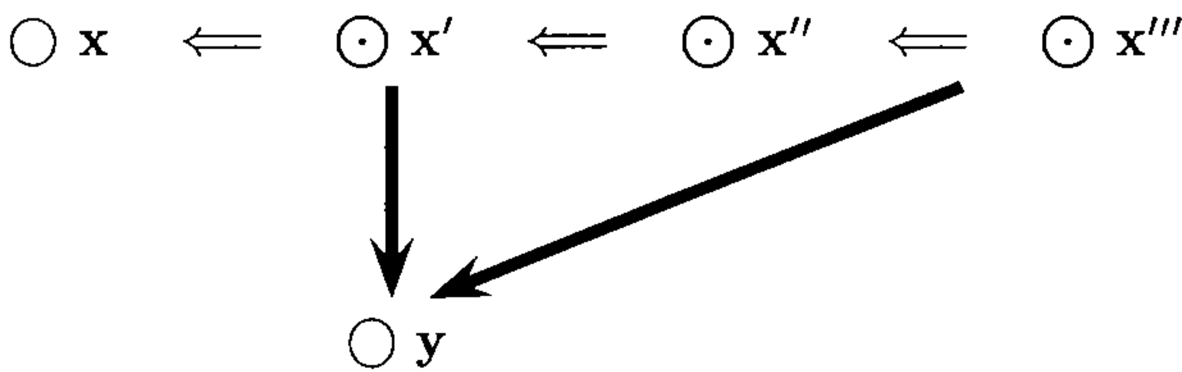
Prima facie, it seems plausible to reason as follows (as we will shortly see, this reasoning turns out to be subject to an important caveat). If $\mathbf{x}$ is in $\mathbf{y}$, then $\mathbf{x}'$, the absence of $\mathbf{x}$, is not in $\mathbf{y}$, and so $\mathbf{x}''$, the absence of $\mathbf{x}'$, is in $\mathbf{y}$. Conversely: if $\mathbf{x}''$ is in $\mathbf{y}$, then $\mathbf{x}'$ is not in $\mathbf{y}$, so $\mathbf{x}$ is in $\mathbf{y}$. Graph-theoretically, we represent this as follows (see next figure):

If this is right, then it follows that an entity and the absence of its absence 'occur' in exactly the same set of loci: for all $\mathbf{y}$, there is an inherence edge ($\mathbf{x}$, $\mathbf{y}$) just in case there is an absential location edge ($\mathbf{x}''$, $\mathbf{y}$). Can we appeal now to a uniqueness condition for absences, and infer that the absence of an absence of



an entity is identical to the entity? The point is controversial, with the majority favouring identification. It is Raghunātha[71] who argues that the identification is unsound, on the ground that nothing can turn an absence into a presence. Here again Raghunātha's intuition agrees with the graph-theoretic reconstruction: the nodes **x** and **x″** are connected to other nodes by means of different types of edge. So they cannot both represent entities of the same type. Moreover, as we shall see in more detail below, the Naiyāyikas do not accept that it is generally valid to infer from the occurrence of **x** in **y** to the occurrence of **x″** there, although they do allow the converse. This is the caveat in the line of reasoning with which I began this paragraph. The implication is that **x** and **x″** need not, after all, share the same set of loci.

Let us repeat the procedure once more. If **x′**, the absence of **x**, is in **y**, then **x″** is not in **y**, and so **x‴**, the absence of **x″**, is in **y**. Conversely: if **x‴** is in **y**, then **x″** is not in **y**, so **x′** is in **y**. Graph-theoretically:



It follows that a first-order absence and the absence of *its* absence reside in exactly the same set of loci. But here we *can* appeal to the uniqueness condition,

[71] Raghunātha, *Padārthatattvanirūpaṇa*, p. 55. Daniel Ingalls, *Materials for the Study of Navya-Nyāya Logic* (Cambridge, Mass.: Harvard University Press, 1951), p. 68. Bimal. K. Matilal, *Logic, Language and Reality* (Delhi: Motilal Banarsidass, 1985), p. 149. Roy W. Perrett, "Is Whatever Exists Knowable and Nameable?" *Philosophy East & West* 49.4 (1999), pp. 410 414, esp. 408–9. I disagree here with the idea of Matilal and Perrett that there is only an intensional difference between an object and the absence of its absence. For me, a type difference in the graph means a type difference in categories of thing.

because the edges are all of the same type. So x''' is identical to x', as Raghunātha himself allows.[72] Similarly, x'''' is identical to x'', and so on. There are no absential nodes of order higher than two. The argument is summed up by Annambhaṭṭa in the *Tarkasaṃgraha* [§89]:

> The view of the early thinkers is that the absence of an absence is nothing but a presence; it is not admitted as a new absence for there would then be an infinite regress. According to the new school, however, the absence of an absence is a distinct absence, and there is no regress as the third absence is identical to the first.

Recall that we defined the absence $\mathbf{x}'$ as a node such that $\mathbf{x}'$ is absentially located in $\mathbf{y}$ if there is no edge between $\mathbf{x}$ and $\mathbf{y}$. That definition was adequate for the introduction of first-order absences, because there is only one kind of edge in the original graph, namely the inherence edge. The expanded graph has another sort of edge, however: the absential edge. So the notion of a second-order absence is underdetermined by our original definition. The new definition we need is:

Rule for Absence:

> An absence $\mathbf{x}'$ is absentially located in $\mathbf{y}$ if $\mathbf{x}$ does not inhere in $\mathbf{y}$.

Rule for Higher Order Absence:

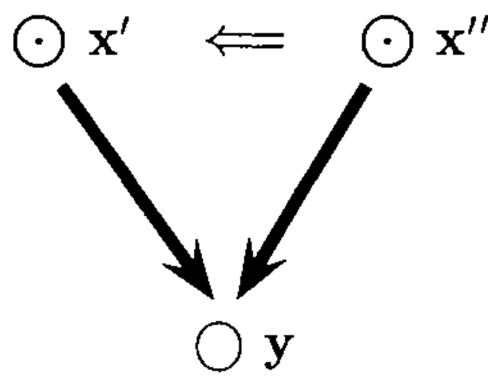
> For $i > 1$, an absence $\mathbf{x}^i$ is absentially located in $\mathbf{y}$ iff $\mathbf{x}^{i-1}$ is not absentially located in $\mathbf{y}$.

The second clause implies that absence is a classical negation for $i > 1$, and so, in particular, that an absence of an absence of an absence is identical to an absence. A double negation, however, is a mixture — a negation defined on inherence edges followed by a negation defined on absential qualifier edges – and for that reason behaves non-classically. What I will show in the next section is that Navya-Nyāya logic rejects the classical rule of Double Negation Introduction — the rule that licenses one to infer from p to $\neg\neg p$. What replaces it is a weakened rule — infer from $\neg p$ to $\neg\neg\neg p$. This is because negation is a procedure for filling 'gaps' in the graph: whenever there is no edge between two nodes, the rule for negation licenses us to insert an absential node between them. The classical rule for Double Negation Elimination – the rule that licenses one to infer from $\neg\neg p$ to p — remains valid in Nayva-Nyāya logic (i.e. if $\mathbf{x}'$ is not in $\mathbf{y}$, then $\mathbf{x}$ is in $\mathbf{y}$).[73] The effect of this weakening in the rule for Double Negation Introduction is that one is no longer

[72] *Padārthatattvanirūpaṇa*, pp. 67–69. Daniel Ingalls, *Materials*, pp. 68–69; Bimal. K. Matilal, *Logic, Language and Reality*, pp. 149-150.

[73] Daniel Ingalls draws a comparison between Navya-Nyāya and intuitionist logic (*Materials*, p. 68, n. 135), claiming that it is the elimination rule for double negation that is rejected. However we are able, in Navya-Nyāya logic, to infer from the absence of the absence of an entity to the presence of that entity; conversely, we are not able to infer from the presence of an entity to the absence of its absence — the non-pervasive node is a counter-example.

entitled to infer that if $\mathbf{x}$ is in $\mathbf{y}$, then $\mathbf{x}'$ is not in $\mathbf{y}$. One effect of this is to block the equivalence of a positive entity with the absence of its absence. We can say that $\mathbf{x}'$ is the absence of $\mathbf{x}''$, but we cannot say that $\mathbf{x}$ is the absence of $\mathbf{x}'$. Graph-theoretically, connections of the form $\bigcirc\mathbf{x} \Rightarrow \odot\, \mathbf{x}'$ are prohibited, since a positive entity cannot be the absence of *anything*. Also prohibited are triangles of the form below, because negation behaves classically within the domain of absences. What is stranger, however, is the effect the weakened rule has of *permitting* a positive entity to be co-located with its absence. For we are no longer in a position to assert that the presence of an entity is inconsistent with its absence. Let us see how the Nyāya philosophers arrive at the conclusion that one must allow for such an unusual possibility.



Whenever something inheres in a compound substance, the question arises: does it also inhere in the parts? An entity is said to be of 'locus-pervading' occurrence just in case it inheres in all the parts of its locus (as well as in the locus itself).[74] It *saturates* its locus. A sapphire is red through-and-through, and sesame oil pervades every part of the seed; but a painted vase is blue only on the outside. Let us say then that x is locus-pervading with respect to y just in case x inheres in y and if z is a part of y then x inheres in z.[75] The only things that have parts are substances, and substances inhere in their parts and in nothing else. So x is locus-pervading with respect to y just in case x inheres in y and if y inheres in z then x inheres in z. Certain types of quality pervade their loci, according to the classical Vaiśeṣika authors.[76] Examples include weight, viscosity, and fluidity. A thing is heavy just in case every part of it is heavy. Colours, tastes, smells can pervade their loci but need not do so.[77] And a compound substance is locus-pervading with respect to each of its parts, if 'part of' is a transitive relation.

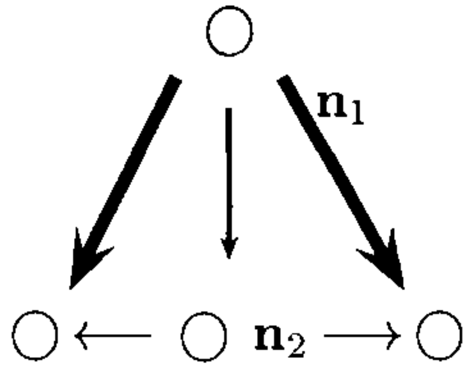
[74] Ingalls (1951: 73-74); Bimal Matilal, *The Navya-Nyāya Doctrine of Negation* (Cambridge, Mass.: Harvard University Press, 1968), p. 53, 72, 85; Matilal (1985: 119-122).

[75] Frege's notion of 'divisibility' is formally rather analogous. Gottlob Frege, *The Foundations of Arithmetic*, translated by J.L. Austin (Oxford: Basil Blackwell, 1950), p. 66: "The syllables "letters in the word three" pick out the word as a whole, and as indivisible in the sense that no part of it falls any longer under the same concept. Not all concepts possess this quality. We can, for example, divide up something falling under the concept 'red' into parts in a variety of ways, without the parts thereby ceasing to fall under the same concept 'red."'

[76] Karl Potter ed., "Indian Metaphysics and Epistemology – The Tradition of Nyāya-Vaiśeṣika up to Gaṅgeśa", in *The Encyclopedia of Indian Philosophies*, vol. 2 (Delhi: Motilal Banarsidass, 1977), pp. 114–119.

[77] Raghunātha, *Padarthatattvanırupaṇa*, pp. 44 6.

The notion of a locus-pervading entity has a distinctive graph-theoretic correlate. An edge ($\mathbf{n}_1$, $\mathbf{n}_2$) is locus-pervading just in case there is an edge from $\mathbf{n}_1$ to any node in any path from $\mathbf{n}_2$.



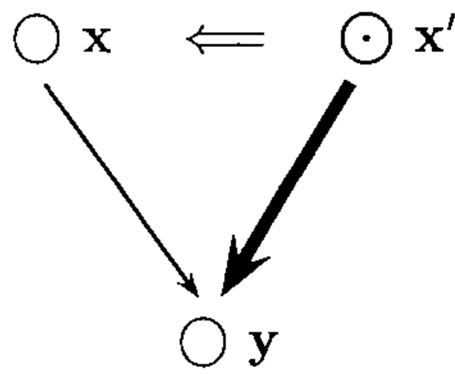
While locus-pervading nodes are straightforwardly definable in the system as so far developed, the concept of 'unpervaded occurrence' (*avyāpya-vṛttitva*) marks a theoretical innovation. The classic Buddhist refutation of realism about wholes is that wholes must be the bearers of contradictory properties. For if some parts of a vase are red and other parts are not red, and if the vase as a whole has a colour in virtue of its parts having colour, then one seems forced to admit either that the whole is both red and not red, or that it has no colour at all.[78] The traditional Nyāya-Vaiśeṣika solution is less than satisfactory – it is to say that the whole has a new shade of colour called 'variegated'! Recognising the *ad hoc* nature of such a response, later Naiyāyikas try instead to make sense of the idea that a property can be co-located with its absence.[79] The idea is to capture the sense in which one says that the vase is red, because its surface is red, allowing at the same time that it is not red, because its insides are some other colour. A favourite Nyāya example involves the relation of contact: the tree enjoys both monkey-contact (there is a monkey on one of its branches) and also the absence of monkey-contact (its roots and other branches are in contact with no monkey). This defence of realism is what motivates later writers to allow there to be such a thing as unpervaded occurrence, defined to be an occurrence that is co-located with its absence. That is, an unpervading node is a node **x** such that there is an edge (**x**, **y**) and an edge (**x**′, **y**). Triangles such as the following are now deemed to be permissible in the graph:

The strangeness of such a possibility is ameliorated if one says, as some Naiyāyikas do, that x occurs in y as 'delimited' by one part, and its absence occurs in y as 'delimited' by another part.[80] Gaṅgeśa nevertheless goes to considerable lengths to reformulate logic and the theory of inference in Navya-Nyāya in a way that per-

[78]Dharmakīrti, *Pramāṇavārttika* II, 85-86; Kamalaśīla, *Pañjikā under Tattvasaṃgraha* 592-598.

[79]Udayana, *Ātmatattvaviveka*, pp. 586–617. Prabal Kumar Sen, "The Nyāya-Vaiśeṣika Theory of Variegated Colour (*citrarūpa*): Some Vexed Problems", *Studies in Humanities and Social Sciences* 3.2 (1996), pp. 151–172.

[80] Ingalls (1951: 73–4); Bimal Matilal, *The Navya-Nyāya Doctrine of Negation* (Cambridge, Mass.: Harvard University Press, 1968), pp. 71–73.



mits the co-location of an entity with its absence. The phenomenon of unpervaded occurrence is not regarded as a minor curiosity in Nyāya, but as the occasion for serious revision in their analysis.[81]

4.3 *Definitions of logical consequence*

With the introduction of absence, the graph-theoretic ontologies can serve as semantic models for a propositional language. A sentence 'p' is assigned, let us stipulate, an ordered pair of nodes ($\mathbf{x}$, $\mathbf{y}$). The sentence is true if that pair is an edge in the graph, false if it is not.[82] The negation of that sentence, '$\neg p$', is true if ($\mathbf{x}'$, $\mathbf{y}$) is an edge, false if it is not. Again, '$\neg\neg p$' is true if ($\mathbf{x}''$, $\mathbf{y}$) is an edge, false if it is not. If triangles like the one above are possible, then the truth of 'p' does not imply the truth of '$\neg\neg p$', since ($\mathbf{x}$, $\mathbf{y}$) is an edge but not ($\mathbf{x}''$, $\mathbf{y}$). So the propositional logic being modelled is, as we have already observed, one in which Double Negation Introduction does not hold. In this theory, we still have these correspondences between truth-value and negation:

(R1) if $\neg\mathrm{T}\alpha$ then $\mathrm{T}\neg\alpha$. from Rule for Absence

(R2) $\mathrm{T}\neg\neg\alpha$ iff $\neg\mathrm{T}\neg\alpha$ from Rule for Higher Order Absence

What we no longer have is:

(R3) if $\mathrm{T}\neg\alpha$ then $\neg\mathrm{T}\alpha$

The reason, as I said before, is that negation is an operation that fills 'gaps' in the graph — it tells us nothing when there is already an edge between two nodes. So the truth of a proposition is consistent, in Navya-Nyāya logic, with the truth of its negation. This element of dialetheism in the theory does not, however, mean that anything is provable or that anything follows from anything else — the correspondences R1– R2 are enough to prevent the system collapsing. Let us see why.

[81] Matilal's property-location language, in which properties have both a 'presence range' and an 'absence range' and the two ranges are permitted to overlap, is a different way to capture the same idea; Matilal (1985: 112–127).

[82] Gaṅgeśa, *Tattvacintāmaṇi*, I, *pramālakṣaṇa*, p. 401.

In the modern analysis of valid inference, an inference is valid just in case it is impossible for the premises to be true without the conclusion also being true. In the logic of classical India, validity is a matter of property-substitution, and the problem is to determine the conditions under which the occurrence of a reason property at a location warrants the inference that a target property occurs there too ("*Ta* because *Ra*"). The leading idea is that such property substitutions are valid just in case the reason does not 'wander' or 'deviate' from the target (*avyabhicāra*). In a famous passage called the *vyāptipañcaka*, Gaṅgeśa suggests five ways to make sense of this idea:[83]

> Now, in that knowledge of a pervasion which is the cause of an inference, what is pervasion? It is not simply non-wandering. For that is not
>
> 1. nonoccurrence in loci of the absence of the target, nor
> 2. nonoccurrence in loci of the absence of the target which are different from loci of the target, nor
> 3. non-colocation with difference from a locus of the target, nor
> 4. being the absentee of an absence which resides in all loci of absence of the target, nor
> 5. nonoccurrence in what is other than a locus of the target,
>
> since it is none of these where the target is maximal.

A 'maximal' property is a property resident in everything (*kevalānvayin*). Gaṅgeśa dismisses the five provisional analyses on the grounds that all are formulated in terms of 'absence of the target', and that that phrase is undefined when the target is maximal (the absence of a maximal property — assumed here not to be of unpervaded occurrence — would occur in nothing and so be 'unexampled', contradicting a basic condition of connectedness). In his preferred definition, Gaṅgeśa exploits a trick to overcome this problem.[84] He says that any property whose absence is colocated with the reason is not identical to the target. This implies that the target is not a property whose absence is colocated with the reason, but the contraposed formulation avoids the use of the troublesome phrase 'absence of the target'.

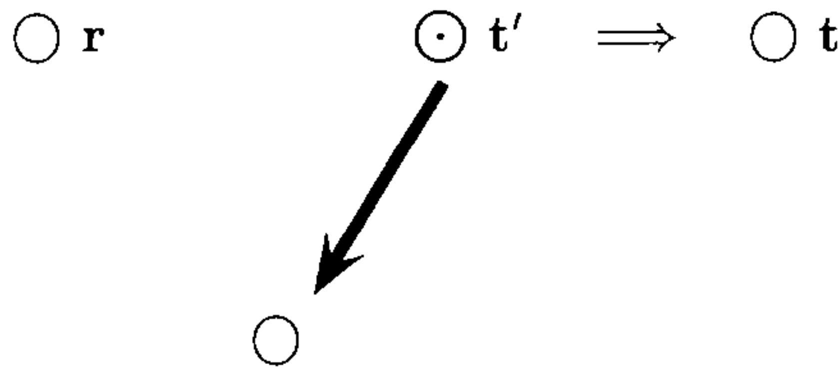
Consider now the difference between the first and second analyses in the list of five. Graph-theoretically, what the first analysis states is that, if **r** is the node representing the reason, and **t** is the node representing the target, then **r** is present in no node where **t** is absent —

But what happens if the target has nonpervaded occurrence? Then the first analysis is too strong.[85] For it is not a necessary condition on valid inference that

[83] Gaṅgeśa, *Tattvacintāmaṇi*, II, *vyāptipañcaka*, pp. 27–31.

[84] Gaṅgeśa, *Tattvacintāmaṇi*, II, *siddhānta-lakṣaṇa*, p. 100.

[85] I follow here the explanation of Raghunātha. *Vyāptipañcakadīdhiti* text 3 4 (Ingalls (1951: 154)).



the reason not be present wherever the target is absent, if there are nodes where the target is present as well as absent. What validity precludes is the presence of the reason without the presence of the target. So the proper definition is that the reason is not present wherever the target is not present (and so also absent). This is exactly what the second analysis states. We can make the point in terms of our earlier definitions of truth and negation. The premise in an inference is the statement that the reason occurs in a certain location, the conclusion the statement that the target occurs in that location. What our first analysis asserts is that the premise is not true if *the negation of the conclusion is true* (= absence of target in the location). The second analysis states instead that the premise is not true if the conclusion is *false* (= denial of presence of target in the location). Ironically, then, it is the very element of dialetheism of the Navya-Nyāya system which forces Gaṅgeśa to disambiguate the definition of validity, and to distinguish the correct definition from the one that had been preferred before.

Let α = "the reason r inheres in x", β = "the target t inheres in x". Then $\alpha \models \beta$ iff t pervades r. The problem is to solve for 'pervades'. The first solution in Gaṅgeśa is:

1. whatever the value of x, α is *not true* if $\neg\beta$ is true, i.e.

   $\alpha \models \beta$ iff under any assignment of value to x, $T\neg\beta \rightarrow T\neg\alpha$

His second solution is:

2. whatever the value of x, α is not true if β is not true, i.e.

   $\alpha \models \beta$ iff under any assignment of value to x, $\neg T\beta \rightarrow \neg T\alpha$.

What we have seen is that (2) and not (1) is the correct analysis of logical consequence if R3 is rejected.[86]

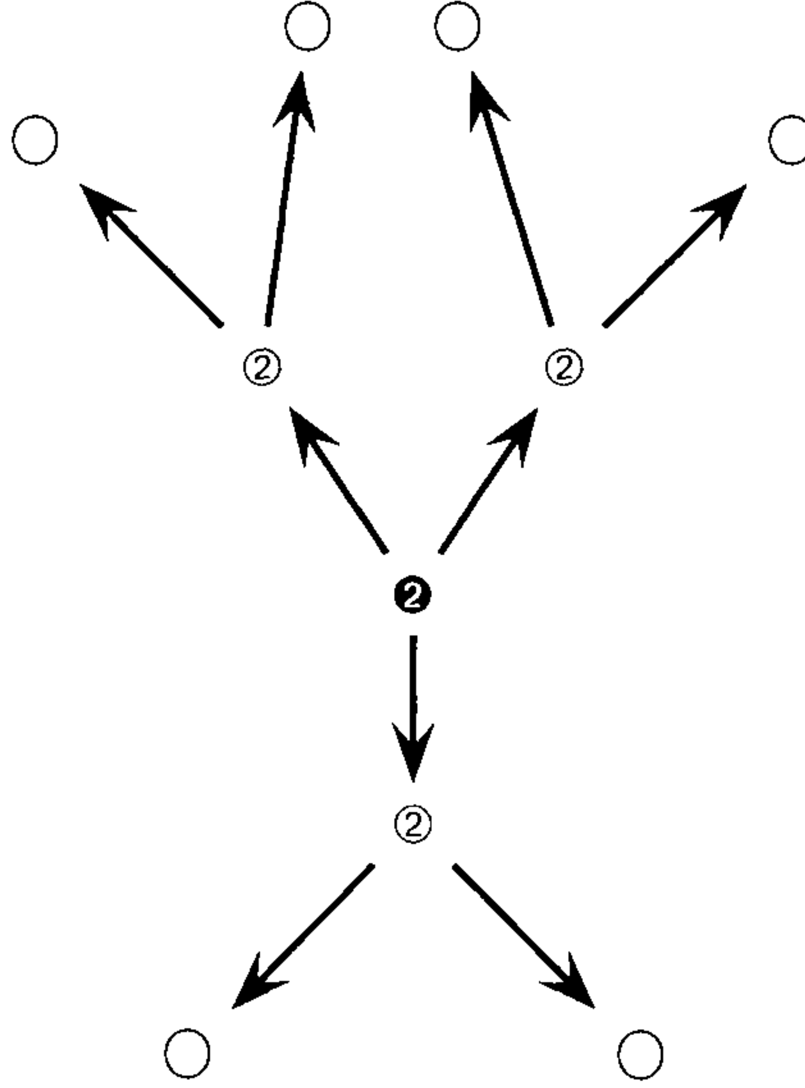
4.4 Number

The classical Vaiśeṣika theory of number is that numbers are qualities of substances.[87] A quality 'two' inheres in both members of a pair of substances, another

[86] R3 is what Graham Priest calls the 'exclusion principle.' For a semantic theory without this principle, see his *In Contradiction: A Study of the Transconsistent* (Dordrecht: Martin Nijhoff Publishers, 1987), chapter 5.

[87] *Vaiśeṣikasutra* 1.1.9, 7.2.1 8.

quality 'two' inheres in another such pair, and all the qualities 'two' have inhering in them a single universal 'twohood' (see graph on page 388.[88]



❷ the universal "twohood"
② the quality-particular "two"

Bhāsarvajña and Raghunātha, as usual, lead the reforming move. Bhāsarvajña's theory[89] is that numbers are not qualities at all, but relations of identity and difference. Thus the sentence 'a and b are one' means simply that $a = b$, while 'a and b are two' means that $a \neq b$. Bhāsarvajña's analysis is echoed, very much later, in Gadādhara's (c. AD 1650) comments on the meaning of the word 'one'.[90] Gadādhara states that the meaning of 'one F' is: an F as qualified by being-alone, where 'being alone' means 'not being the absentee of a difference resident in something of the same kind.' In other words, 'one F' is to be analysed as saying of something which is F that no F is different to it. If this is paraphrased in a

[88]For a more detailed description of the classical account: Jonardon Ganeri, "Objectivity and Proof in a Classical Indian Theory of Number," in *Synthese*, 129 (2001), pp. 413 437.

[89]*Nyāyabhūṣaṇa*, p. 159.

[90]*Śaktivāda with Kṛṣṇa Bhaṭṭa's Mañjūṣa, Mādhava Bhattācārya's Vivṛtti and Sāhitya Darśanācārya's Vinodini*, edited by G. D. Sastri (Benares: Kashi Sanskrit Series no. 57, 1927). p. 189.

first-order language as 'Fx & $\neg(\exists y)(Fy$ & $y \neq x)$' then it is formally equivalent to a Russellian uniqueness clause 'Fx & $(\forall y)(Fy \rightarrow y = x)$'. The idea that 'one' expresses uniqueness is in the spirit of Bhāsarvajña's idea that it denotes the identity of a thing. In any case, it is clear that, for Gadādhara, 'one' has a logical role similar to that of the definite article.

Raghunātha is more radical still.[91] The central problem is that things in *any* category in the Vaiśeṣika ontology can be numbered, and Raghunātha concludes that numbers must belong in a new category of their own:

> Number is a separate category, not a kind of quality, for we do judge that there is possession of that [number] in qualities and so on. And this [judgement we make that qualities have number is] not an erroneous one, for there is no [other] judgement which contradicts it.

Raghunātha puts pressure at exactly the right place. The 'is-the-number-of' relation is not reducible to the relation of inherence or any relation constructed out of it, for it is a relation between numbers and *any* type of thing. What is this new relation? Raghunātha points out that while inherence is a distributive relation (*avyāsajya-vṛtti*), the number-thing relation has to be collective (*vyāsajya-vṛtti*). The distinction occurs in the context of sentences with plural subjects. An attributive relation is distributive if it relates the attribute to every subject — if the *trees* are old, then each individual tree is old. A relation is collective if it relates the attribute to the subjects collectively but not individually — 'the trees form a forest' does not imply that each tree forms a forest. Number attributions are collective; if one says that there are two pots here, one does not imply that each pot is two. Inherence, however, is a distributive relation, and so cannot be the relation of attribution for numbers. This new relation is called the 'collecting' (*paryāpti*) relation by Raghunātha:[92]

> The collecting relation, whose existence is indicated by constructions such as "This is one pot" and "These are two", is a special kind of self-linking relation.

His commentator Jagadīśa explains:

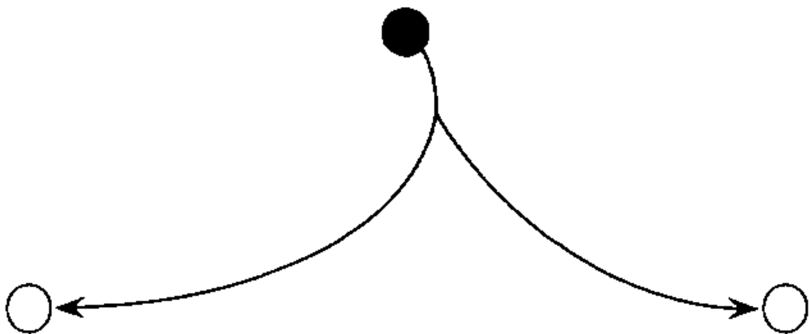
> It might be thought that the collecting relation is [in fact] nothing but inherence...So Raghunātha states that collecting [is a special kind of self-linking relation]. ... In a sentence like "This is one pot", collecting relates the property pot-hood by delimiting it as a property which resides in only one pot, but in a sentence like "These are two pots", collecting relates the property twoness by delimiting it as a property which resides in both pots. Otherwise, it would follow that there is no difference between saying "These are two" and "Each one possesses twoness".

[91] *Padārthatattvanirūpaṇa*, pp. 86–87.

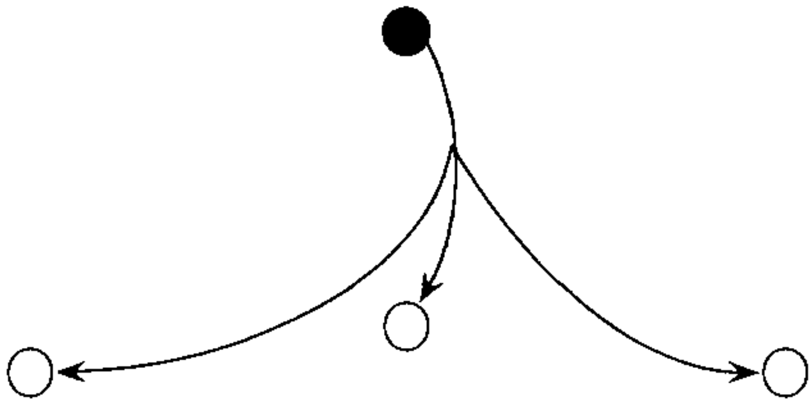
[92] *Avacchedakatvanirukti with Jagadīśa's Jāgadīśī*, edited by Dharmananda Mahabhaga (Varanasi: Kashi Sanskrit Series 203), p. 38.

Thus the number two is related by the collecting relation to the two pots jointly, but not to either individually. Raghunātha's idea is clear in the graph-theoretic context. The introduction of numbers requires one final expansion of the graph. We introduce another new domain of nodes (**1**, **2**, **3**,..) and another new type of edge from these nodes. Like ordinary edges, this new type of edge is an ordered pair whose first member is a node, but now the second member is set of nodes. The new edge connects the node **2** with every pair of nodes (**x**, **y**). Likewise, it connects the node **3** with every triple of nodes (**x**, **y**, **z**), and so on. The node **2**, then, is that node from which all edges to pairs begin, the node **3** the node from which all edges to triples begin, and so forth. This is enough to individuate number-nodes graph-theoretically (see graph on 390:

NUMBER 2



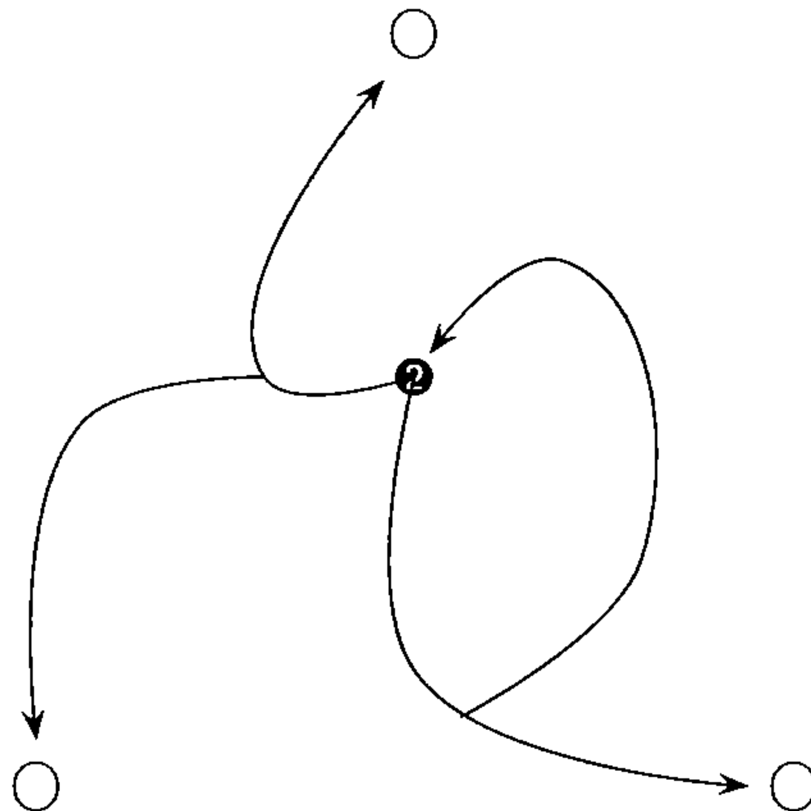
NUMBER 3



The nodes to which the new edge can connect a number-node can be of any type. In particular, they can themselves be number-nodes. Indeed, the new edge connects **2** with pairs of nodes one of whose members is **2** itself (see graph on page 391:

This solves the cross-categorial problem. Number-nodes are related by the new kind of bifurcating edges to nodes of any and every type in the graph, including number-nodes themselves.

The graph-theoretic approach is, I think, full of potential. It offers a new way to read and interpret Navya-Nyāya logic. One might proceed by looking for further situational constraints on what constitutes a permissible graph and applying graph theory to analyse the structure of those graphs. One might also try to establish



the relationship between such graphs and classical or nonclassical logics. The treatment of negation suggests a comparison with dialetheic logic, and the idea of self-linking nodes perhaps with non-wellfounded set theory. My aim here has been to expose the logical basis of Vaiśeṣika theory, and to draw a conclusion about the nature of logical thinking in India. The conclusion is simply this. The idea that nature instantiates mathematical structure is not remote from the Indian understanding of natural philosophy, contrary to what has generally been believed, but is in fact a fundamental aspect of it.

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THE MEGARIANS AND THE STOICS

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1 INTRODUCTION

In the opinion of Carl Prantl, the nineteenth century historian of logic, the Stoics were no logicians at all, but merely confused plagiarists who peddled second-rate versions of Peripatetic and Megarian doctrines. Chrysippus of Soli, touted by many as the greatest logician of the Hellenic age, was a special target of Prantl's baleful attacks, as witness the following assessment of his logical skills:

> Chrysippus created nothing really new in logic, for he only repeated details already known to the Peripatetics or pointed out by the Megarians; his activity consisted in this, that in the treatment of the material he descended to a pitiful degree of dullness, triviality, and scholastic quibbling. ... It is to be considered a real stroke of luck that the works of Chrysippus were no longer extant in the Middle Ages, for in that extensive morass of formalism, the tendency (weak as it was) toward independent investigation would have been completely eliminated.[1]

Although Chrysippus may have borne the main thrust of Prantl's assault, there is no doubt that his criticisms were meant to have general application among the philosophers of the Stoa.

It is difficult to discern the motivation behind Prantl's *ad hominem* attacks on the Stoics in general and on Chrysippus in particular, and one has to look back to Plutarch's polemic in *De communibus notitiis* and *De Stoicorum repugnantiis* to find anything remotely similar in tone. On the other hand, it is not so difficult to see, as Mates argues [Mates, 1953, pp. 87–88], that one can safely discount virtually all of Prantl's judgements on Stoic logic, not only because there is lacking any argument to substantiate them, but also because Prantl himself seems to have had little understanding of what the Stoics were about (cf. Bocheński 1963, 5). Unfortunately, Prantl's estimation of Stoic logic, echoed by Eduard Zeller with somewhat more scholarly decorum, but with little more skill and understanding, was to remain for the most part unchallenged until late in the first half of the twentieth century when it was called into question, first, by Jan Łukasiewicz, and later, by Benson Mates.

[1] *Geschichte der Logik im Abendlande*, 1.404. Translated by Mates [1953, p. 87].

Handbook of the History of Logic. Volume 1
Dov M. Gabbay and John Woods (Editors)

Prantl's work, according to Łukasiewicz, although "indispensable ... as a collection of sources and material,... has scarcely any value as an historical presentation of logical problems and theories" [Łukasiewicz, 1967, p. 67]. Moreover, since neither Prantl, nor Zeller, nor any other of the older historians of logic, had any understanding of the difference between the logic of terms, which was Aristotle's logic, and the logic of propositions, which was the logic of the Stoa, there exists no history of the logic of propositions, and thus, no complete understanding of the history of formal logic. For this reason, "the history of logic must be written anew, and by an historian who has fully mastered mathematical logic" [Łukasiewicz, 1967, p. 67].

In the interest of setting the historical record straight, one cannot help but endorse this project, and we ought to be ever grateful not only to Łukasiewicz, but also to Mates, Bocheński, the Kneales, and others for their contributions toward this end. It should be noted, however, that there is a danger that the historian of logic possessing this requisite mastery of mathematical logic may allow his or her familiarity with the discipline to obscure, or even distort, the historical enterprise. When viewing the past from the perspective of contemporary doctrines, it is sometimes all too easy to succumb to the appeal of a 'convergence' theory of history, and to assume that one's predecessors, if only they had got it right, would have come to the same place we now occupy. At any rate, there seems to have been a tendency toward such a view among several modern commentators. These writers seem to presume that the Stoics had in mind to develop a formal logic along the lines of the modern propositional calculus, and their respective appraisals of Stoic logic might be seen as dependent on their estimates of the degree of success to which this goal was carried through.

It must be admitted, however, that there are some texts which would seem to justify such a presumption; on the other hand, all of these texts can be, and have been, called into question. To consider two examples, there are texts which may be taken to indicate that, in general, the Stoics defined those logical constants having a role in their syllogistic as binary connectives; other texts may be taken to support the view that Stoic logic was a formal logic—formal, that is, in the specialised sense in which mathematical logic is formal, namely, the substitutional sense.

As to the first of these suppositions, it is surely false that the logical connectives which appear in the Stoic syllogisms were in general defined as binary connectives. In particular, consider διεζευγμένον , which is evidently the notion of disjunction occurring in the fourth and fifth Stoic syllogisms.[2] This disjunction, represented at the linguistic level by the connective particle 'ἤ' (or 'ἤτοι'), is assumed by not a few writers to be the 'exclusive' disjunction defined by the matrix 0110.[3]

[2]The fourth syllogism may be represented by the *schema* 'α or β; but α; therefore, not β', and the fifth by the *schema* 'α or β; but not α; therefore, β'.

[3]Łukasiewicz [1967, p. 74]; Bocheński [1963, p. 91]; Mates [1953, p. 51]; Kneale and Kneale [1962a, p. 148]; Gould [1970, p. 72].

The reason for this assumption, according to some writers at least,[4] is that an inclusive notion of disjunction will not support both the fourth and fifth Stoic syllogisms, whereas the exclusive disjunction will. It can be shown, however, that διεζευγμένον, the Greek notion of disjunction which validates both the fourth and the fifth syllogisms, is not the exclusive disjunction of modern logic in which the connective is defined as a binary operator satisfying the matrix 0110.

Now it is not at all clear whether the Stoics viewed διεζευγμένον as a purely truth-functional notion of disjunction, but it seems evident from the texts which mention a disjunction consisting of more than two disjuncts that if the Stoic disjunction were to be characterized truth-functionally, then the general truth condition (i.e. the truth condition for the occurrence of two or more disjuncts) would be that it is true whenever exactly one of its clauses is true.[5] On the other hand, it can be shown by means of a simple inductive proof that the general truth condition for the modern exclusive disjunction is that an odd number of disjuncts be true. As a consequence of this result, it is apparent that even if the Stoic disjunction is given a truth-functional interpretation, its truth will coincide with that of the 0110 disjunction only in the two-disjunct case.

It seems evident, then, that we can rule out the assumption that the Stoic disjunction and the modern exclusive disjunction are identical; hence, we can not take their alleged identity as evidence for the conclusion that the Stoic disjunction was viewed as a binary connective. Furthermore, there seems to be nothing in the texts to force the interpretation of διεζευγμένον as even having a fixed 'arity',[6] and in particular, nothing to force the interpretation that it has arity two. But if a Stoic disjunction has no fixed arity, then we are not required—indeed, we are not *allowed*—to treat an n-term disjunction as a two-term disjunction by the use of some bracketing device. This proscription has consequences for the idea that Stoic logic was formal in the substitutional sense, an issue which we shall take up later on in this article.

If it is neither true that the logical connectives occurring in the Stoic syllogisms are in general binary operators, nor that Stoic logic is a formal system in the substitutional sense of formal, then the modern historians and commentators who have affirmed the contrary have misunderstood and misrepresented Stoic logic no less than have the earlier historians of logic, such as Prantl and Zeller. The obstacle for both the later and earlier writers, it seems to us, is that they have allowed their preconceptions to obscure their understanding. It is obvious from their writings that Prantl and Zeller held to the general conviction of their era that the intellectual achievements of the Hellenic period in Greece were few indeed. And no doubt this view would have affected their ability to provide a balanced account of Stoic philosophy in general, and Stoic logic in particular. As for the writers who published in the early and middle years of this century, they no doubt

[4]Mates [1953, p. 52]; Bocheński [1963, p. 91].

[5]cf. Aulus Gellius *Noctes Atticae*. 16.8.13-14; Galen *inst. log.* 12; PH 2.191.

[6]The 'arity' of a binary operator is 2; that of a ternary operator, 3; of a quaternary operator, 4; and so on.

were influenced by the tendency, experienced at some time or other by most of us familiar with modern formal logic, to suppose that there really is no other notion of logic.

Be that as it may, we are nevertheless indebted to these logicians and historians of logic for having rescued the logic of the Stoa from the lowly status to which it was relegated at the hands of Prantl and his contemporaries. That having been said, it needs also to be said that we ought now to move out from the shadows of Łukasiewicz, Bocheński, and Mates, and attempt an interpretation of Stoic logic less coloured by a reverence for modern formal systems, and more in harmony with what the texts seem to indicate as being the place of logic in the Stoic system as a whole. This point of view is well expressed by Charles Kahn:

> We may not have an accurate picture of Chrysippus' enterprise in "dialectic" if we see it simply as a brilliant anticipation of the propositional calculus. No doubt it could not be accurately seen at all *until* it was seen in this way, again by Lukasiewicz and then more fully by Mates. But now that their insights have been assimilated, I think it is time to return to a more adequate view of Stoic logic within the context of their theory of language, their epistemology, their ethical psychology, and the general theory of nature [Kahn, 1969, p. 159].

The elements of Stoic philosophy mentioned by Kahn— epistemology, theory of language, ethical psychology, and a general theory of nature—are just the elements viewed as extraneous to the logician's enterprise by the early modern commentators. Indeed, the creation of contemporary formal logic by Frege required, in the words of Claude Imbert, "[the] gradual and piecemeal disintegration of a logical structure built by or borrowed from the Stoics" [Imbert, 1980, p. 187]. On this account, it seems evident that any attempt to understand Stoic logic as a formal calculus must fail; moreover, it would seem that anyone wishing to provide an adequate understanding would be constrained to do so as part of a reconstruction of the logical edifice built by the Stoics. The present essay is one attempt to formulate such an interpretation.

Leaving aside the matter of setting straight the historical record, one might ask what the worth of studying an ancient logic such as that of the Stoics might be. The answer to this question, it seems to us, lies in how one views the nature of Stoic logic itself. For our part, we believe that Stoic logic developed out of a desire to provide an account of the inferences one could make concerning the natural course of events, such inferences depending on premisses based in the perceptual knowledge of the occurrence of particular events or states of affairs, and in a general knowledge of relationships discovered in nature between events or states of affairs of certain types.

The relationships between events or states of affairs which the Stoics referred to as 'consequence' (ἀκολουθία) and 'conflict' (μάχη), are represented in the Stoic syllogistic in the major premisses of four of their five basic inference *schemata*. Particular events or states of affairs are represented in the minor premisses. Knowl-

edge of these relationships and particular events is based on certain perceptual structures called 'presentations' (φαντασίαι). Associated with these presentations as their content are conceptual structures called *pragmata* (πρᾶγματα), and associated with the *pragmata* are 'propositional' structures called *lekta* (λεκτά). According to the Stoic theory, we proceed from language and thought to the world, and to language and thought from the world, through the media of these various structures.

This theory suggests a different paradigm from that of present-day formal logic. It would seem to imply an understanding of logic as a human linguistic practice—a theory of inference rather than of inferability. Given the difficulties encountered so far in attempts to develop automated inference systems based on the paradigm of modern formal logic, it may be worthwhile to attempt a formalisation of the Stoic semantic theory in the hope that such a formalisation would provide a more successful alternative. The first step in such an enterprise would be to try to develop as clear an understanding of the Stoic theory as is possible.

2 HISTORICAL SURVEY

On the assumption that more than a few readers will be unfamiliar with early Stoic philosophy, and since this essay is an interpretation of certain logical doctrines of the Old Stoa, it would seem appropriate to present first a brief historical sketch of the Stoic School, and, in particular, of the philosophers of the early Stoa.

2.1 The influence of Stoicism

The first Stoic was Zeno of Citium who founded the school some time near the turn of the century between the third and fourth centuries B.C. The last Stoic, according to Eduard Zeller [Zeller, 1962, p. 314], was Marcus Aurelius, the Roman emperor who died in 180 A.D. Of course, as J. M. Rist points out [Rist, 1969c, p. 289], one ought to understand this claim not in a literal sense, for there were Stoics who came after Aurelius, but rather in the sense that with his passing the school came to an end as a recognisable entity. Hence the Stoic School was extant for a period of almost five hundred years, a remarkable achievement by any standard. But it is not only the longevity of the school which would seem exceptional, for it might be claimed, as at least one scholar has done, that "Stoicism was the most important and influential development in Hellenistic philosophy" (A. A. Long [1986], 107). The basis for this claim lies in part at least in the far-reaching domain of Stoic doctrines. For according to Long, not only were Stoic teachings prevalent among a large segment of the educated population in the Hellenic era, but also their influence is apparent in various intellectual spheres during the early post-Hellenic period as well as from the Renaissance up to fairly recent times. The tenets of Christian philosophy, for example, exhibit certain evidence of Stoic bias, as do the moral precepts of Western civilisation in general. Moreover, the manifestations of such influence would seem apparent in the realm of secular literature and thought

as well, reaching a peak, according to Long, between 1500 and 1700 (1986, 247). One scholar even perceives the presuppositions of Stoic logical principles at work in the writings of the Alexandrian novelists and poets (Claude Imbert [1980]). In later philosophy, according to Long, Stoic canons are evident in the writings of men as diverse in their beliefs as the religious philosopher Bishop Butler and the metaphysicians, Spinoza and Kant (1986, 107). And according to Imbert: "[The] gradual and piecemeal disintegration of a logical structure built by or borrowed from the Stoics was a necessary preliminary to Frege's formulation of a sentential calculus and to the conception ... of such a calculus as an independent system" [Imbert, 1980, p. 187].

But if Stoicism has indeed found expression in such widespread and various areas of thought, then why, one might ask, does it seem that in the English-speaking world at least, Stoic philosophical theories have been so little studied in recent times as compared with the treatises of Plato and Aristotle—aside, that is, from the chiefly moralistic writings of the later Stoics such as Marcus Aurelius. The answer no doubt lies, at least in part, in the fact that virtually none of the records of the early Stoics has survived the vagaries of time. We are fortunate to have mined in their extant writings an abundant source from which we may develop a deep appreciation of the philosophical thought of Plato and Aristotle, but there is no mother lode of philosophical literature from which to develop a rich understanding of Stoic thought. Those direct sources which have survived consist in a few badly damaged papyri salvaged from the ruins of Herculaneum. For the most part, however, one must rely on the reports of various commentators of uneven reliability, authors such as Sextus Empiricus,[7] Diogenes Laërtius,[8] Galen, Cicero, Stobaeus, Plutarch, Alexander of Aphrodisias, and Aulus Gellius, to name a few. Many of these reports are presented in the form of doxography, and many are second-hand; some, however, possess the merit of having direct quotations appearing in them, and it is from these quotations that the most reliable information can be gleaned. But even having quotations available is no guarantee that one is getting an unbiased account. For much of the commentary by authors such as Plutarch,

[7]Sextus Empiricus (*circa* A.D. 200) was a Greek physician and sceptic philosopher who succeeded Herodotus of Tarsus as head of the Sceptic School. Sextus' critique of Stoic philosophical doctrine is covered in a series of eleven books under the general title of *Adversus Mathematicos* (abbreviated AM), and a series of three books under the title *Outlines of Pyrrhonism* (abbreviated PH). Stoic logical doctrine is contained for the most part in Books 7 and 8 of AM and in Book 2 of PH. Physical tenets are covered in Books 9 and 10 of AM and in Book 3 of PH. Ethical teachings are criticised in Book 11 of AM and in Book 3 of PH. Sextus' account of Stoic Philosophy is probably one of the most extensive of the ancient commentaries. However, because of it polemic nature, its value is perhaps less than it might been.

[8]After Sextus Empiricus, Diogenes Laërtius *(circa* A.D. 200-250?) provides the next most extensive account of Stoic doctrine. Much of what he writes on the Stoics corroborates what is written by Sextus Empiricus, but in contrast to the latter's account, Diogenes' report has the advantage that it is not in the least polemical. It is fortunate that in Book 7 of *Lives and Opinions of Eminent Philosophers*, the section in which he covers the Stoic School, Diogenes draws on a handbook written by Diocles of Magnesia. His account of Stoic logic, therefore, is widely considered to be reliable. Since he is not in general considered a reliable source, it might have been otherwise.

Alexander, Sextus Empiricus, Cicero, and to some extent, Galen, is polemical in tone and clearly inimical to various Stoic doctrines. Hence, one suspects that quotations are often chosen not to illustrate a point in a positive mode, but rather to show up perceived absurdities and inconsistencies in Stoic thought. Given problems of this kind, it is clear that the road to an understanding of Stoicism could not be as free from impediment as that to an understanding of Plato and Aristotle.

A further reason why Stoic philosophy has been by comparison so little studied in recent times might be the bad press it received at the hands of the nineteenth century historians of philosophy, Prantl and Zeller.[9] Prantl apparently took to heart Kant's pronouncement that "since Aristotle ... logic has not been able to advance a single step, and is thus to all appearance a closed and completed body of doctrine."[10] Hence, in his *Geschichte der Logik*, he was especially critical of Stoic logical doctrines, attacking with a vehemence curiously out of place in what is supposed to be a scholarly work, not only these doctrines themselves, but also the men who authored them. Nor did Stoic logic fare much better at the hands of Zeller, although it must be admitted that his critique lacks to some extent the intense personal animosity characteristic of Prantl's writings. According to Zeller, "no very high estimate can be formed of the formal logic of the Stoics" [Zeller, 1962, p. 123], and "the whole contribution of the Stoics in the field of logic consists in ... clothing the logic of the Peripatetics with a new terminology" [Zeller, 1962, p. 124]. But it was not only the field of Stoic logic that received such negative assessment from Zeller, for in his judgement the Hellenistic era was characterised by a general decline in the quality of intellectual life, and by the particular decline in the virtue of the philosophical enterprise (cf. Long [1986, p. 10; 247]). There would seem to be little doubt, according to Long, that the estimates of Prantl and Zeller in favour of Platonic and Aristotelian philosophy and against Hellenistic philosophy carried a good deal of weight among succeeding historians of the subject [Long, 1986, p. 10]. Perhaps no further reason than this need be sought to account for the want of a general interest in Stoic philosophy in the modern era.

2.2 The philosophers of the Early Stoa

Traditionally, the Stoic era has been divided into three periods: the Early Stoa (often the Old Stoa), which begins with Zeno and ends with Antipater; the Middle Stoa, which covers the leadership of Panaetius and Posidonius; and the Late Stoa, which is represented by Seneca, Epictetus, and Marcus Aurelius.[11] Since this

[9]We have relied on the commentaries of Benson Mates [1953, pp. 86–90] and I. M. Bocheński [1963, pp. 6 8]; [1963, pp. 5 6] for an assessment of Prantl's contribution to the history of Stoic logic.

[10]*Critique of Pure Reason*, Unabridged Edition. Translated by Norman Kemp Smith. Macmillan, 1929. New York: St. Martin's Press, 1965.

[11]See, for example, A. A. Long, [1986, p. 115]; Jonathan Barnes, *Oxford History of the Classical World*, p. 368.

essay will be concerned mainly with the logical doctrines of the Early Stoa, and in particular the doctrines of Zeno, Kleanthes, and Chrysippus, we shall have little to say in this introduction about the Middle and Late periods of Stoic history; indeed, given these restrictions, we shall not have much to say about those Stoics such as Diogenes of Babylon, Antipater of Tarsus, and several others, who came after Chrysippus but yet belong to the Old Stoa. Furthermore, the scope of this introduction will not permit more than a cursory glimpse of the lives of Zeno and his two immediate successors, nor will it allow more than a brief and somewhat arbitrary survey of their philosophical doctrines.

Zeno (333-261 B.C.)

As mentioned above, the Stoic School was founded by Zeno, a native of Citium on the island of Cyprus. Born around 333/2 B.C., Zeno is reported to have arrived in Athens at about the age of twenty-two and to have begun his teaching about 300/301 B.C.[12] By way of establishing himself as a teacher, he chose to discourse with his followers in the 'Painted Colonnade' (*Stoa Poikilē*), where Polygnotus' paintings of the Battle of Marathon were displayed. Hence, his followers, known first as the 'Zenonians', came to be known as 'the men of the Stoa' or 'the Stoics' (DL 7.5).

In the period between his arriving in Athens and his starting the Stoic School, Zeno engaged in the development of his formal philosophical education. He apparently began this training by studying with the Cynic philosopher Crates (DL 7.4), but though it would seem fairly certain that Zeno's first formal training in philosophy was in the doctrines of the Cynics, it is possible that he had prepared himself for the study of philosophy by reading the many books on Socrates which his father, a merchant, had brought home from his trips to Athens (DL 7.31). After the period of study with Crates, Zeno became a pupil of the Megarian philosopher Stilpo and of the Dialectician Diodorus Cronus (DL 7.25). As well, he "engaged in careful dispute" with the Dialectician Philo of Megara, who was also a student of Diodorus Cronus (DL 7.16). He is further reported to have studied with the Academic philosophers Xenocrates and Polemo (DL 7.2), but given the chronology cited above, there is some doubt that he actually did study with the former (cf. Zeller [1962, 37n1]). On the other hand, there is confirmation that he was a pupil of Polemo, for Cicero explicitly states that this was so (*de fin.* 4.3), and Diogenes Laërtius attests that he was making progress with Diodorus in dialectic when he would enter Polemo's school (7.25). It may be, however, that his attendance at

[12]Much of the information on the lives and doctrines of the early Stoics come from Diogenes Laërtius' *Lives and Opinions of Eminent Philosophers*. However, we have also relied heavily on Reichel's *Stoics, Epicurians, and Sceptics*, which is translated from the third volume of Zeller's *Die Philosophie der Griechen*, as well as on Long's *Hellenistic Philosophy*, on Long and Sedley's *The Hellenistic Philosophers*, on Sandbach's *The Stoics*, on Rist's *Stoic Philosophy*, and on Barnes' article in *The Oxford History of the Classical World*, pp. 365-85.

It is evident from Diogenes account that there was some dispute concerning the important dates in Zeno's life. By way of reconciling this problem, we will report the alternatives which Long and Zeller suggest to be the most reliable.

Polemo's lectures was somewhat surreptitious, for according to Diogenes, Polemo is said to have reproached him thus: "You slip in, Zeno, by the garden door—I am quite aware of it—you filch my doctrines and give them a Phoenician make-up."[13] However that may be, it is in light of Cicero's further testimony that Zeno's association with Polemo is significant, for he maintains that Zeno had adopted Polemo's teaching on the primary impulses of nature (*de fin.* 4.45), as well as on the doctrine that the *summum bonum* is 'to live in accordance with nature' (*de fin.* 4.14).

There is some controversy among modern scholars whether the Peripatetics had any influence on Zeno's philosophical education, and, assuming they did have, in what it might have consisted. Some commentators propose that there was such influence,[14] but it would seem that such proposals should be regarded as conjecture, for unlike the situation with the Cynics, the Megarians, the Dialecticians, and those from the Academy, "there is no ancient evidence concerning Zeno's relationship with Theophrastus or other Peripatetics" Long [1986, p. 112].[15] In addition, many writers insist that certain Stoic doctrines must have developed either as an extension of Aristotelian theories, or, as a reaction to them, and underlying this contention is the assumption that the early Stoics must have had available, and made a close study of, the corpus of Aristotelian literature which we have available to us.[16] In his monograph "Aristotle and the Stoics," F.H. Sandbach conducts a careful study of these claims and of the assumptions which underlie them. He contends that this investigation supports the theory that, barring a few exceptions, this corpus of Aristotle's works (which he refers to as the 'school-works') was not available to the early Stoics [Sandbach, 1985, p. 55]. On the other hand, where certain Stoic doctrines are similar in content to passages in the school-works, this similarity may be explained by the hypothesis that an analogous passage occurred in an 'exoteric work',[17] and some Stoic had read it there. Alternatively, the early Stoics may have come across these teachings in oral form, either as explicitly attributed to Aristotle or as recognised Peripatetic doctrine. Another possibility, one apparently acknowledged by few, if any, writers, is that the Stoics may simply have thought of these doctrines independently of Aristotle [Sandbach, 1985, p. 55]. Sandbach concludes that for the most part the evidence will not support the probability, let alone the certainty, that Aristotle was an influence on the origin

[13]Sandbach: "... Citium, once a Greek colony, was [at the time of Zeno's birth] predominantly Phoenician in language, in institutions, and perhaps in population. Zeno's contemporaries who called him a Phoenician may have been justified in so doing, but he must be imagined as growing up in an environment where Greek was important" [Sandbach, 1975, p. 20].

[14]Andreas Graeser, for example, in his *Die logischen Fragmente des Theophrast* assumes that Zeno was a pupil of Theophrastus (44).

[15]Cf. F.H. Sandbach's reference "the striking absence of explicit evidence that the early Stoics took an interest in the work of Aristotle or of his following in the Peripatos" [Sandbach, 1985, p. 55].

[16]This view is expressed by Sandbach in his concluding paragraph [Sandbach, 1985, p. 55], and evidence for it is well-documented throughout his monograph.

[17]Sandbach's explanation of this term is as follows: "... Aristotle did write some works, now lost, of which some were dialogues, intended for a wider public than the students who were attached to his school. Later scholars, and probably Aristotle himself, referred to these as 'exoteric"' [Sandbach, 1985, p. 1].

of Stoic doctrines [Sandbach, 1985, p. 57]. Moreover, he quite forcefully expresses the view that "it is a mistake to proceed on the a priori assumption that the Stoics must have known the opinions expressed in his school-works, must have understood his importance sub specie aeternitatis, and must therefore have been influenced by him" [Sandbach, 1985, p. 57]. It is important to see that Sandbach is not ruling out the possibility of Aristotelian or Peripatetic influence, but rather urging a more careful and less biased approach to the question.

As might be expected, the foundations for many of the central tenets of Stoicism can be discerned in the philosophical education of Zeno, and though it would be interesting to trace the sources of the full range of Stoic doctrines, such pursuits will be restricted in this introduction to considering those influences which would seem likely to have affected the origins of Stoic logical theory. Probably Zeno's most significant contribution to Stoic logic is his work in epistemology.

It has been widely accepted among modern commentators that Zeno received his education in logical techniques directly from the Megarians, and in particular, from Diodorus Cronus (cf. Long [1986, p. 111]; Mates [1953, p. 5]; Kneale and Kneale [1962a, p. 113]; Rist [1978, p. 388]). An article by David Sedley, however, would seem to raise some doubt that this assumption can be maintained. What is called into question is not whether Zeno was indeed a student of Diodorus, for this would seem to be beyond dispute, but rather whether Diodorus himself can be established as belonging to the Megarian School.

The Megarian School was founded by Euclides, a pupil of Socrates (DL 2.47) and a native of Megara on the Isthmus (DL 2.106). He was succeeded as head of the school first by Ichthyas and later by Stilpo, also a native of Megara in Greece (DL 2.113). Evidently, since Diodorus can trace his philosophical lineage back to Euclides through Apollonius Cronus and Eubulides (DL 2.110-11), it has been generally thought that he also was a member of the Megarian school; hence, the Megarian connection with respect to the source of Zeno's logical doctrines would seem assured. Sedley, however, has presented what seems to us a convincing argument to the effect that Diodorus belonged rather to a rival school which was called the Dialectical School (Sedley, [1977, pp. 74–75]; cf. Sandbach, [1985, p. 18]).

At 2.106 Diogenes reports that the followers of Euclides were called Megarians after his birthplace. Later they were called Eristics, and later still, Dialecticians. Sedley argues for the possibility that these remarks should not be interpreted, as they usually are, to mean that this was one and the same school known at different times by different names, but rather that these names designated splinter groups whose *raisons d'être* were different enough from that of the Megarian School to warrant viewing them as distinct schools [Sedley, 1977, p. 75]. According to Sedley, several sources inform us that the Dialecticians recognised Clinomachus of Thurii, a pupil of Euclides, as the founder of their school [Sedley, 1977, p. 76]. However, since the name 'Dialectician' was first coined for the school by Dionysius of Chalcedon (DL 2.106), an "approximate" contemporary of Diodorus (Sedley [1977, p. 76]), it seems more likely not that Clinomachus actually founded the

school, but rather that he was recognised by its members as the source of the ideas foremost in their teachings [Sedley, 1977, p. 76]. As Sedley points out, practically nothing is known of Clinomachus' doctrines, except for the fact, reported by Diogenes Laërtius (2.112), that he was the first to write about *axiōmata*[18] (ἀξιώματα) and predicates (κατηγορήματα). But this fact is of "utmost significance" [Sedley, 1977, p. 76], for both of these notions, as will become apparent in the sequel, are fundamental constituents of the conceptual apparatus in Stoic logic.

The Dialectician Diodorus and his pupil Philo apparently engaged in a famous debate about the criterion according to which the consequent of a conditional *axiōma* (proposition) 'follows' from the antecedent (PH 110-12; AM 112-17).[19] The controversy between these logicians was apparently of interest not only to themselves, for Sextus Empiricus reports that Challimachus, who served in mid-third century B.C. as the chief librarian at the great library in Alexandria, wrote to the effect that even the crows on the rooftops, having repeatedly heard the debate, were cawing about the question which conditional *axiōmata* are sound (AM 1.309-10). The debate was extended by the introduction of two additional accounts, one of which is attributed to Chrysippus on fairly strong evidence (e.g., Gould, [1970, pp. 74–76]), but the other not attributed to any particular philosopher or even to any particular school. Given Zeno's association with Diodorus and Philo it seems fairly certain he would have taken part in the debate; moreover, he wrote a book *On Signs* (περὶ σημείων) (DL 7.4), and a 'sign' is defined by the Stoics as "the antecedent *axiōma* in a sound conditional, capable of revealing its consequent" (AM 8.245). Hence he would have had an interest in the criterion for a sound conditional axiōma, perhaps even offering a view of his own. We will argue in the sequel (see page 142) that given his motivation and particular interests in logic, it is unlikely that he would have opted for the view of either Diodorus or Philo. And since the third account can be attributed to Chrysippus with some certainty, one may conjecture that Zeno supported the fourth view. Furthermore, if Long and Sedley are correct, there may be no significant difference between the third and fourth statements of the criterion [Long and Sedley, 1990, 1.211], so that one might suppose, as long as we are in the realm of speculation, that the third criterion represents a tighter version of the fourth account.

Evidently, then, there is some reason to believe that Zeno was influenced by Diodorus and Philo through a familiarity with these ideas, and that it was the Dialectic School rather than the Megarian School which was the important inspiration for the development of Stoic logic (Sedley [1977, p. 76]). It may be, nevertheless, that Stilpo played some part as an influence on Zeno's logical education, and indeed, there may be some overlap in certain areas, for after all, if what has been argued above is correct, the source of the logical doctrines of both

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[18] *Axiōmata* are somewhat akin to propositions, but differ in some important ways. For a discussion see section 6 on page 463.

[19] For the details on this debate, see Section 8.1.

the Megarians and the Dialecticians is, for the most part, one and the same.[20] Given that Stilpo was one of those who came to listen to Diogenes the Cynic (DL 6.76), it is possible, as Sandbach submits [Sandbach, 1975, p. 22], that it was not Stilpo's logical tutoring but rather his moral teachings which attracted Zeno to the Megarian School, for these instructions were probably not unlike those of the Cynics. On the other hand, J.M. Rist puts forward the hypothesis that Zeno became dissatisfied with Cynic ethical doctrine and its rather circumscribed concept of 'life according to nature', and so was looking to develop an account of nature with a basis in physical theory—any such account being rejected by the Cynics [Rist, 1978, pp. 387-88]. The difficulty in such an undertaking is summed up by Rist as follows:

> In Zeno's time and before ... the problems confronting a philosopher who has come to the conclusion that he must embark on the study of nature ... is that Parmenides and his Eleatic successors had attempted to rule out such study altogether, and before it could be taken up, philosophers deemed it desirable to propose ways by which Parmenides' ban could be overcome [Rist, 1978, p. 388].

Hence "for the would-be φυσικός the acquisition of a certain familiarity with Eleatic procedures would be a *sine qua non*" [Rist, 1978, p. 388]. According to the testimony of Diogenes Laërtius, Euclides "applied himself to the writings of Parmenides" (2.106), and since Stilpo was a pupil of Euclides (or at least, of one of Euclides students) (2.113), he would no doubt be familiar with the arguments of Parmenides; hence, it is possible that Zeno was attracted to him for this reason. And certainly in the Stoic theory of a coherent and continuous universe held together by a pervasive *pneuma* immanent in all matter, (e.g., Alexander *de mixtione* 216.14 17) there is some evidence that Zeno took up Parmenides' thesis of the unity of being.

Another possibility, not necessarily an alternative, is that Zeno was attracted by Stilpo's fame in the posing of logical problems and fallacies. One account has it that Zeno once paid twice the asking price for seven dialectical forms of the 'Reaper Argument', so great was his interest in such things (DL 7.25). Probably of more reliability, we have Plutarch's testimony that Zeno would spend time solving sophisms and would encourage his pupils to take up dialectic because of its capacity to assist in this endeavour (*de Stoic repugn.* 1034e). Furthermore, Stilpo

[20]It should be noted in passing that it cannot be assumed that either Sextus Empiricus, or any other late commentator or doxographer, in using the term '*hoi dialektikoi*', is referring specifically to the Dialectical School. Sedley points out at least two reasons for supposing that this is so: first, the term '*dialektikos*', was commonly used to designate anyone who used the method of argumentation from which the Dialectical School got its name, that is, the method of putting an argument in the form of question and answer (75; cf. DL 2.106); moreover, to quote Sedley, "by the time of Chrysippus, in the late third century, [dialektikos] is the standard term for 'logician"' [Sedley, 1977, p. 75]. Now since the Stoics were recognised for their logical acumen above and beyond any rival school, it seems more likely that when Sextus refers to '*hoi dialektikoi*', for the most part, he means the Stoics.

is reported by Diogenes Laërtius to have "excelled all the rest in the invention of arguments and in sophistry" (DL 2.113). The story goes that during a banquet at the court of Ptolemy Soter he addressed a dialectical question to Diodorus Cronus which the latter was unable to solve. Because of this failure, Diodorus was reproached by the king and subsequently received the derisive name 'Cronus'. This caused him so much anguish that he wrote a paper on this logical problem and "ended his days in despondency" (DL 2.112). Even taking into account the likelihood that this story might be somewhat apocryphal, it probably can be taken as a reliable indicator of Stilpo's skill as an inventor of logical arguments and puzzles. As an aside, recalling Sedley's argument cited above, it would also seem to point to a certain tension between Stilpo and Diodorus, a tension that one would not normally expect if they had been members of the same school.

Another logical doctrine in which Zeno may have been influenced by Stilpo is that concerning his rejection of the 'forms' or 'ideas'. According to Diogenes Laërtius, Stilpo used to demolish the forms or universals, saying that whoever asserts the existence of Man refers to nothing, for he neither refers to this particular man nor to that; hence, he refers to no individual man (DL 2.119). And according to the testimony of Stobaeus. similar opinions were held by Zeno, for he and his followers relegated such 'ideas' or 'concepts' (ἐννοήματα) to a sort of "metaphysical limbo", referring to them as 'pseudo-somethings' (ὡσανεί τινα) (*eclog.* 1.136.21). It should be noted that this stance does not represent a rejection of all those things which we refer to as 'universals'. Common nouns such as 'man' or 'horse' were taken to refer to the essential quality and all qualities are *corporeal* which made something either a man or a horse (DL 7.58). It is if we were to use the term 'man' to refer to the genetic material which differentiates us from other creatures.

Kleanthes (331-232 B.C.)

After the death of Zeno in 261/2 B.C., Kleanthes, a native of Assos on the Troad, became head of the Stoic School (DL 7.168). According to the historian Antisthenes of Rhodes, Kleanthes was a boxer before taking up philosophy (DL 7.168). Upon his arrival at Athens he fell in with Zeno and was introduced to Stoic teachings which, in spite of having no natural aptitude for physics and of being extremely slow (DL 7.170), he studied "right nobly", remaining faithful to the same doctrines throughout (DL 7.168). Zeno compared him to those hard waxen tablets which are difficult to write on but which retain well the characters written on them (DL 7.37). Kleanthes was perhaps the most religious of the Stoics, as witness his well-known *Hymn to Zeus.*[21] He was acclaimed for his industry and perseverance (DL 7.168), for it was said that he came to Athens with only four drachmas in his possession (DL 7.168), and so was forced to work drawing water for a gardener by night in order to support himself while studying philosophy by day (DL 7.169).

[21] The Greek text is in Stobaeus *Eclogae Physicae et Ethicae*, vol. 1, page 25, line 12 to page 27, line 4. The text is translated in Long and Sedley [1990, 1.326–327]. There is also a translation in somewhat more archaic (poetical?) language in Sandbach [1975, pp. 110–111].

Apparently he never got far beyond this impecunious state, for the story goes that he was too poor even to buy paper, and so used to copy Zeno's lectures on oyster shells and the blade-bones of oxen (DL 7.174). At some point, however, he must have gained access to a supply of writing material, for Diogenes has compiled a list of his writings which includes about sixty books.

Of these sixty books catalogued, a series of four is listed under the title *Interpretations of Heraclitus* (DL 7.174). A connection with Heraclitus is also indicated by the report of Arius Didymus to the effect that Kleanthes, comparing the views of Zeno with those of other natural philosophers, says that Zeno's account of the soul or *psychē* (ψυχή) is similar to that of Heraclitus (fr. 12 DK; DDG 470.25–471.5). Other than a book under the title *Five Lectures on Heraclitus* attributed to the Stoic Sphaerus, a pupil of Kleanthes (DL 7.178), there is little other direct evidence to support the hypothesis of the influence of Heraclitus on Stoicism and on Zeno in particular. This hypothesis is expressed, for example, in the following statement by A.A. Long: "Heraclitus' assumption that it is one and the same *logos* which determines patterns of thought and the structure of reality is perhaps the most important single influence upon Stoic Philosophy" [Long, 1986, p. 131]. It is also expressed by G.S. Kirk, but with reservations:

> Although Zeno must have based his physical theories particularly upon Heraclitus' description of fire, he is never named in our sources as having quoted Heraclitus by name; while Kleanthes evidently initiated a detailed examination of Heraclitus with a view to the more careful foundation of Stoic physics upon ancient authority. . . . and there is reason to believe that he made some modification of Zeno's system in the light of his special knowledge of the earlier thinker ... [Kirk, 1962, pp. 367 68].

It is not a straightforward matter to see what can be made out from these circumstances. What seems likely is that Zeno tacitly appealed to Heraclitus in formulating his views on physics and cosmology, and that it was left to Kleanthes to make explicit this appeal, modifying the theory where it seemed appropriate to do so.

We can also look to the catalogue of Kleanthes' writings reported by Diogenes Laërtius for assistance in giving an account of Kleanthes' contribution to Stoic logical theory. There is a set of three books under the title *περὶ τοῦ λόγου* (DL 7.175) which one might take to be about logic, especially since Hicks translates this title as *Of Logic*. It seems to us, however, that *Of the Logos* would be equally possible. In light of Kleanthes' interest in Heraclitus, and in light of the apparent debt—pointed out by Long in the passage above—which the Stoics owe to Heraclitus for their concept of the *logos*, it seems less likely that these books of Kleanthes are about logic than that they are about the *logos*. There are three other titles, however, which would appear to be uncontroversially on logical topics. These are: *Of Dialectic, Of Moods or Tropoi,* and *Of Predicates* (DL 7.175). As to the first title, not much can be said, for there is little in the sources to indicate Kleanthes'

particular thoughts on dialectic as such. However, more can be said about the subjects of the other two books, for it is of some interest that Kleanthes wrote about them.

Concerning the book about *tropoi*, Galen reports that logicians (*dialektikoi*) call the *schemata* of arguments by the name 'mode' or '*tropos*' (τρόπος). For example, for the argument which Chrysippus calls the first indemonstrable (ὁ πρῶτος ἀναπόδεικος) and which we would call *modus ponens*, the mode or *tropos* on the Stoic account is as follows: If the first, the second; but the first; therefore, the second (*inst. log.* 15.7). Now according to Galen, since the major premisses (προτάσεις)[22] ιν σψλλογισμς οφ τηις σορτ (ιν τηις ςασε, τηε ςονδιτιοναλ πρεμισς) αρε δετερμινατιε (ἡγεμονικαί) of the minor premisses (προσλήψεις), Chrysippus and his followers call such a proposition or *axiōma* not only determinative but also *tropic* (τροπικόν).[23] What is of interest here is that Kleanthes' concern with *tropoi* may be an indication that he had some knowledge of the so-called indemonstrables, that is, the five argument *schemata* which Chrysippus took as the basis of the Stoic theory of inference.[24] As to the book about predicates (κατηγορήματα), we also have a passage of Clement of Alexandria which maintains that Kleanthes and Archedemes called predicates *lekta* (λεκτά) (*strom.* 8.9.26.3-4). According to Michael Frede, this testimony indicates that Kleanthes was the first philosopher to use the term '*lekton*' ([Frede, 1987b], 344). This innovation is quite significant, for the concept of the *lekton* is well recognised as possibly the most fundamental notion in Stoic semantic theory. Frede suggests, however, that this passage is evidence that the concept may have been introduced by the Stoics in the ontology of their causal theory rather than in their philosophy of language ([Frede, 1987a, p. 137]; cf. Long and Sedley [1990, 2.333]). At any rate, one might conjecture that Kleanthes had some hand in the development of this concept.

We have the testimony of Epictetus of a book on a logical topic written by Kleanthes but not recorded by Diogenes. Although he does not give its name, this work, according to Epictetus, was on the so-called 'Master Argument' of Diodorus Cronus (*disc.* 2.19.9). The Master Argument was apparently posed by Diodorus in order to establish his definition of the possible (*disc.* 2.19.2; cf. Alexander *in*

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[22]The term *protasis* (πρότασις) is used in Sextus Empiricus to refer to the major premiss of a categorical syllogism (PH 2.164; 195). Galen seems here to be extending the use of this term to refer to the major premiss of a Stoic hypothetical syllogism as well. Thus it would be interchangeable with the term *lēmma* (λῆμμα), which, according to Diogenes Laërtius, the Stoics used to refer to the major premiss of an argument (DL 7.76).

[23]In his commentary on this section of the *Institutio*, Kieffer provides the following explanation: "Galen's point in calling the major premiss in a hypothetical syllogism determinative of the minor is that the minor premiss is either one of the members of the hypothetical major or its contradictory" ([Kieffer, 1964], 92). Thus in the case where the major premiss of the hypothetical syllogism is a conditional, the minor premiss will be either the antecedent of the conditional (as in *modus ponens*) or the negation of the consequent (as in *modus tollens*). Note that 'hypothetical syllogism' here covers any syllogism which is not categorical, and for the Stoics this includes not only syllogisms with a conditional as major premiss but also those with either a disjunctive or a negated conjunction.

[24]For a more complete account of the Stoic argument *schemata* called the 'indemonstrables', see page 474.

an. pr. 184.5), this definition being 'The possible is that which either is or will be' (ὃ ἢ ἔστιν ἢ ἔσται (*in an. pr.* 184.1).[25] A definition of the possible attributed to the Stoics both by Diogenes Laërtius (7.75) and by Boethius (*in de interp.* 234.27) is that the possible is 'that which admits of being true and which is not prevented by external factors from being true' (DL 7.75). Now both Kleanthes and Chrysippus rejected Diodorus' interpretation of the Master Argument (Epict. *disc.* 2.19.6) and presumably, therefore, would have also rejected his account of the possible, and since the Stoic characterisation given by Diogenes and Boethius is not attributed to any specific Stoic, it is open to debate whether to credit it to Chrysippus or to Kleanthes. There is, however, a passage in Plutarch which would seem to indicate that Chrysippus warrants the attribution (*de Stoic repugn.* 1055d-e). However that may be, it is evident that Kleanthes had an interest in questions about modality and no doubt gave some account of the possible and the necessary.

Chrysippus (circa 282–206 B.C.)

Chrysippus of Soli[26] succeeded Kleanthes in 232 B.C. to become the third leader of the Stoic School (DL 7.168; 1.15-16). There is not much information on his early life. Hecato, the Stoic, says that he came to philosophy because the property he had inherited from his father had been confiscated by the king (DL 7.181). And there is a story that he was a long distance runner before taking up philosophy as a pupil of Kleanthes (DL 7.179). Even as a student he seemed to possess a good deal of confidence in his abilities, especially in logic, for he used to say to Kleanthes that he needed to be instructed only in the doctrines; the proofs he would discover himself (DL 7.179). His relationship with Kleanthes was somewhat troublesome to him at times. On the one hand, he showed a great deal of respect for Kleanthes, deflecting to himself the attacks of certain presumptuous dialecticians who would attempt to confound Kleanthes with their sophistical arguments. Chrysippus would reproach them not to bother their elders with such quibbles, but to direct them to his juniors (DL 7.182). On the other hand, he himself would sometimes contend with Kleanthes, and whenever he had done so, would suffer a good deal of remorse (DL 7.179).

Chrysippus apparently left the Stoic school while Kleanthes was still alive, becoming a philosopher of some reputation in his own right (DL 7.179), and on the authority of the historian Sotion of Alexandria, Diogenes tells us that he also studied philosophy for some period under Arcesilaus and Lacydes at the Academy (DL

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[25] cf. Boethius *in de interp.* 234.22 26: *quod aut est aut erit.* As Benson Mates points out, Boethius in this passage also gives definitions of the related terms 'impossible', 'necessary', and 'non-necessary', and based on the construction of these other definitions, one can conjecture that the above definition of the possible was "slightly elliptical." The full definition should have been 'that which is or will be *true*' [Mates, 1953, p. 37].

[26] According to Zeller, the view of most writers was that Chrysippus was born at Soli in Cilicia; however, since his father emigrated to Soli from Tarsus, it is possible that Chrysippus was born there instead ([Zeller, 1962], 45n5).

7.183). This would explain, according to Diogenes, his arguing at one time for common experience (συνηθεία),[27] ανδ ανοτηερ τιμε αγαινστ ιτ (ΔΛ 7.184). Βψ τηις ρεμαρκ Διογενες ις νο δουβτ ρεφερρινγ το τηε φαςτ τηατ ἤρψσιππυς ωροτε α σεριες οφ σιξ βοοκς υνδερ τηε τιτλε *Αγαινστ δμμον Εξπεριενςε* (Κατὰ τῆς συνηθείας), as well as a series of seven books under the title *In Defence of Common Experience* (Ὑπὲρ τῆς συνηθείας) (DL 7. 198). In this regard, according to Cicero, some later Stoics complained against him for providing Carneades and the Academy with arguments with which to assail against the whole of common experience, as well as against the senses and their clarity and against reason itself (*acad.* 2.87; cf. Plutarch *de Stoic repugn.* 1036b-c).

Chrysippus was an extremely prolific writer. Diogenes Laërtius reports that in all he wrote seven hundred and five books (DL 7.180), of which three hundred and eleven were on logic (DL 7.198). And Diogenes provides an inventory of about three hundred and seventy five of them, the majority of these being books on logic (DL 7.189-202). Diogenes also cites the testimony of Diocles Magnes who claims that Chrysippus wrote about five hundred lines a day (DL 7.181). It would seem, however, that in the opinion of many, such a profusion of material did not come without a price, for the ancients, according to Zeller, were unanimous in putting forward a litany of complaints against the literary style of these texts [Zeller, 1962, pp. 47–48]. However, this criticism is somewhat mitigated by Zeller's comment that "with such an extraordinary literary fertility, it will be easily understood that their artistic value does not keep pace" [Zeller, 1962, p. 47]. But whatever are the merits of these criticisms, one cannot help but speculate on how different would have been our understanding of Hellenistic philosophy had even a few of these works survived.

With the death of Kleanthes, Chrysippus returned to Stoicism to become leader of the school. In that capacity he was, "in the opinion of the ancients, ... the second founder of Stoicism" Zeller, [1962, p. 45], for it was said, according to Diogenes Laërtius, that "if there had been no Chrysippus, there would have been no Stoa" (DL 7.183). Gould takes this to refer to the belief that Chrysippus "revived the Stoa after the crushing blows dealt it by Arcesilaus and other Academics" [Gould, 1970, p. 9]. He continues:

> In antiquity, then, even outside the school, Chrysippus was regarded as an eminently capable philosopher, as an extraordinarily skilful dialectician, and as one who came to the defense of the Stoa in a crucial moment, namely, when it was about to encounter its death blow from a rival school in Athens, the Academy, which had then become the stronghold of scepticism [Gould, 1970, p. 9].

[27]Here, and in Plutarch's *De Stoicorum repugnantiis* at 1036c, as well as in Epictetus' *The Discourses* at 1.27.15-21, the term 'συνηθεία' seems to have the meaning 'common experience'; on the other hand, in Diogenes Laërtius at 7.59 it would appear to mean 'ordinary language', and in *De Stoicorum repugnantiis* at 1048a, to mean 'common use of language' (cf. the entry in Liddel and Scott, II.2).

It was no doubt his skill as a dialectician which enabled Chrysippus to defend so well the doctrines of the Stoa, for he was considered by many to have been one of the foremost logicians of Hellenic Greece. In fact, according to Diogenes Laërtius, he was so renowned for his logical acumen "that it seemed to most people, if dialectic was possessed by the gods, it would be none other than that of Chrysippus" (DL 7.180). Perhaps there would be no more fitting way to conclude this section on Chrysippus than to quote the words of Long and Sedley in their source book *The Hellenistic Philosophers*: "In the period roughly from 232 to 206 [Chrysippus] was to ... develop all aspects of Stoic theory with such flair, precision and comprehensiveness that 'early Stoicism' means for us, in effect, the philosophy of Chrysippus" [Long and Sedley, 1990, 1.3].

3 PRELIMINARIES

Several of our sources attest to a tripartite division of philosophy by the Stoics. These branches are logic, physics, and ethics (DL 7.39; Aëtius *plac.* DDG 273.11; Plutarch *de Stoic repugn.* 1035a). According to Diogenes Laërtius, Zeno of Citium, in his book *On Discourse* (*περὶ λόγου*), was the first of the Stoics to make this division (DL 7.39). Diogenes also informs us that Zeno arranged these topics with logic first, physics second, and ethics third (DL 7.40), although it is somewhat unclear whether the standard for this arrangement is according to intrinsic importance or according to teaching priorities. Perhaps what it reflects is the relationship between these parts as it is represented in one of the many similes that the Stoics drew upon for illustration. Their philosophical system, they said, is like a fertile field with logic as the surrounding wall, physics as the soil or trees, and ethics as the fruit (DL 7.40). It is clear on this conception that logic is given the task of protecting the system from external threats —the first line of defence, as it were, and the aspect of logic emphasised is skill in dialectic in the sense of mustering counter-arguments and solving sophisms (cf. Plutarch *de Stoic repugn.* 1034e). But there is another aspect of logic in which the sense of dialectic stressed is that in which it signifies "the testing of hypotheses and the quest for ultimate principles or true definitions, which are the essential procedures of every metaphysician" (Long and Sedley 1.189). In *The Discourses*, Epictetus surmises that the reason why the philosophers of the Old Stoa put logic first in the exposition of their doctrine is that it is in the study of logic that one comes to understand the criterion by which one judges in other pursuits what is true. So it is, according to Epictetus, that:

> ... in the measuring of grain we put first the examination of the measure. And if we should neither first define what a *modius* is, nor first define what a scale is, how shall we be able to measure or weigh anything? So with the subject of logic, how shall we be able to investigate accurately and understand thoroughly anything of other subjects if we neither thoroughly understand nor accurately investigate that which is

> the criterion of other subjects and that through which other subjects are thoroughly understood? (*disc.* 1.17.7–8)

In the first part of this section, we explore the hypothesis that the development of the Stoic system as it is discernible in the philosophical education of Zeno followed the reverse order to that envisaged above: it evolved from a primary interest in ethics and thence to physics and logic. And the aspect of logic cultivated in this succession is that characterised both in the quotation from Long and Sedley and in the quotation from *The Discourses*: that is, logic as concerned with truth, knowledge, definitions, and other elements of reason, and, as Zeno says, with understanding "what sort of thing each of them is, how they fit together and what their consequences are" (Epictetus *disc.* 4.8.12). Given this understanding of Zeno's development of the Stoic system, we shall suggest, in the second part of this section, an interpretation of Stoic logical doctrines which may be perceived as being motivated by this evolution, doctrines propounded either by Zeno himself or by his successors.

3.1 Stoic Ethics: the Motivational Basis

From Crates and the Cynic School Zeno doubtless inherited the foundation for his moral theories. But the Cynics apparently devoted themselves only to ethics, choosing to do away with the topics of logic and physics (DL 6.103). Zeno, on the other hand, clearly did not reject these topics, for it is uncontroversial that he laid the foundations not only for Stoic ethics, but for physics and logic as well. It would seem evident, therefore, that at some point he broke ranks with the Cynics, choosing a philosophical curriculum richer than that of his mentors.[28]The point on which Zeno diverged from the Cynic path is the premiss described by Long and Sedley as "the bastion of Stoic ethics," that is, "the thesis that virtue and vice respectively are the sole constituents of happiness and unhappiness" [Long and Sedley, 1990, 1.357].[29]Αςςορδινγ το τηις ιεω, σομε τηινγσ—φορ εξαμπλε, ηεαλτη, ωεαλτη, βεαυτψ, ανδ πηψσιςαλ στρενγτη—ςαν νειτηερ βενεφιτ (ὠφέλεῖ) nor harm

[28]This thesis is put forward by J.M. Rist in his essay "Cynicism and Stoicism" which appears as Chapter 4 of his *Stoic Philosophy*.

[29]The terms which Long and Sedley render as 'happiness' and 'unhappiness' are 'εὐδαιμονία' (*eudaimonia*) and 'κακοδαιμονία' (*kakodaimonia*) respectively. Now although these are standard translations, it has been suggested that they fail to capture the notion which they are intended to express. Sandbach has the following to say on this point:

> *eudaimonia*, although something experienced by the man who is *eudaimōn*, is (perhaps primarily) something objective, that others can recognise—having a good lot in life. ... Thus the Stoics did not attempt to describe *eudaimonia* as a subjective feeling, but identified it with such things as 'living a good life', 'being virtuous', or 'good calculation in the choice of things that possess value' ... For the Stoic, who confines the word 'good' to the morally good, it is consistent that a good life is a morally good life and the well-being indicated by *eudaimonia* is unaffected by what is morally indifferent, however acceptable [Sandbach, 1975, p. 41].

In another place Long himself renders *eudaimonia* as 'self-fulfilment' ([Long, 1971], 104), a translation which probably better comprehends the meaning, although still not completely.

(βλάπτει): they are neither necessary nor sufficient for a virtuous—which is to say a *moral*—life, and thus can have no effect on one's *eudaimonia*; hence, they are called 'indifferent' (ἀχιάφορος) (DL 7.102–03). This thesis was undoubtedly an endowment to Zeno from the Cynics, for we learn from Diogenes Laërtius that for the Cynics virtue is sufficient in itself to secure happiness (DL 6.11), so that "the end for man is to live according to virtue" (τέλος εἶναι τὸ κατ' ἀρετὴν ζῆν), a credo that was echoed by the Stoics (DL 6.104; cf. Stobaeus *eclog.* 2.77.9). We also learn from Diogenes that the Cynics count as 'indifferent' whatever is intermediate between virtue and vice (DL 6.105).

The Cynic stance toward the 'indifferents' can perhaps best be understood by considering the views of Ariston of Chios, a pupil of Zeno's whom we might call a 'neo-Cynic', since he "greatly simplified Stoicism, so that it was hardly distinguishable from the attitude of the Cynics" (Sandbach [1975, p. 39]; cf. Rist [1969c, pp. 74–80]). Ariston recommended complete indifference to everything between virtue and vice, recognising no distinctions among them and treating them all the same, for the wise man, according to Ariston, "is like a good actor, who can play the part of both Thersites and Agamemnon, acting appropriately in each case" (DL 7.160). A problem for this credo is summarised by Cicero, who is setting out the opinions of the Stoic Cato:

> If we maintained that all things were absolutely indifferent, the whole of life would be thrown into confusion, as it is by Aristo, and no function or task could be found for wisdom, since there would be absolutely no distinction between the things that pertain to the conduct of life, and no choice need be exercised among them (*de fin.* 3.50; trans. Rackham).

This criticism, which was no doubt a standard reproach among the ancients, might be expressed by the observation that "[Aristo's] position robbed virtue of content" [Sandbach, 1975, p. 38]. It is unclear whether Zeno himself subscribed to this assessment; however, it would seem likely that he did. He was, after all, in attendance at Polemo's lectures, and there he would no doubt have become familiar with the view, ascribed by Cicero to Xenocrates as well as to his followers, that the 'end of goods' (*finis bonorum*) is not limited to virtue alone, but includes just those things which belong to the class which the Cynics and Ariston held to be 'indifferents' (*de fin.* 4.49; *Tusc. disp.* 5.29–30).[30]

At any rate, although he may have agreed with this critique of Cynic views to the extent of granting that such things as "health, strength, riches, and fame" (*de fin.* 4.49) have *some* value, Zeno was nevertheless unwilling to depart from Cynic tenets to the point of admitting that any of the indifferents were required for *eudaimonia*. His solution to this dilemma was to introduce a classification

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[30]The doctrine of the Academy would appear to go back to Plato himself, for in *Laws* 661a–d, he has the Athenian say that things such as health, beauty, wealth, and acute sensibility all are to be counted as goods, but only in the possession of the just and virtuous; in the possession of one who is not so, however, all these things are rather evils than goods.

of the indifferents distinguishing those which are 'according to nature' (τὰ κατὰ φύσιν), those which are 'contrary to nature' (τὰ παρὰ φύσιν), and those which are neither (Stobaeus *eclog.* 2.79.18). Indifferents which are according to nature (*ta kata physin*) are such things as health, strength, sound sense faculties, and the like (*eclog.* 2.79.20). All indifferents which are *kata physin* have 'value' (ἀξία), whereas those contrary to nature have 'disvalue' (ἀπαξία) (*eclog.* 2.83.10). A categorisation of indifferents which would seem to coincide with this division is that which distinguishes them according to those which are preferred' (τὰ προηγμένα), those 'rejected' (τὰ ἀποπροηγμένα), and those neither preferred nor rejected (DL 7.105; Cicero *de fin.* 3.51; Stobaeus *eclog.* 2.84.18). According to Diogenes Laërtius, the Stoics teach that those indifferents which are preferred (*ta proēgmena*) have value (*axia*), whereas those rejected have 'disvalue' (*apaxia*) (7.105). Thus it would appear that the class of those indifferents which are '*kata physin*' is coextensive with '*ta proēgmena*'.

Value is defined by the Stoics as having three senses, only two of which are relevant in this context.[31] Foremost, it is the property of contributing to a harmonious life, this being a characteristic of every good (ἀγαθά) (DL 7.105). However, since no goods are among the preferred (Stobaeus *eclog.* 2.85.3), this connotation of *axia* must refer only to goods and not to the preferred, and hence designates value in an absolute sense; indeed, the things which have value in this sense are called 'τιμὴν καθ' αὑτό' (eclog. 2.83.12), which may be rendered as 'value *per se*' (cf. Cicero *de fin.* 3.39 and 3.34). The second sense is that of some faculty or use which contributes indirectly (μέση) to life according to nature (DL 7.105). Things which have value according to this sense of *axia* are 'selected' (ἐκλεκτικός), in Antipater's phrase, on which account, when circumstances permit we choose these particular things rather than those: for example, health against illness, life against death, and wealth against poverty (Stobaeus *eclog.* 2.83.13). The notion of value among those indifferents having 'preferred' status, as well as the responsibility of the moral agent with respect to these notions, is well summarised by Long and Sedley in the following passage:

> This 'selective value', though conditional upon circumstances (contrast the absolute value of virtue), resides in the natural preferability of health to sickness etc. That is to say, the value of health is not based upon an individual's judgement but is a feature of the world. The role of moral judgement is to decide whether, given the objective preferability of health to sickness, it is right to make that difference the paramount consideration in determining what one should do in the light of all the circumstances. In the case of those indifferents of 'preferred' status, there will be 'preferential' reason for selecting these 'when circumstances permit'. It is up to the moral agent to decide, from knowledge of his situation, whether to choose actions that may

[31] In the third sense, according to Diogenes, "value is the worth set by an appraiser, which should be fixed according to experience of the facts, as, for example, wheat is said to be exchanged for barley plus a mule (DL 7.105).

> put his health at risk rather than preserve it, but the correctness of sometimes deciding in favour of the former does not negate the normal preferability of the latter [Long and Sedley, 1990, 1.358].

The hypothesis of *ta kata physin* as a sub-class of the indifferents, then, will permit Zeno to maintain the thesis that only virtue is good, and at the same time provides a content for his ethics. A concomitant result of this hypothesis will be to lead to the reinstatement of physics and logic as legitimate components of a philosophical education. The theory may be represented as positing three levels of maturation in the moral agent (Cicero *de fin.* 3.20-21; 4.39; Aulus Gellius 12.5),[32] so that *ta kata physin* contribute content for Zeno's theory in accordance with the level of moral development of the agent (cf. Edelstein and Kidd, 155–57). In each stage of development *ta kata physin* are associated with a category of acts to which Zeno has given the name 'appropriate acts' (τὰ καθήκοντα) (DL 7.108). At the first level, instanced by babies and young children in whom the faculty of reason or *logos* (λόγος) has not yet evolved, the agent is concerned with 'primary' things according to nature (τὰ πρώτα κατὰ φύσιν) (Stobaeus *eclog.* 2.80.7; Aulus Gellius 12.5.7). These are the things toward which natural impulse (ὁρμή) inclines us in order to preserve and enhance our own constitution (Cicero *de fin.* 3.16; Seneca *epist.* 121.14). Hence at this level an appropriate act would be just to carry out those desires which impulse urges that we do; moreover, such acts would entail no consequences for morality, since moral choice and responsibility requires rationality.

At the second stage, subsequent to the emergence of the faculty of reason in the agent, *ta kata physin* are the base (*proficiscantur ab initiis naturae*) (Cicero *de fin.* 3.22) and the impetus or *archē* (ἀρχή) (Plutarch *comm. not.* 1069e) for those acts which reason convinces us to do, or those for which, when done, a reasonable justification can be given (DL 7.108; Stobaeus *eclog.* 2.85.14). *Ta kata physin* are themselves still the objects of appropriate acts, but now it is *logos* rather than *hormē* (impulse) which is active in the agent, directing his choices (Cicero *de fin.* 3.20). Concomitantly with the emergence of *logos*, comes the capability to form 'conceptions' or *ennoiai* (ἔννοιαι). The gradual accumulation of the appropriate stock of *ennoiai* will, in the end, endow the agent with the capacity to discern the order and harmony in nature and to act in accordance with them (*de fin.* 3.21).

[32]Long and Sedley note that in *De finibus* 3.17 and 20-21, Cicero "envisages five progressive stages, each of which is represented as performance of 'proper functions' as these could evolve for a human being" [Long and Sedley, 1990, 1.368]. Edelstien and Kidd, on the other hand, stresses *De finibus* 4.39 where Cicero gives a threefold division of *ta kata physin* (*Posidonius I: the Fragments* 155). It seems to us that there is a clear relationship between this division of *ta kata physin* and the five stages of *ta kathēkonta* as they are summarised at *de fin.* 3.20, wherein the first two stages of *ta kathēkonta*, i.e. "to preserve oneself in one's natural constitution" and "to retain those things which are in accordance with nature and to repel those that are the contrary," are associated with the first division of *ta kata physin*; the second two stages, i.e. "choice conditioned by reasoned action" and "such choice becoming a fixed habit," with the second division; and the last stage, i.e. "choice fully rationalised and in accordance with nature," with the third division. Hence in our presentation we have exploited this relationship by merging these categorisations into one three-fold differentiation.

Having reached this state, the agent is at the threshold of the third stage of moral development. At this third level, *ta kata physin* are no longer the impetus for choice, but are merely the 'material' or *hyle* (ὑλή) of *ta kathēkonta* (Plutarch *comm. not.* 1069e; cf. Cicero *de fin.* 3.22-23 and Galen SVF 3.61.18); moreover, the object of *ta kathēkonta* at this level is not the attainment of *ta kata physin*, but rather wisdom of choice, which is, in effect, choice in accordance with virtue (*de fin.* 3.22); hence, it is the moral character of the agent that determines the appropriateness of the act (*de fin.* 3.59; Sextus Empiricus AM 11.200; Clement SVF 3.515). Thus, even though there may be "a region of appropriate action which is common to the wise and unwise" (Cicero *de fin.* 3.59), the appropriate acts of the wise man, unlike those of the unenlightened, are consistently motivated by reason. Appropriate acts at this stage are 'perfect' and are referred to as 'right actions' (κατορθώματα), since they contain all that is required for virtue (*de fin.* 3.24; Stobaeus *eclog.* 2.93.14).

It is evident, given the above characterisation of a 'perfect' *kathēkon* or 'right action' (*katorthōma*), that the Stoic sage will be the only one to reach the third level of moral development, for only the wise man performs right actions (Cicero *de fin.* 4.15). It would seem, therefore, that a function or task for wisdom, the lack of which was seen as a shortcoming of the Cynic view as represented by Ariston (see page 416), is found in the choices the wise man makes among *ta kata physin*. Where this Stoic version of moral preference differs from what might be called the 'common sense' account is that the object of such choices is not the attainment of particular things which accord with nature—things such as health, strength, wealth, and so on—but rather the attainment of a virtuous disposition which functions consistently in the making of such choices (Cicero *de fin.* 3.22; 3.32). Thus, even though this feature of the Stoic position was much maligned in ancient times (e.g., *de fin.* 4.46-48; Plutarch *comm. not.* 1060e), there is no doubt that Zeno's innovation provided the improvement he desired over Cynic doctrine (Cicero *de fin.* 4.43).

A related shortcoming of the Cynic view is the doctrine to the effect that "virtue needs nothing except the strength of a Socrates" and that "virtue is concerned with deeds, requiring neither a host of rules nor education" (DL 6.11). One might suspect that for Zeno the difficulty with this credo would have been the problem that it rendered the content of virtue as quite arbitrary, dependent only on the will of the wise man;[33] moreover, one might also surmise that Zeno would have questioned how the ordinary person, not endowed by nature with the strength of a Socrates, would be supposed to go about improving himself with respect to moral rectitude. This difficulty was evidently addressed by the thesis that virtue is the outcome of a developmental process. The latter premiss suggests that it should

[33]So Sandbach: "What Zeno was probably afraid of was that what might be dignified with the name of acts of will might in fact be acts of whim and caprice. Since virtue itself seemed so difficult to understand or describe, the danger of this was very real indeed. That is why so many of the Cynics give the impression of being merely irresponsible exhibitionists" [Sandbach, 1975, p. 71].

be possible to learn to be virtuous, and, indeed, the idea that virtue is or can be taught is explicitly reported as Stoic doctrine in several places (DL 7.91; Clement SVF 3.225). Now although Zeno is not mentioned as advocating this view, both Kleanthes and Chrysippus are; hence, there would seem to be no good reason not to attribute it to Zeno as well. The details of the Stoic account of how virtue is learned may be inferred from the sources;[34] what is relevant here, however, is the contrast between this thesis and the preceding description of Cynic doctrine.

Given the account of moral development outlined above, it would seem that Zeno would have been in a position to augment the Cynic dictum reported by Diogenes Laërtius that 'the *telos* for man is to live in accordance with virtue' (see page 416). The right actions or *katorthōmata* performed by the wise man are mentioned in several places by Stobaeus as being actions performed according to 'right reason' (ὀρθὸς λόγος) (*eclog.* 2.66.19; 93.14; 96.18); moreover, right reason is described by numerous sources as being equivalent to virtue (Plutarch *de virt. mor.* 441c; Cicero *Tusc. disp.* 4.34; Seneca *epist.* 76.11). Hence, the virtue of the wise man consists in the perfection of his rationality with respect to the choices he makes among *ta kata physin* (Seneca *epist.* 76.10; Cicero *de fin.* 3.22). These choices are made in accordance with his own rational nature and in accordance with the *logos* of the universe, the rationality of which he shares (DL 7.87-88; Cicero *de nat. deorum* 1.36-39; 2.78; 133; 154; Seneca *epist.* 124.13-14).

According to Diogenes Laërtius, Zeno's position concerning the *summum bonum* was that "the *telos* for man is to live harmoniously with nature" (τέλος εἶπε τὸ ὁμολογουμένως τῇ φύσει) (DL 7.87); Stobaeus, however, reports that this formulation is due to Kleanthes, whereas Zeno's statement is simply that the *telos* is "to live harmoniously" (τὸ ὁμολογουμένως ζῆν). This means to live in accordance with a single harmonious *logos*, since those who live in conflict with this are not *eudaimones* (*eclog.* 2.75.6-76.6). Chrysippus', on the other hand, said that the *telos* is "to live in accordance with experience of what happens by nature" (ζῆν κατ' ἐμπειρίαν τῶν φύσει συμβαινόντων) (*eclog.* 2.76.6-8). There would seem to be no good reason to suppose that these definitions are incompatible in any way, for according to Stobaeus, the augmentations to Zeno's statement were proposed not because Kleanthes and Chrysippus disagreed with him, but rather because they assumed that his formulation was an 'incomplete predicate' (*eclog.* 2.76.2-3) and they wished to make it clearer (*eclog.* 2.76.7).[35]

Cicero reports a further statement which the Stoics declared to be equivalent to Zeno's representation. It asserts that the *telos* is "to live in the light of a knowledge of the natural sequence of causation" (*vivere adhibentem scientiam earum rerum quae natura evenirent*) (*de fin.* 4.14). The justification for this equivalence can evidently be inferred from several passages in the sources. First, the Stoics called the natural sequence of causation '*heimarmenē*' (εἱμαρμένη), usually translated as 'fate' or 'destiny' (Cicero *de div.* 1.125-26; Aulus Gellius 7.2.3). In addition, Stobaeus reports that *heimarmenē* is the *logos*, or rational principle of

[34]E.g., Cicero *de fin.* 3.33; Seneca *epist.* 120.4. See also Long [1986], 199-205.
[35]See Sandbach's discussion in *The Stoics*, [Sandbach, 1975, pp. 53 55].

the universe (*kosmos*). It is "the *logos* according to which past events have happened, present events are happening, and future events will happen" (λόγος καθ' ὅν τὰ μέν γεγοντότα γέγονε, τὰ δὲ γινόμενα γίνεται, τὰ δὲ γενησόμενα γενήσεται); furthermore, Stobaeus informs us that the rational principle, in addition to being called the *logos,* is also referred to as 'truth' (ἀλήθεια), 'explanation' (αἰτία), 'nature' (φύσις), or 'necessity' (ἀνάγκη) (*eclog.* 1.79.1-12; cf. Alexander *de fato* 192.25). The identification of *heimarmenē* with *logos*, the rational principle of the *kosmos*, and the fact that this principle is also referred to as *physis*, would seem to establish the basis for the equivalence in question.

Thus Zeno's interpretation of the Cynic doctrine that the end for man is to live according to virtue can be formulated first by the statement that the *telos* is 'to live according to right reason', since virtue and right reason are taken to be equivalent. What this means, given the account of the development of the virtuous man, is that the end for man is 'to live in accordance with his own rational nature and in accordance with the *logos* of the universe'. This latter version can be summarised in turn by the statement that the *telos* for man is 'to live harmoniously with nature', or equivalently, 'to live in the light of a knowledge of the natural sequence of causation'.

Evidently, if someone thought that to be a wise man one ought to live according to nature in the sense of living according to a knowledge of the natural sequence of causation, then he most likely would also think that the study of physics and logic would be a requirement of a philosophical education. It is quite probable, therefore, that Zeno's concept of the *telos* would have led him to adopt a philosophy in which physics and logic were as much a part of the curriculum as was ethics. The discussion in the last few paragraphs would seem to show that the development of Zeno's notion of the *telos* is a result of his doctrine of *ta kata physin.* This doctrine represented a major break with his Cynic roots inasmuch as it required that some things which the Cynics had classified as absolutely morally 'indifferent', be classified instead as 'preferred', in the sense that they are 'according to nature'. Hence it would seem not only that Zeno's notion of *ta kata physin* itself represented an important break with Cynic doctrine, but also that this notion led to a further breach inasmuch as it induced him to include physics and logic in his philosophical curriculum, contrary to the Cynics.

If the philosopher is to be educated in the study of physics and logic, the question arises concerning the scope and content of his knowledge in these subjects. A passage from Seneca and one from Diogenes Laërtius will be helpful here. Seneca tells us that "the wise man investigates and learns the causes of natural phenomena, while the mathematician follows up and computes their numbers and their measurements" *(epist.* 88.26) In a passage with a similar theme, Diogenes Laërtius tells us that the part of physics concerned with causation is itself divided between the investigation of such things as the *hēgemonikon* (ἡγεμονικόν) that is, the 'leading part' of the soul or *psychē* (ψυχή), of what happens in the *psychē*, of generative principles, and of other things of this sort. This is the province of the philosopher. On the other hand, the mathematician is concerned with such

things as the explanation of vision, the cause of an image in a mirror, the origins of weather phenomena, and similar things (DL 7.133). These passages would seem to suggest that scope of the wise man's knowledge of physics would probably not be of particular data, but rather of general principles.[36]

As to the question concerning the nature of the logical theory that the philosopher would need, we take it that the motivation for such a theory would be the requirements of the ethical doctrine outlined above.[37] Thus, given that the wise man's aim is to live in harmony with a knowledge of the natural sequence of causation, he will need to make correct judgements about the relations between particular states of affairs, based on his knowledge of the general principles governing such connections; moreover, he will need to comprehend the patterns of inference which will allow him to effect such judgements. Evidently, these general principles will be manifestations of the universal *logos*, and as such, given that *logos* is another name for *anankē* (necessity) (Stobaeus *eclog.* 2.79.1–12; Alexander *de fato* 192.25), they will be embodied by necessary connections in nature; moreover, one can surmise that the patterns of inference which emerge will consist in part of representations of such necessary connections.

3.2 Inference and Akolouthia

At 7.62, Diogenes Laërtius reports that, according to Chrysippus, dialectic is about 'that which signifies' and 'that which is signified' (περὶ τὰ σημαίνοντα καὶ τὰ σημαινόμενα). It is clear from the context that 'that which signifies' is a meaningful utterance. Presumably, then, 'that which is signified' is the *significatum* of such an utterance. Diogenes does not elaborate on what belongs to this class, except to say that the doctrine of the *lekton* is assigned to the topic of 'that which is signified'.[38] Ηε τηεν γοες ον το σκετςη αν αςςουντ οφ τηε αριους τψπες οφ *λεκτα* (ΔΛ

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[36]For a more extensive explication of this problem, see Kerferd's article in Rist 1978: "What Does the Wise Man Know?" [Kerferd, 1978b]. See also the article by Nicholas White, "The Role of Physics in Stoic Ethics," *Southern Journal of Philosophy: Recovering the Stoics*, Spindel Conference: 1984 (1985): 57–74. White takes the view that "we are not in a position to be sure why the early Stoics thought that detailed physical and cosmological theory ... would be required by their ethics" [White, 1985, p. 72]. Although White grants the plausibility of the premiss that they might have held such a view, he argues that any actual arguments for it, or explanations of it, are lacking [White, 1985, p. 72].

[37]At the end of his summary of Stoic logic Diogenes has this to say: "The reason why the Stoics adopt these views in logic is to give the strongest possible confirmation to their claim that the wise man is always a dialectician. For all things are observed through study conducted in discourses, whether they belong to the domain of physics or equally that of ethics (DL 7.83).

Compare: "[T]he Stoics, who define dialectic as the science of speaking well, taking speaking well to consist in saying what is true and what is fitting, and regarding this as a distinguishing characteristic of the philosopher, use [the term 'dialectic'] of philosophy at its highest. For this reason, only the wise man is a dialectician in their view" (Alexander in top. 1.8-14).

[38]Contrary to some interpretations (e.g., Kerferd [1978a, p. 260]; Watson [1966, pp. 47–48], we take it that although every *lekton* belongs to the class of *sēmainomena*, not every *sēmainomenon* is a *lekton*. Proper names and common nouns, for example, are *sēmainonta* which signify 'individual qualities' (ἰδία ποιότης) and 'common qualities' (κοινὴ ποιότης) respectively (DL 7.58). Since qualities for the Stoics are corporeal, and since *lekta* are incorporeal, it is evident that both

7.63). Ωε ωιλλ ηαε μορε το σαψ ιν τηε σεχυελ ςονςερνινγ τηε τηεορψ οφ τηε *λεκτον* ανδ τηε μεανινγ οφ τηε τερμ 'τὸ λεκτόν', but for the present it will be sufficient to understood the meaning as 'what is said' or 'what can be said' (cf. Long [1971], 77); in any case, what is of immediate interest is the reported Stoic classification of *lekta*, and in particular, the type of *lekton* called the *axiōma*.

Axiōmata have two characteristic properties which differentiate them from the other lekta: first, they are the *significata* of declarative sentences, and second, they are the only *lekta* which can be true or false (DL 7.68; cf. AM 8.74). Thus, it seems apparent that *axiōmata* are somewhat in accord with what we call propositions (I have already used that term to refer to them in the discussion above); there are, however, several characteristics with respect to which *axiōmata* differ from propositions, so that they cannot be merely identified with them (cf. Kneale and Kneale [1962a], 153 57). It may be, as Long and Sedley propose (1.205), that 'proposition' is the least misleading of the possible translations for *axiōma*; nevertheless, we propose to avoid using 'proposition' and to merely transliterate the term.

Having introduced the notion of an *axiōma*, Diogenes goes on to report that several Stoics, including Chrysippus, divided *axiōmata* into the simple and the non-simple. Simple *axiōmata*, on this account, are those consisting of one *axiōma* not repeated (for example: 'It is day'), whereas non-simple are those consisting either of one *axiōma* repeated (for example: 'If it is day, it is day') or of more than one *axiōma* (for example: 'If it is day, it is light') (DL 7.68–69). Of the non-simple *axiōmata*, the first introduced is the 'conditional', an *axiōma* constructed by means of the connective 'if' (εἰ) (DL 7.71). The Greek word is 'συνημμένον', which might be better rendered 'connexive' in accordance with its etymology; however, even leaving etymological questions aside, translating *synēmmenon* as 'conditional' is somewhat misleading. It seems evident that the Stoic use of the connective 'εἰ' was technical, and hence there are some uses of this connective in ordinary Greek which seem not to be captured by the Stoic understanding of the term.

One problem with taking 'conditional' as the translation of '*synēmmenon*' is that there is a temptation to suppose, as the Kneales seem to do, that what the Stoics had in mind was to give an account of the occurrence in language of the connective 'εἰ' which would be "satisfactory as a general account of all conditional statements" (Kneale and Kneale [1962a, p. 135]). It seems to us that this interpretation gets the matter wrong. The term '*synēmmenon*' denotes a complex *axiōma*, and according to the description of Diogenes Laërtius at 7.66, this is just to say that it denotes a complex state of affairs; moreover, this complex state of affairs is signified by the predication of the relation of *following* between the states of affairs which are the constituents of the complex axiōma. What the Stoics had in mind was to give an account of the inferences that could be made given the knowledge that some particular type of event or state of affairs followed from some other particular event or state of affairs. It seems plausible that they chose 'εἰ' as the syntactic representation of the relation of following because it is

individual qualities and common qualities are *semainomena* which are not *lekta*.

suggestive of that relation, as the arrow is suggestive of the relation of implication in modern syntactic accounts. Hence they chose expressions of the form 'If A, B' to be the canonical representation in their patterns of inference. They might, however, have chosen to express this relation by saying 'B follows from A' rather than 'If A, B', for although they would not recognise the *schema* 'B follows from A; but A; therefore B' as a proper syllogism of their logical system, they were nevertheless willing to view it as being equivalent to the syllogistic *schema* 'If A, B; but A; therefore B', which was an authentic syllogism of their system (Alexander *in an. pr.* 373.29-35). And if they had chosen so to represent it, then no one, we assume, would be tempted to view their characterisation of 'τὰ συνημμένοντα' as an attempt to give a general account of conditional statements. Having said all this, we will nevertheless carry on the tradition of translating '*synēmmenon*' as 'conditional *axiōma*', just so long as it is understood that by so doing we do not assume that in giving a characterisation of '*synēmmenon*' the Stoics supposed themselves to be providing a general account of conditionals.

Now although the Stoics did not use expressions of the form 'B follows from A' as the canonical expression of the relation denoted by '*synēmmenon*', this notion of 'following' in a conditional was of primary importance in their theory of inference, for their fundamental criterion of a valid argument was based on this concept. This canon is the so-called *conditionalisation principle* (cf. Mates [1953, pp. 74–77]). As it is framed by the Stoics, this principle states that an argument is conclusive[39] whenever its *corresponding conditional* is sound (ὑγιέξ: PH 2.137) or true (ἀληςέξ: AM 8.417):[40] that is, the conditional which has the conjunction of the premisses as antecedent and the conclusion of the argument as consequent. Now Sextus Empiricus writes that "the 'dialecticians'[41] all agree that a conditional is sound whenever its consequent 'follows' its antecedent" (AM 8.112; cf DL 7.71). In effect, then, one can say that for the Stoics, an argument is valid (conclusive) just in case its conclusion 'follows' from its premisses, as the consequent follows from the antecedent in a sound or true conditional. In noting this criterion, however, Sextus also outlines a difficulty, for it seems that although the 'dialecticians' were agreed on the standard for a true conditional, there was a controversy as to how the notion of 'following' was to be characterised (AM 8.112; PH 2.110). There were apparently four competing views,[42] only two of which, we shall suggest, would have provided a criterion consistent with the role of inference in Stoic philosophy: the first of these, advocated by those who spoke

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[39] In some places (e.g., PH 2.137, 146) Sextus uses 'συναχτιχόξ' and 'ἀσύναχτοξ' for 'conclusive' and 'inconclusive' (or 'valid' and 'invalid'), whereas at other places (e.g., AM 8.429) he uses 'περαντιχόξ' and 'ἀπέραντοξ'. Hence, as Mates indicates in his glossary ([Mates, 1953], 132-36), these terms appear to be interchangeable. Diogenes Laërtius, however, in his discussion of arguments from 7.77-79 uses 'περαντιχόξ' exclusively for 'conclusive' and 'ἀπέραντοξ' for 'inconclusive'.

[40] See page 477 for a discussion of the use of 'ὑγιέξ' and 'ἀληςέξ' in these contexts.

[41] It is unlikely that by 'dialectician' here Sextus is referring exclusively to a member of the Dialectical School of which Diodorus Cronus and Philo were members. He is probably using it as a synonym for 'logician' (see footnote 20, page 408).

[42] See Section 8.1 for a discussion of the four views.

of 'connexion' (συνάρτησις), required that the contradictory of the consequent 'conflict' (μάχηται) with the antecedent; while the second, advocated by those who spoke of 'implication' (ἔμφασις), required that the consequent be 'potentially contained' (περιέχεται δυνάμει) in the antecedent (PH 2.111-12).[43]

The Greek terms used in these contexts are 'ἀκολουθεῖν' or 'ἑπεῖσθαι', either of which has the sense of 'to follow upon' or 'to be consequent upon'. Now although 'to follow logically' is no doubt one way in which these terms were understood, it also seems evident that this meaning is not the only one they carried. But even if it had been, that fact would not warrant the assumption that for the Stoics 'to follow logically' meant quite what it does in a modern setting. For, given the hypothesis outlined earlier in this section of a motivation for logic grounded in ethics, we take it that the role of logic in Stoic philosophy is primarily to determine the inferential relations between states of affairs and only derivatively (if at all) between sentences of the language. The view we put forward has much in common with that expressed by A.A. Long in the following passage ([Long, 1971], 95):

> The human power of drawing inferences from empirical data presupposes an *ennoia akolouthias*, an idea of succession or consequence. ... And *endiathetos logos*, internal speech (reason), is described as 'that by which we recognise consequences and contradictions' (τὰ ἀκόλουθα καὶ τὰ μαχόμενα) But *akolouthia* is not confined to what we would call 'logical consequence'. The sequence of cause and effect is explained by reference to it, for fated events occur κατὰ τάχιν καὶ ἀκολουςίαν[according to order and consequence] or κατὰ τὴν τω ν αἰτίων ἀκολουςίαν[according to the following of causes]. This use of a common term is exactly what we should expect in view of Chrysippus' methods of inference from actual states of affairs.
>
> Given that the world operates according to a strict causal nexus one of the roles of logic, perhaps its major role in Stoicism, is to make possible predictions about the future by drawing out consequences from the present. The cardinal assumption of the Stoics is that man can put himself in touch with the rational course of events and effect a correspondence between them and his own actions and intentions. This assumption provides the ethical aim of living homologoumenōs [harmoniously]. More particularly, ethics is connected with logic and physics by *akolouthia* and its related words.

If this is an accurate characterisation of the role of logic in Stoic philosophy, then in order that such predictions might be carried out, the Stoic conditional will need to represent the logical as well as the nomic connections, not only between actual events or states of affairs, but also between mooted events or states of affairs. Such connections will need to be manifest in the relation between the content of the antecedent and that of the consequent.

[43]Long and Sedley have commented that the containment view "may not differ significantly" from the conflict view (1.211). We intend to explore this possibility.

In describing these constraints on *akolouthia* in the above quotation, Long refers to Chrysippus' "methods of inference from actual states of affairs" (1971, 95). In a similar vein, J.B. Gould states that "as Chrysippus maintains, one may generalise and affirm that if events of a particular sort occur, then other events of a specified sort will occur. Such generalisations may be expressed in conditional propositions. ... These kinds of generalisations, then, are true only when they denote connections between things or events in nature" ([Gould, 1970], 200-201). Along the same lines, Michael Frede asserts that "the Stoics seem to regard consequence and (possibly various kinds of) incompatibility as the relations between states of affairs or facts in terms of which one can explain that something follows from something" ([Frede, 1987d], 104). We propose to take seriously this talk of 'states of affairs' and 'facts' in order to present an interpretation of the notion of 'following' or 'consequence' in the Stoic system of inference.

If the conception of consequence or 'following from' as expressed in the conditional is a notion of a relation between states of affairs, then to say that a sound conditional is such that the contradictory of the consequent 'conflicts with' the antecedent, as in the 'connexion' theory, or that the consequent is 'potentially contained' in the antecedent, as in the 'containment' theory, would seem to suggest that these conceptions of conflict and of potential containment are also notions of a relation between states of affairs. How is this to be understood? In particular, how are we to understand this talk of 'states of affairs' and the notion of one state of affairs being 'in conflict with' or being 'potentially contained in' another? Unfortunately, there is so little information in the texts about the containment criterion that we cannot attempt to give more than a speculative account of this definition. On the other hand, there are fairly clear indications in the texts about what was meant by 'conflict'. As to the first part of this question, answering it will be the burden of our interpretation of Stoic semantic theory. Interpreting Stoic semantics is in effect to provide an understanding of the theory of the *lekton*. Moreover, since there is a clear indication in the texts of the dependence of *lekta* on 'rational presentations' (φαντασία λογική) (e.g. AM 8.70: DL 7.63), it would seem that an understanding of the *lekton* will require an interpretation of this relation in particular, and of the theory of presentations in general; indeed, it seems to us that there is evidence enough to indicate that one cannot give an adequate account of Stoic logical theory without taking into consideration the theory of *phantasiai*. Such reflections will no doubt entail some involvement in epistemological questions, and though some writers have ruled out concern with such questions as being 'extra-logical' (e.g., Mates [1953, pp. 35–36]; Kneale and Kneale [1962a, p. 150]; Mueller [1978, p. 22]), others see them as an essential component of an understanding of Stoic logical theory (e.g., Imbert [1978, p. 185]; Gould [1970, pp. 49–50]; Kahn [1969, p. 159]).

Our suggestion is that the 'states of affairs' which stand in the relation of conflict or containment in a sound conditional can be thought of as abstract semantic structures the constituents of which correspond to individuals, properties, and relations. They are the objective content of rational presentations as well as of

axiōmata and other types of *lekta*, and as such are the designata of the Stoic term '*pragmata*'.

4 SEMANTICS

4.1 Epistemology: phantasiai and lekta

The connection between the theory of the *lekton* and the Stoic doctrine of presentations is well documented in the texts. Parallel passages in Sextus Empiricus and in Diogenes Laërtius explicitly refer to this connection. The passage written by Sextus at AM 8.70 is somewhat more complete:

> [The Stoics] say that the *lekton* is that subsisting (ὑφιστάμενον) coordinately with a rational (λογική) presentation, and that a rational presentation is one in which it is possible that what is presented be exhibited by means of discourse.[44]

The corresponding passage in Diogenes Laërtius is at 7.63: "They (sc. the Stoics) assert that the *lekton* is that subsisting coordinately with a rational presentation."[45] The Stoics used forms of the verb 'ὑφιστάσθαι' to indicate a 'mode of being' which is something less than the being that material bodies possess (see page 462); hence, the use of 'ὑφιστάσθαι' to describe the being of the *lekton*, makes it seems evident that one can take the *lekton* as somehow dependent for its being on the corresponding rational presentation. On this understanding of the relationship between the *lekton* and the presentation, it is evident that an exploration of the semantic role of the *lekton* cannot leave the doctrine of presentations out of account. As we understand the theory, that which is presented (τὸ φαντασθέν) in a rational presentation, and which is capable of being exhibited by means of discourse, is a *pragma* (πρᾶγμα). Moreover, we take it that the term '*pragma*' is used in Stoic semantics to mean a 'state of affairs' which is the unarticulated objective content of a rational presentation. And since we also understand the *axiōma* to be what is said in the assertion of a *pragma*, it would seem necessary to refer to the Stoic theory of presentations in order to present a characterization of the notion of *akolouthia* as a relation between *axiōmata*.

From the point of view of some modern logicians and historians of logic, this proposal presents a difficulty inasmuch as it implies a merging of epistemological (and psychological) concerns with logical concerns. Benson Mates, for example, states in his *Stoic Logic* that "the criterion for determining the truth of presentations ... is an epistemological problem and not within the scope of this work" [Mates, 1953, pp. 35–36]. According to the Kneales, "the theory of presentations belongs to the epistemology rather than the logic of the Stoics" [Kneale and Kneale, 1962a, p. 150]. And Mueller expresses the view that considerations of the

[44] λεκτὸν δὲ ὑπάρθειν φασὶ τὸ κατὰ λογικὴν φαντασίαν ὑφιστάμενον, λογικὴν δὲ εἱ ναι φαντασίαν χας' ἥν τὸ φαντασςὲν ἔστι λόγῳ παραστη σαι.

[45] φασὶ δὲ [τὸ] λεκτὸν εἶναι τὸ κατὰ φαντασίαν λογικὴν ὑφιστάμενον.

possibility of knowledge of "necessary connections between propositions" (which, we take it, are a component of the theory of presentations) "would take us outside the domain of logic and into epistemology" [Mueller, 1978, p. 22]. In other words, there seems to be a strong bias among some contemporary commentators in favour of the supposition that epistemological considerations could have had no part in the logic of the Stoa, and that those who cultivated Stoic logic shared this aversion in common with those who developed the propositional calculus. But not all present-day commentators agree with this assessment. Some believe, in the words of Claude Imbert, that "the ancients... judged matters differently" [Imbert, 1980, p. 185].

Josiah B. Gould, for example, cites textual evidence from Sextus Empiricus and from Epictetus to support the claim that the Stoics placed their epistemological theory "squarely within the confines of logic" ([Gould, 1970], 49-50). In his treatise against the logical doctrines of 'the dogmatists', Sextus tells us that the logical branch of Stoic philosophy includes the theory of criteria and proofs (AM 7.24).[46] According to this theory, things which are evident can be apprehended directly, either through the senses or through the intellect, in accordance with a criterion of truth.[47] Things non-evident, on the other hand, can be apprehended only indirectly through the means of signs and proofs by inference from what is evident (DL 7.25). The inclusion of such a theory—which, as Gould points out [1970, p. 49], is that of Chrysippus himself—in the logical division of their philosophy seems clearly to commit the Stoics to consideration of epistemological concerns within the purview of their logic. As for Epictetus, according to him the philosophers of the Old Stoa held that logic "has the power to discriminate and examine everything else, and, as one might say, to measure and weigh them" (*disc.* 1.17.10). Thus it is "the standard of judgement for all other things, whereby they come to be known thoroughly" (*disc.* 1.17.8). This power, it seems to him, is the reason why these philosophers put logic first in the development of their doctrine (cf. AM 7.22; DL 7.40). In Gould's estimation [Gould, 1970, p. 49], these comments of Epictetus seem to provide further evidence of a close relationship between logic and knowledge in early Stoic philosophy.

As the quotation above would appear to indicate, Claude Imbert is another recent commentator who thinks we have evidence that the Stoics took epistemological questions to be within the province of logic. She cites the passage at 7.49 in Diogenes Laërtius to support this thesis. In an earlier passage Diogenes has presented a summary of the Stoic logical doctrine (7.39-48), and he proposes now to give in detail what has already been covered in this introductory treatise. He begins with a quotation from the book *Synopsis of the Philosophers* by Diocles of Magnesia, the passage cited by Imbert:

[46] ὁ δὲ γε λογικὸς τοπός τὴν περὶ τῶν κριτηρίων καὶ τῶν ἀποδείξεων θεωρίαν περιεῖχεν.

[47] According to Rist, "the overwhelming body of evidence that we shall consider [concerning the Stoic criterion of truth] suggests that the normal Stoic answers to the question What is the criterion of truth? are either Recognition [κατάληψις], or Recognizable Presentation [καταληπτικὴ φαντασία]" [Rist, 1969b, p. 133].

> The Stoics accept the doctrine that the account of presentation and sensation (αἴσθησις) be ranked as prior [in their logical theory], both inasmuch as the criterion by which the truth of states of affairs (πράγματα) is determined is of the genus presentation, and inasmuch as the account of assent (συγκατάθεσις), apprehension (κατάληψις), and the process of thought (νόησις), although preceding the rest [of their logical theory], cannot be framed apart from presentation. For presentation comes first, then thought, being capable of speaking out, discloses by means of discourse that which is experienced through the presentation (DL 7.49).

This passage, according to Imbert, indicates that "however obscure it may seem to modern logicians, it is undeniable that the Stoics derived their methods of inference from certain presentational structures" [Imbert, 1980, p. 185]. Moreover, since it indicates that the criterion of truth is itself a presentation, it also implies that Stoic logical theory contains an epistemological component.

The difference of opinion among these scholars concerning the content of Stoic logic is no doubt a reflection of a more fundamental disagreement about its general nature. Some writers seem to view the Stoic system as an attempt to develop a calculus of propositions with a truth-functional semantics in accordance with the model of the propositional calculus which emerged in the twentieth century. For such writers the inclusion of epistemological (or psychological) components in a logical system would no doubt be seen as a flaw, a reason to discount such a system as a genuine logic and to view the attempt at its development as misguided. And of course, if Stoic logic really were an attempt to develop such a calculus, they would be right. Other writers, however, seem to proceed with no such presuppositions about the nature of the Stoic system, or at least, with different ones. This latter approach is well summarised by C.H. Kahn in a passage which we cited in the introductory section (see page 400). According to Khan, our picture of Stoic logic will be distorted if we see it merely as a precursor to the propositional calculus. A more adequate view would require that we take into account the relationship in Stoic philosophy between 'dialectic' (logic) and their epistemology, semantics, ethical psychology, and general theory of nature [Kahn, 1969, p. 159]. Since we are in agreement with this assessment of what is required to construct an adequate interpretation of Stoic logic in general, we could perhaps appeal to these remarks as independent justification for including the doctrine of presentations in an interpretation of the notion of *akolouthia*. However, assuming that *akolouthia* is a relation between *axiōmata*, it would seem that the connection outlined above between *axiōmata*, *lekta*, and rational presentations is sufficient in itself to justify this inclusion.

4.2 Phantasiai

'Presentation' (φαντασία), according to Stoic doctrine, is an 'impression' (τύπωσις) on the soul or *psychē* (ψυχή) (AM 7.228). Sextus Empiricus attests that this doc-

trine was put in place by Zeno himself (AM 7.2.30; 36), but that it was interpreted somewhat differently by Kleanthes and Chrysippus (AM 7. 2.28 31). Kleanthes apparently took the meaning of the term 'impression' quite literally, understanding it in the sense that a signet-ring makes an impression in wax (AM 7.228). Chrysippus objected to this interpretation, arguing that not only would this model make simultaneous impressions impossible, but also it would imply that more recent impressions would obliterate those already in place. Since experience would seem to show that various impressions can occur simultaneously, and that prior impressions can coexist with more recent ones, this model cannot be correct (AM 7.228-30). The model to which Chrysippus appealed was that of the air in a room, which, when many people speak at once, receives many different impacts and undergoes many alterations (AM 7.231). The model is apt, as we shall see, since the soul, according to Chrysippus, is composed of *pneuma* or 'natural breath'. Accordingly, he defined *phantasiai* as 'alterations' or 'modifications' occurring in the *psychē*, revealing both themselves and that which has caused them:[48]more specifically, they are modifications of the *hēgemonikon* (ἡγεμονικόν), the 'governing part' of the *psychē* (AM 7.233).[49]

We are informed by several sources that the *psychē* itself has eight parts.[50] Aside from the *hēgemonikon* already mentioned, it consists in the five senses, the faculty of speech (τὸ φωνητικόν), and the generative or procreative faculty.[51] In the account of Diogenes Laërtius, the term 'ἡγεμονικόν' does not appear; instead, he uses the term 'διανοητικόν', i.e., the intellectual faculty, "which is the mind itself" (DL 7.110).[52]The suggestion implicit in this passage is that we can understand the *hēgemonikon* to be the mind itself; moreover, this interpretation is verified by Sextus Empiricus in the passage at AM 7.232: "[Presentations occur] only in the mind or governing part of the *psychē*."[53]Thus it would seem that presentations, according to the Stoics, are modifications or alterations of (or in) the mind. In the parlance of present-day philosophy of mind, we (at least some of us) would refer to presentations as 'mental states' and associate them with corresponding 'brain states'. But for the Stoics there was no need to postulate this kind of dualism (cf. Sandbach [1971b, p. 10]). For according to them, the *psychē* is constituted by '*pneuma*' or 'breath' (πνεῦμα), itself composed of the elements fire and air "which are blended with one another through and through" (Galen SVF 2.841).[54]It seems evident on this account that since fire and air are material elements *par excellence*, the soul must also be a material entity.

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[48]Sextus Empiricus AM 7.230; Aëtius SVF 2.54. Sextus uses the term 'ἡτεροιώσειξ', which we have rendered as 'alteration' or 'modification' (cf. Bury). In the corresponding passage, Aëtius uses 'gpa'joc', which might be rendered 'affection' (cf. Long and Sedley [1990, 1.237]).

[49]This expanded definition was put forward in order to forestall certain objections that not all modifications of the *psychē* could be presentations.

[50]Nemesius SVF 1.143; Chalcidius SVF 2.879; DL 7.110.

[51]τὸ γεννητικόν: DL 7.110; τὸ σπερματικόν: Nemesius SVF 2.39.22.

[52]ὅπερ ἐστιν αὐτὴ ἡ διάνοια.

[53]ἀλλὰ περὶ τῇ διανοίᾳ μόνον καὶ τῷ ἡγεμονικῷ.

[54]δι' ὅλων ἀλλήλοις κεχραμμένα.In his *Alexander of Aphrodisias on Stoic Physics*, Robert B. Todd discusses the Stoic theory of total blending as it is reported by Alexander in *De mixtione*.

The Stoics held that there are two principles (ἀρχαί) in the universe: the passive (τὸ πάσχον), which is substance without quality (οὐσία ποιά), or prime matter (ὕλη), and the active (τὸ ποιοῦν), which is the *logos* inherent in matter, or God (DL 7.134). By the nature of the properties ascribed to the *pneuma*, it would appear that the active principle is embodied in it. First, the *pneuma* is the force which maintains the universe as a unity. Chrysippus, for example, holds that "the whole of substance is unified because it is entirely pervaded by a *pneuma*, by means of which the universe is held together, is maintained, and is in sympathy with itself."[55]Second, the *pneuma* invests with qualities the undifferentiated matter (ὕλη) in which it inheres (Plutarch *de Stoic repugn.* 1054a-b). And third, the *pneuma* is constitutive of the souls of human beings. According to Chalcidius, Zeno and Chrysippus put forward similar arguments for this thesis. Chrysippus argues thus: "It is certain that we breathe and live with one and the same thing. But we breath with natural breath (*naturalis spiritus*). Therefore we live as well with the same breath. But we live with the soul. Therefore, the soul is found to be natural breath" (SVF 2.879).[56]

According to the Stoics, then, the *psychē* is corporeal, and hence just as much a material entity as is the substantial body of which it is a part. Further arguments for this thesis are set out both by Kleanthes (SVF 1.518) and by Chrysippus (SVF 2.790); moreover, there is no doubt that they followed Zeno in this view (SVF 1.137; 138; 141). The details neither of these arguments nor of the underlying physical theory need concern us here; what is of interest, however, is the implication that when the Stoics speak of modifications or changes in the mind, they are not speaking metaphorically. A modification of the mind would appear to be a determinate change of state of the mind-substance or *pneuma* (πνεῦμα). Hence a presentation, since it is such an alteration, would be a physical event (cf. [Sandbach, 1971a, p. 10]), as much a physical entity as the *pneuma* itself.[57]

At 7.51, Diogenes Laërtius provides evidence that the Stoics observed a distinction among presentations between those which are sensory (αἰσθητικαί) and those which are non-sensory (οὐκ αἰσθητικαί). "Sensory impressions," according to Diogenes' account, "are those which are apprehended (λαμβανόμεναι) through one or more of the sense organs; non-sensory, on the other hand, are those which apprehended through thought or by the mind (διὰ τῆς διανοίας), such as those of the incorporeals (ἀσώματα) and other things apprehended by reason (DL 7.51). We take it that this distinction is designed to describe presentations in accordance with the character of their immediate sources. Obviously, a sensory presentation

[55]ἔστι δὲ ἡ Χρυσίππου δόξα περὶ κράσεως ἥδε ἡνῶσθαι μὲν ὑποτίθεται τὴν σύμπασσαν οὐσίαν, πνεύματός τινος διὰ πάσης αὐτῆς διήκοντος, ὑφ' οὗ συνέχεταί τε καὶ συμμένει καὶ συμπαθές ἐστιν αὐτῷ τὸ πᾶν(Alexander *de mixtione* 216.14-17).

[56]Zeno's corresponding argument is also recorded by Chalcidius (SVF 1.138). cf. Tertullian (SVF 1.137).

[57]One fragment seems to indicate that not only presentation, but also assent (συγκατάθεσις), impulse (ὁρμή), and reason (λόγος) are qualities of the *psych* (Iamblichus *de anima, apud* Stobaeus *eclog.* 1.368.12-20; cf. AM 7.237), and qualities, according to the Stoics, are corporeal (SVF 2.376-98).

is one whose immediate source is an actual state of affairs, and this state of affairs is also its cause. It is somewhat unclear what the immediate source of non-sensory presentations might be, but since they are "apprehended through thought," perhaps the most likely candidate would be another presentation. Moreover, there would seem to be nothing to stand in the way of this second presentation's being the cause of the first, for presentations, as was noted above, are 'bodies' *(sōmata)* and hence can enter into causal relationships (Stobaeus *eclog.* 138.24; AM 9.211). Indeed, there would seem to be no reason why one could not envisage a sequence of presentations forming a causal chain.

Ultimately, however, there must be a presentation whose cause is not another presentation, but rather some external state of affairs. This requirement would not be a problem in the case of some non-sensory presentations: for example, the Stoics hold that "it is not by sense-perception (αἴσθησις) but by reason (λόγος) that we become cognizant of the conclusions of demonstrations, such as of the existence of the gods and of their providence" (DL 7.52). Presumably, since the gods themselves are evidently corporeal entities,[58] it would be as a result of their actions that one would become cognizant of their existence, and the presentation in which one apprehends that existence would have its cause in the gods themselves. But in the case of other non-sensory presentations, such as those of the incorporeals, there is a difficulty in seeing how to give an account of the causal basis of such a presentation. For the Stoics hold that the class of incorporeals, which includes *lekta*, void, place, and time (AM 10.218), are *asōmata* (literally 'without body') and hence cannot enter into causal relationships (AM 8.263). A plausible solution to this difficulty is suggested by Long and Sedley, who propose that "perhaps we should connect [this relation between asōmata and presentations] with 'transition' [μετάβασις], a method by which incorporeals are said to be conceived" ([Long and Sedley, 1990], 1.241). They go on to suggest that "this refers ... to the mind's capacity to abstract, e.g., the idea of place from particular bodies" [Long and Sedley, 1990, 1.241]. In an earlier work, however, Long renders *metabasis* as "a capacity to frame inferences" [Long, 1971, p. 88], and there are several texts which would seem to confirm this reading.[59] Presumably, such a capacity would be seated in the mind (*hēgemonikon*) or intellectual faculty (*dianoētikon*) and thus ultimately in the soul itself (cf. DL 7.110).

According to Iamblichus as quoted by Stobaeus, "those philosophers who follow Chrysippus and Zeno and all those who conceive of the soul as body, bring together

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[58] Aristocles SVF 1.98; Chalcidius 293, L & S 44E; Galen *hist. phil.*, DDG 608; DL 7.134. One reading of DL 7.134 (Suidas) has it that the *archai* are ἀσώματα(incorporeal). The reading of the codices, however, has it that they are σώματα(corporeal). Long and Sedley prefer the reading 'σώματα' [1990, 2.226], their reasons being (1) that this interpretation is supported by other texts, and (2) the corporeality of the principles follows by implication from the Stoic view that only bodies are capable of acting and being acted upon [Long and Sedley, 1990, 1.273–74]. But since the active principle is explicitly identified with God (DL 7.134), then it would seem to follow that God (or the gods) is corporeal.

[59] AM 8.194;275;3.25;DL 7.53. Sandbach [1971a, p. 26] translates '*metabasis*' in DL 7.53 as 'inference'.

the powers (αἱ δυνάμεις) of the soul as qualities in the substrate (ὑποκείμενον), and posit the soul as substance (οὐσία) already underlying the powers" (*eclog.* 1.367.17). Moreover, there are several texts which report that the Stoics characterise qualities as corporeal,[60] and at least one passage specifically reports that they describe the qualities of the *psychē* as such (Alexander *de anima* 115.37). Given their corporeal nature, one might suppose that the various capacities of the *psyche* would have causal powers, and this supposition gains credence from a passage in which Zeno is reported to hold that prudence (φρόνησις) is the cause of 'being prudent' (τὸ φρονεῖν), and temperance (σωφροσύνη) is the cause of 'being temperate' (τὸ σωφρονεῖν) (Stobaeus *eclog.* 1.138). Thus one might plausibly conjecture that the Stoics could give an account of the causal basis of non-sensory presentations, such as those of the incorporeals, by invoking, presumably along with the data of experience, the causal powers associated with *metabasis.* And one might further suppose that some such account would throw light on the Stoic explanation that "presentations are formed because *of* [the incorporeals] and not *by* them,"[61] and that they are perceived not by the senses, "but in a certain manner by the senses" (*sed quodam modo sensibus*) (Cicero *acad.* 2.21). If so, then we would not have to suppose with Long and Sedley that these explanations represent an attempt by the Stoics "to find a relationship other than causal to fit the case" [Long and Sedley, 1990, 1.241].

Whatever may be the difficulties involved in providing a causal basis for non-sensory presentations, no comparable problems exist for sensory presentations, for the source of such presentations is an actual state of affairs. We can probably take a passage of Aëtius to imply that sensory presentations are the primary means by which a person develops the stock of conceptions which comprise the content of memory and experience.

> When a man is born, the Stoics say, he has the commanding-part of his soul like a sheet of paper ready for writing upon. On this he inscribes each one of his conceptions (ἔννοιαι). The first method of inscription is through the senses. For by perceiving something, e.g., white, they have a memory of it when it has departed. And when many memories of a similar kind have occurred, we then say we have experience (ἐμπειρία). For the plurality of similar impressions is experience. Some conceptions arise naturally in the aforesaid ways and undesignedly, others through our own instruction and attention. The latter are called 'conceptions' only, the former are called 'preconceptions' (προλήψεις) as well. Reason, for which we are called rational, is said to be completed from our preconceptions during our first seven years.[62]

[60]For example, Plutarch *de comm. not.* 1085e; Galen SVF 2.377; 410; Simplicius *in cat.* 271.20; *in phys.* 509.9. Long and Sedley argue indexLong, A. A.that "the corporeality of qualities is one of many Stoic theses implied by the corporeality of the principles" [Long and Sedley, 1990, 1.274] (see 432, footnote 58).

[61]ἐπ' αὐτοῖς φαντασιουμένου καὶ οὐχ ὑπ' αὐτῶν (AM 8.409).

[62]Aëtius *plac.* 4.11.1 4, DDG 400 = SVF 2.83. The translation is that of Long and Sedley

Another distinction among presentations which is relevant at this point is that between presentations which are rational (λογικαί) and those which are irrational (ἄλογοι). Rational presentations, according to Diogenes Laërtius, are those of rational creatures. They are processes of thought (DL 7.51), and they have an objective content which can be expressed in language (AM 8.70). It looks as though the 'preconceptions' mentioned in the quotation from Aëtius are those which 'arise naturally' from sensory presentations. Since these preconceptions are a requisite for rationality, it is apparent that our first sensory presentations are preconceptual and hence non-rational. Evidently, rational presentations are possible only when a person has acquired the preconceptions which go to make up the content of such presentations. Since the preconceptions would seem to provide a fairly basic conceptual apparatus (e.g., colour concepts), the rational presentations based on them would also be fairly basic. Diogenes Laërtius lists several ways that more complex conceptions may be brought about.

> Of these [complex conceptions] some are acquired by direct experience, some by resemblance, some by analogy, some by transposition, some by composition, and some by contrariety. ... Some things are conceived by inference (μετάβασις), such as *lekta* and place. The conception of what is just and good comes naturally. And some things are conceived by privation, such as the idea of being without hands (DL 7.52-53).

Sextus Empiricus gives a similar list of ways by which conceptions are grasped, and it is notable that he precedes this list with the comment, apparently having its basis in Stoic doctrine, that "in general it is not possible to find in conception that which someone possesses not known by him in accordance with direct experience" (AM 8.58). Of the rational presentations which are primary, an important sub-class are those presentations which are called 'apprehensive' or 'cognitive' (αἱ καταληπτικαὶ φαντασίαι).We follow F.H. Sandbach in rendering 'αἱ καταληπτικαὶ φαντασίαι' as 'cognitive presentations' [Sandbach, 1971a, p. 10]. Presentations belonging to this class play a central role in the Stoic theory of knowledge.

The material nature of the mind in Stoic psychology is an important component in what seems to us a plausible interpretation of the notion of an 'apprehensive' or 'cognitive' presentation. The interpretation we have in mind is that presented by Michael Frede in his essay "Stoics and Skeptics on Clear and Distinct Impressions" [Frede, 1987e, 151 76]. Cognitive presentations (Frede calls them 'cognitive impressions') were deemed by the Stoics to be "the criterion of truth" (τὸ κριτήριον

[1990, 1.238]. According to Sandbach, "the claim that reason is made up in the first seven years is surprising and conflicts with all other sources, which give 14 as the age when it is established. ... Au)'tius seems to have confused the beginning of the growth of reason in the first seven years of life with its completion round about the age of fourteen" [Sandbach, 1985, 80n118]. Jamblichus, for example, reports that "the Stoics say that reason is not immediately implanted, but is assembled later from sense perceptions and presentations about the fourteenth year" (*de anima, apud* Stobaeus *eclog.* 1.317.20). cf. Inwood: "Reason ... begins to be acquired at or about the age of seven and is 'completely acquired' at or about the age of fourteen" [Inwood, 1985, p. 72].

τῆς ἀληθείας) (DL 7.54), and as such played a foundational role in the Stoic account of the development of an individual person's knowledge of the world. According to the definition given both by Sextus Empiricus (AM 7.248) and by Diogenes Laërtius (7.46), cognitive presentations arise only from that which is real and are imaged and impressed in accordance with that reality.[63] In his account, Sextus Empiricus adds a third condition to this definition: a cognitive presentation cannot have its source in that which is not real (AM 7.248). This last condition was apparently added to forestall certain objections of the Academics. According to Sextus (AM 7.252), the Stoics thought that a cognitive presentation would possess a distinctive feature (ἰδίωμα) by which it could be distinguished from all other presentations, such a feature reflecting a corresponding distinction in the object from which the cognitive presentation arises. The Academics, on the other hand, denied that any presentation could have such a feature. According to them, a false presentation can always be found which is similar in all respects to any given presentation (AM 7.402-10). According to Frede, both the Stoic and Academic schools probably agreed that cognitive presentations, "in order to play the role assigned to them by the Stoics, would have to satisfy the third condition too" [Frede, 1987e, pp. 165–66].

A problem for the Stoics, then, is to give an account of how one could tell that a presentation satisfied this condition, or, as Sandbach expresses it, "How could the bona fides of a cognitive presentation be established?" [Sandbach, 1971a, p. 19]. The difficulty is that any sort of test one might make to determine whether a given presentation is cognitive will itself depend on a presentation. But then a test will be required to determine whether the latter presentation is cognitive, and so on. Evidently such a process will lead to an infinite regress, a criticism expressed by Sextus Empiricus (AM 7.429), and probably derived from the early Academics, Arcesilaus and Carneades (cf. Long and Sedley, [1990, 1.249]).

One sort of reply to this criticism is that proposed by Sandbach: "There must be a point to call a halt. There must be some presentations that are immediately acceptable, that are self-evidently true. That is what constitutes a cognitive presentation. It is the attitude of common sense that most presentations are of this sort" [Sandbach, 1971a, p. 19]. It is not clear how far this reply will go to convince the sceptic. At any rate, if there were to be such self-evidently true presentations, it seems a plausible supposition that they would be sensory presentations having fairly basic conceptions as content. From these basic cognitive presentations the corresponding conceptions would be derived, and from these, in turn, more complex presentations. Thus, through the development of more and more complex notions, a complete grasp of things would eventually be gained, such grasp being expressed in general conditionals such as this: "If a thing is a human being, it is

[63]We are rendering 'τὸ ὑπάρχον' as 'that which is real' or 'reality' rather than as 'the real object'. We will argue in the sequel that although the Stoics took "objective particulars" as their "fundamental existents" (cf. Long [1971, p. 75]), they nevertheless thought that reality consists not only in such objective particulars, but also in the properties and relations of these objects.

a rational mortal animal" (*Si homo est, animal est mortale, rationis particeps*). This seems to be the developmental process envisaged by Antiochus in his defence of Stoic epistemology (Cicero *acad.* 2.21). Frede conveys the idea with the remark that "the Stoics take the view that only perceptual impressions are cognitive in their own right. Thus other impressions can be called cognitive only to the extent that they have a cognitive content which depends on the cognitive content of impressions which are cognitive in their own right" [Frede, 1987e, p. 159].

Frede suggests that these basic presentations which are self-evidently true are so because they possess a causal feature which acts on the mind "in a distinctive way" thus bringing about recognition of the veridicality of the presentation [Frede, 1987e, p. 168]. "It is in this sense," according to Frede, "that the mind can discriminate cognitive and noncognitive impressions" [Frede, 1987e, p.168]. The plausibility of this suggestion, it seems to us, depends in no small measure on the material nature of the mind in Stoic psychology. Previously in this section we saw that the *pneuma* which pervades all substance is also constitutive of the minds of human beings. Now according to the Stoics, causal interactions between bodies occur either through spatial contact (Simplicius *in cat.* 302.31) or through the medium of the *pneuma* (Aëtius *plac.* 1.11.5, DDG 310). Hence, the feasibility of a causal interaction between the mind and some distinctive feature of a state of affairs is not *prima facie* out of the question; moreover, such an interaction would evidently result in a unique presentation.

5 *LEKTA*

5.1 Signifier and Signified

Traditionally, one of the more celebrated texts providing evidence for Stoic semantic theory is that presented by Sextus Empiricus at AM 8.11–12. Just before this passage he has given an account of a controversy between the Epicureans and the Stoics as to whether the true is that which is perceptible only to the senses or only to the intellect. He continues:

> Such, then, is the character of the first disagreement concerning what is true. But there was another controversy according to which some located both the true and the false in that which is signified, some in the utterance, and some in the process of thought. The Stoics, moreover, put forward the first opinion, saying that three things are connected: that which is signified (τὸ σημαινομένον), that which signifies (τὸ σημαῖνον), and the subject of predication (τὸ τυγχάνον). Of these, that which signifies is the utterance (φωνή), for example, 'Dion'. That which is signified, that is, what is indicated (τὸ δηλούμενον) by the utterance, is the state of affairs itself (αὐτὸ τὸ πρᾶγμα) which we apprehend as subsisting coordinately with (παραφισταμένου) our thought, but which the Barbarians, although hearing the utterance, do not comprehend. The subject of predication is the external substrate (τὸ ἐκτὸς

> ὑποκείμενον) as, for instance, Dion himself. And of these (three things) they say that two are corporeal, namely, the utterance and the subject of predication; whereas one is incorporeal and spoken, or able to be spoken (λεκτόν), namely, the state of affairs (πρᾶγμα) signified, precisely that which also becomes (γίνεται) true or false. And these (*pragmata* which are spoken or can be spoken) are not all of a kind, but some are incomplete (ἐλλιπῆ), while others are complete (αὐτοτελῆ). And of the complete, one is called *axiōma*, which they also describe by saying "The *axiōma* is that which is true or false."

The above passage provides a point of reference for the discussion of various questions which play a central role in the interpretation of Stoic semantic theory. It is our intention that an understanding of this theory will emerge as a result of discussing these various issues. Since we will frequently refer to this passage in what follows, it will be convenient for such reference to call it 'Passage A'.

Sextus informs us in this passage that the Stoics develop their theory of what is true or false by distinguishing three kinds of items which are connected. These are 'that which signifies' (τὸ σημαῖνον), 'that which is signified' (τὸ σημαινομένον), and 'the subject of predication' (τὸ τυγχάνον). He goes on to provide a more specific delineation of each of kind of item. That which signifies (*to sēmainon*) is characterised as 'the utterance' (ἡ φωνή). The term 'φωνή' is standardly rendered as 'sound' or 'speech', but it seems to us that in certain contexts it has a somewhat more ambiguous meaning for the Stoics, this meaning being better captured by the indeterminate sense that 'utterance' has as it is used in modern philosophy. For instance, 'utterance' on this account would encompass writing as well as speech (cf. DL 7.56). As an example of *to sēmainon*, Sextus provides the utterance of the name 'Dion'. This example, it seems to us, is not only completely inappropriate for the context, but is also inappropriate at a more fundamental level. We shall have more to say about this problem presently. That which is signified (*to sēmainomenon*), that is, what is indicated by the utterance, is characterised as 'the state of affairs itself' (αὐτὸ τὸ πρᾶγμα).[64]Sextus implies that on hearing the utterance a Greek speaker will apprehend the *pragma* as 'subsisting coordinately with thought', but the Barbarian or non-Greek-speaker will not apprehend the *pragma*, even though he hears the same utterance. Recalling that rational presentations are 'processes of thought' (DL 7.51) having an objective content which can be expressed by discourse (AM 8.70), this sounds very much like a description of how a rational presentation would be induced in the mind of the Greek speaker by the utterance, with the content of the presentation being the state of affairs signified by the utterance.

[64]We follow several authors in translating 'τὸ πρᾶγμα' as 'the state of affairs: e.g., Long [1971, 107n10]; Long and Sedley [1990, 1.195, 202]; Reesor [1989, Ch. 3]. The term certainly can have this meaning in ordinary Greek; it seems evident, however, that the Stoics gave it a technical meaning in the context of their semantic theory. We take it that in this context the term referred to a semantic structure which corresponded in structure either to a real state of affairs, or to a mooted state of affairs.

To return to the problem of Sextus' example 'Dion', the context of Passage A is an account of what it is to which the Stoics ascribed the property of having a truth value, and we are told that it is *to sēmainomenon.* We are also told that *to sēmainomenon* is the *pragma* or state of affairs indicated by the utterance. Now it seems evident that the utterance of 'Dion' will not indicate a state of affairs which is either true or false. Hence, the example seems to be inappropriate in the context of the discussion. A more suitable example would be something like the utterance of the sentence 'Δίων περιπατεῖ' (Dion is walking).[65]

At a more fundamental level, the example is problematic inasmuch as it seems to suggest that in Stoic semantics a proper name signifies a 'meaning' or 'sense' and refers to the object named. According to Diogenes Laërtius, however, Diogenes the Babylonian defined a name as a part of speech (μέρος λόγου) indicating an individuating quality (ἰδία ποιότης) (DL 7.58). This teaching would appear to have its basis in certain epistemological and metaphysical concerns, in particular, in the doctrine of cognitive presentations, in the principle of the identity of indiscernibles, and in the theory of change and identity. Recall that one of the functions of the *pneuma* in Stoic philosophy is to invest undifferentiated matter (ὕλη) with qualities (see page 430). Certain of these qualities serve not only to differentiate portions of prime matter from the rest, but also to individuate them as unique entities. The matter invested with an 'individuating quality' (ἰδία ποιότης), along with the quality itself, together comprise 'that which is individually qualified' (ὁ ἰδίως ποιόν), that is, the uniquely qualified individual which serves as the substrate for further qualities and for the predication of attributes (Simplicius *in cat.* 48.11). An essential feature of this doctrine is that although the substance (οὐσία) of which an individual entity is comprised is constitutive of that entity, it is not identical with it (Stobaeus *eclog.* 1.178.21-179.17). Thus, the Stoics were able to defend the idea of something which remains constant and serves as the basis for change, for although the substance of which an entity is comprised might undergo constant 'alteration' (ἀλλοίωσις) and so never be the same from moment to moment, the individuating quality remains constant (Stobaeus *eclog.* 1.177.21-178.21; Plutarch *comm. not.* 1083c). As well, this notion of a uniquely qualified individual is no doubt the basis of the Stoic thesis of the identity of indiscernibles which held that "no hair or grain of sand is in all respects the same as another hair or grain" (Cicero *acad.* 2.85), and which served in the defence of the theory of cognitive presentations (*acad.* 2.83-85).

According to this doctrine, then, the utterance of 'Dion' signifies the portion of *pneuma* individuating that part of the substrate (τὸ ὑποκείμενον) which is constitutive of the qualified individual (ἰδίως ποιός), Dion. And even when Dion has died and it is no longer possible to refer to him by means of a demonstrative,

[65] For similar commentary on Sextus' example, see Long and Sedley [1990, 2.197]; Long [1971, p. 77 and 107n11]; Frede [1987b, p. 349]. Kerferd, on the other hand, argues that the conclusion that 'Dion' signifies a *lekton* is straightforward in spite of the many passages suggesting that only *axiōmata* are true or false [Kerferd, 1978a, pp. 260 61]. He does not mention the difficulty posed by Diogenes' passage which says that names signify corporeal qualities, not incorporeal *lekta.*

it is still possible to refer to him by name (Alexander *in an. pr.* 177.31), since the name picks out not the substance of Dion, but the individuating quality. At any rate, the point is that the Stoics already have an adequate theory of signification for names which links the utterance of the name directly to what it signifies, and there is no need, therefore, to posit an incorporeal 'meaning' or 'sense' as the signification of a name.

Returning to Sextus' account of the three connected items, the 'subject of predication' (τὸ τυγχάνον) is characterised as 'the external substrate' (τὸ ἐκτὸς ὑποκείμενον). According to Simplicius, the Stoics, as well as earlier philosophers, held that the substrate is twofold: primarily it is unqualified matter (ἄποιος ὕλη), which is what Aristotle named it; and secondly, it is that which is commonly or individually qualified (ὃ κοινῶς ποιόν ἢ ἰδίως). In this latter case, the qualified substrate serves as the substrate for further qualities and as the subject of predication (*in cat.* 48.11-16). Since it seems evident that the utterance signifies the *pragma* and predicates a property or quality of the external substrate, we have translated 'τὸ τυγχάνον' as 'the subject of predication'.[66]

Having given this more specific characterisation of the three connected items, Sextus then reports the Stoic doctrine that two of these items are 'corporeals' (σώματα), which is to say, bodies or material entities, while the other is 'incorporeal' (ἀσῶμα), literally, 'without body'. It seems obvious that the Stoics would have classed the external substrate as corporeal. Moreover, since they viewed the utterance as a body, it seems clear that they would also have classed it as corporeal. It is Stoic doctrine that whatever produces an effect is a body; hence, the utterance is evidently a body, for it produces an effect as it proceeds from the speaker to the hearer (DL 7.55-56). In addition, since a written utterance (φωνὴ ἐγγράμματος)—which, according to Diogenes the Babylonian, is speech (λέξις) (DL 7.56)—is also capable of producing an effect, they no doubt would have counted it as corporeal as well. On the other hand, given that the Stoics held that "bodies alone are existents" (Plutarch *comm. not.* 1073e), the conception of the *pragma* as incorporeal does seem to be problematic, for, as Gerard Watson puts it, "'incorporeal' is an extraordinary concept in a materialist universe" [Watson, 1966, p. 38]. We shall have more to say about the ontological status of the *pragma* or *lekton* in Subsection 5.5. For the moment, however, we discuss the semantic considerations which might have prompted the Stoics to posit such an item.

At the end of Passage A Sextus makes it clear that there are several different types of complete *pragmata*, and in a later passage (AM 8.70-74) he provides a list of them. In this later text, however, he does not write, as he does in Passage A, of complete *pragmata* which are spoken or can be spoken (λεκτόν), but rather of complete *lekta*. It would appear, therefore, that we can take complete *lekta* to be complete *pragmata* which are spoken or can be spoken. From what was said earlier

[66]Long and Sedley translate 'τὸ τυγχάνον' as 'the name-bearer'. For an explanation and discussion of this translation, see Long and Sedley [1990], 1.201 and 2.197. In an earlier work, Long translated 'τὸ τυγχάνον' as 'the object of reference' [Long, 1971, p. 76, 107n9]. For Michael Frede's interpretation, see his [1987b, pp. 349 50].

in Passage A, the *lekton* is the signification of an utterance, and from discussion in the text at AM 8.70-74, as well as in the text of Diogenes Laërtius at 7.65-68, the type of the *lekton* is evidently determined by the type of speech act which is its signifier. One type of *lekton*, for example, is signified by the utterance of a command, another type, by the utterance of a question (DL 7.66; AM 8.71). The *axiōma*, as one might expect, is apparently signified by an assertion, which is to say, the utterance of a declarative sentence (DL 7.65-66).

Now it would seem that a difficulty becomes manifest when one asks what is the character of the *lekton*, or, as it might be expressed, what is the character of 'that which is signified'. This difficulty is relevant to every type of *lekton*, but one can get a general idea of the problem by considering the *axiōma* in particular. At the beginning of Passage A Sextus informs us that the Stoics rejected the view that the true and the false are in the utterance, as well as the view that they are in the process of thought. They themselves put forward the thesis that the true and the false are located in 'that which is signified' (τὸ σημαινομένον). At the end of Passage A, we are told that of the various types of complete *lekta*, the *axiōma* is the one which the Stoics say is true or false, and according to Diogenes Laërtius (7.65-66), the *axiōma* is signified by an assertion. Hence, according to the Stoics, when someone makes an assertion, i.e., utters a declarative sentence, he signifies an *axiōma*, and the *axiōma* is either true or false. The problem, then, which in general can be expressed as 'What is the nature of "that which is signified"?', can be expressed with respect to the *axiōma* as 'What is the nature of "that which is true or false"?'.

It might be helpful at this point to consider how the Stoics define something as being true. Sextus reports in one place, for example, that they hold the definite *axiōma* 'This man is sitting' or 'This man is walking' to be true (ἀληθές) whenever the predicate 'to sit' or 'to walk' corresponds to the attribute falling under the demonstrative (AM 8.100).[67]Similarly, Diogenes Laërtius relates that on the Stoic account, someone who says 'It is day' seems to make a claim that it is day, and the *axiōma* set forth is true (ἀληθές) just in case it really is day, otherwise, it is false (ψεῦδος) (DL 7.65).[68]

Now suppose that someone utters the sentence 'Dion is walking', and suppose further that Dion really is walking. Evidently, on the above account, what is signified by the utterance would be true, and it seems tempting in such a situation

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[67]καὶ δὲ τὸ ὡρισμένον τοῦτο ἀξίωμα, τὸ 'οὗτος κάθηται' ἢ 'οὗτος περιπατεῖ,' τότε φασὶν ἀληθὲς ὑπάρχειν ὅταν τῷ ὑπὸ τὴν δεῖξιν πίπτοντι συμβεβήκῃ τὸ κατηγόρημα, οἷον τὸ καθῆσθαι ἢ τὸ περιπατεῖν.

[68]Note that in English, the *axiōma* is mentioned by setting of the corresponding sentence in single quotation marks. In the Greek, it is often mentioned by similar means—usually with double quotation marks, and examples are often introduced by the term 'οἷον', with or without quotation marks. Also, the *axiōma* is sometimes mentioned by nominalising the corresponding sentence by means of the definite article. Hence, the *axiōma* which we represent in English as 'Dion is walking', may be represented in the Greek as 'τὸ Δίων περιπατεῖ'. At any rate, the same means are used to mention sentences, both in English and in the Greek, respectively, and it would appear that some ancient commentators, as well as some recent translators, do not always keep the distinction in mind.

to think that what is signified is the actual state of affairs, which, on the Stoic view, could be described as Dion's *hēgemonikon* in a certain state. And since the *hēgemonikon*, which part of the soul, is constituted by *pneuma* and so is corporeal (see page 430), what would be signified on this understanding would be something corporeal, and hence unproblematic for Stoic materialism.[69] Suppose, however, that someone utters the sentence 'Theon is walking', and that Theon is actually sitting at the Stoa listening to Zeno's lecture. Evidently, what is signified by the utterance in this case will be false; moreover, there is no temptation to think that what is signified is an actual state of affairs consisting in the non-walking Theon. Nevertheless, the utterance is significant, and whether Theon is actually walking or not, a Greek speaker who hears the utterance of the sentence 'Θέων περιπατεῖ' will experience in either case a rational presentation according to which he will apprehend the same *pragma* signified and spoken. In other words, what will be signified by the utterance will be the same in either case. It seems evident, therefore, that what is signified is not the actual state of affairs. What is suggested by this commentary is that the Stoics were persuaded by theoretical considerations to admit items into the ontology of their theory of language for which they could not give a materialist account.

It may be, however, that because of reflections on their theory of causality, the *lekton*, or at least, the incomplete *lekton*, had already been admitted as an item in their ontology. Although the weight of evidence adduced by many modern commentators would seem to support the view that the *lekton* was posited by the Stoics in their semantic theory, Michael Frede has recently proposed that "it is not clear ... that the notion of a *lekton* was introduced by the Stoics in the context of their philosophy of language rather than their ontology" [Frede, 1987a, p. 137]. The evidence for this proposal comes from a passage in Clement's *Stromata* (8.9.4) in which it is claimed that Kleanthes called predicates (κατηγορήματα) *lekta*. As far as we know, according to Frede, this is the first use by a Stoic of the term *'lekton'* [Frede, 1987a, p. 137]. In order to bring out the significance of this passage with respect to the present concern, it will be necessary to consider briefly the role of the predicate *(katēgorēma)* in the Stoic theory of causality.

It would seem that for the Stoics "the canonical representation of the causal relation was ... as a three-place relation between a body and another body and a predicate true of [the second] body" (Frede [1987a, p. 137]). Thus a knife (or a scalpel) is the cause for flesh of being cut (τῇ σαρκὶ τοῦ τέμνεσθαι) (AM 9.211; Clement *strom.* 8.9.30.3), and fire is the cause for wood of burning (τῷ ξύλῳ τοῦ καίεσθαι) (AM 9.211). In representing the causal relation in this manner the Stoics were no doubt influenced by their conception of the universe as a dynamic continuum. Such a view would seem to presuppose a theory of causality in which events rather than particular entities are seen as the effects of causes. For on this conception, the universe just is the totality of events which occur as the result

[69]It is evident that the Stoics thought that the attribute 'walking' is real when possessed by someone or something, even though they also thought that the predicate 'walking' is incorporeal (see footnote 84, page 448).

of causal interactions between bodies *(sōmata)*, either through spatial contact or through the medium of the *pneuma*.[70] According to Sextus Empiricus, the Stoics characterise such interactions as follows: "Every cause is a body which becomes responsible to a body of something incorporeal" (AM 9.211).[71] Thus an effect, on this account, is something which happens to a body as a result of some action of another body. This 'something which happens', however, is not itself a body, but is something 'incorporeal', that is, a predicate. We shall return to the topic of the predicate and its role in Stoic semantic theory, but for the moment we intend to give some consideration to the various other types of *lekta* recognised by the Stoics.

In the last part of Passage A, Sextus writes that the *pragmata* which are signified by the utterance are not only incorporeal, but also spoken or able to be spoken (λεκτόν). They are differentiated first of all between those which are 'complete' (αὐτοτελῆ) and those which are 'incomplete' (ἐλλιπῆ). Of the complete *pragmata*, one is called the *axiōma*, and it is this which is either true or false. In similar texts strongly suggestive of a common source, Sextus and Diogenes Laërtius each render an account of the various kinds of complete *pragmata* (AM 8.70-74; DL 7.65-68). In these passages, however, they write of *lekta* rather than of *pragmata* which are spoken or can be spoken. In the text at AM 8.70, Sextus reiterates some of the things he mentioned in Passage A. In particular, he tells us that the Stoics maintain in common that the true and the false are in the *lekton*. He goes on to report that according to them, the *lekton* subsists coordinately with a rational presentation, and that a rational presentation is one in which it is possible that what is presented be exhibited in discourse. Further on, he also mentions again that the Stoics call some *lekta* incomplete, and others, complete. He then provides a list of several different kinds of complete *lekta* (AM 8.71-73).

Diogenes Laërtius, after relating that Chrysippus takes the subject of dialectic to be that which signifies (τὸ σημαίνοντα) and that which is signified (σημαινόμενα), also reports that the lekton is that which subsists coordinately with a rational presentation. He provides a brief summary of the doctrine of the *lekton*, saying that this theory is arranged under the topic of *pragmata* and *sēmainomena*, and includes complete *lekta*, such as *axiōmata* and syllogisms, as well as incomplete *lekta*, namely predicates (κατηγόρημα), both direct (ὀρθά) and indirect (ὕπτια) (DL 7.63).[72]He then gives an account of incomplete *lekta*, or predicates (7.63-65),

[70]"As physical events are transmitted by nearby action, either through direct contact of bodies or by the pneuma, this must be true also for cause-effect relations. Contiguity is therefore an essential attribute of causality, and causes are bodies acting upon other bodies either in spatial contact with them or through the medium of the pneuma" (Sambursky [1959, p. 53]).

[71]εἴγε στωικοὶ μὲν πᾶν αἴτιον σῶμά φασι σώματι ἀσωμάτου τινὸς αἴτιον γίνεσθαι.

[72]Hick's note here is somewhat misleading. He writes that "'Direct Predicate' answers to our Active Verb, 'Predicate Reversed' to our Passive" (DL 7.63 note a). This seems to point out a fairly fundamental misunderstanding of the concept of a *lekton* in Stoic semantics. As *lekta*, predicates are incorporeal, verbs are parts of speech, and as such, are corporeal. A verb (ῥῆμα), according to Diogenes the Babylonian, is a part of speech signifying an uncombined predicate (DL 7.58).

which leads into a summary of the various kinds of complete *lekta* (7.65-68). He begins this synopsis with the following characterisation of the *axiōma*:

> An *axiōma* is that which is either true or false, or a complete *pragma* (αὐτοτελὲς πρᾶγμα) such as can be asserted (ἀποφαντόν) in itself. Thus Chrysippus says in his *Dialectical Definitions*, "An *axiōma* is that such as can be asserted in itself, as, for example, 'It is day', 'Dion is walking'" (DL 7.65).[73]

There is some question concerning the meaning of 'ἀποφαντόν' in this passage. Hicks renders it 'capable of being denied', but Mates argues that this adjective is derived from ἀποφαίνω, not from ἀποφάσκω or ἀπόφημι, and so should be translated as 'asserted' or 'capable of being asserted' [Mates, 1953, p. 28]. In accord with Mates' view, Diogenes writes, just after the text quoted above, that the term 'ἀξίωμα' is derived from the verb 'ἀξιοῦσθαι'. This has the meaning 'to be asserted' or 'to be claimed'.[74]

As for the other kinds of complete *lekta*, both authors present a similar catalogue. There are discrepancies, however, inasmuch as some kinds of *lekta* appear on one list but not on the other, and inasmuch as some kinds are not denoted by the same terminology on both lists. What we report here is inclusive of both lists and ignores the differences in terminology. This comprehensive list includes questions of two kinds: interrogations (ἐρώτηματα), i.e., those which require only a 'yes' or 'no' reply, and inquiries (πύσματα), i.e., those which require an explanatory reply. It also includes imperatives (προστατικοί), prayers (εὐκτικαί) and curses (ἀρατικαί), oaths (ὁρκικοί), hypotheticals (ὑποθετικοί), vocatives (προσαγορευτικοί), declaratives (ἀποφαντικαί), a kind of rhetorical question (ἐπαπορητικοί), and finally, a *lekton* which Diogenes calls a *quasi-axiōma* (ὅμοιον ἀξίωμα) (DL 7.67). For example, although the sentence 'Priam's sons are like the cowherd' signifies an *axiōma*, the sentence 'How like to Priam's sons the cowherd is!' signifies a *quasi-axiōma*; or, as Sextus puts it, "something more than an *axiōma*, but not an *axiōma*" (AM 8.73). We have noted that these texts would seem to suggest that the type of *lekton* is determined by the type of speech act which is its signifier, and it is tempting to conjecture that there is some parallel with the speech act theories of Searle, Hare, and Austin. At any rate, we shall say more about this possibility in the next subsection.

Not much textual information has come down to us concerning the Stoic treatment of these various kinds of *lekta* other than *axiōmata*. We do know, however, that Chrysippus had an interest in developing a theory of at least some of them,

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[73] Αξίωμα δέ ἐστιν ὅ ἐστιν ἀληθὲς ἢ ψεῦδος ἢ πρᾶγμα αὐτοτελὲς ἀποφαντὸν ὅσον ἐφ' ἑαυτῷ, ὡς ὁ Χρύσιππός φησιν ἐν τοῖς Διαλεκτικοῖς ὅροις 'ἀξίωμά ἐστι τὸ ἀποφαντὸν ἢ καταφαντὸν ὅσον ἐφ' ἑαυτῷ'. Compare the definition given by Sextus at PH 2.104: 'καὶ τὸ μὲν ἀξίωμά φασιν εἶναι λεκτὸν αὐτοτελὲς ἀποφαντὸν ὅσον ἐφ' ἑαυτῷ'. These descriptions differ on in that Sextus has '*lekton*' instead of '*pragma*'. And Aulus Gellius reports that he found this definition of the *axiōma* in his Greek books: 'λεκτὸν αὐτοτελὲς ἀπόφαντον ὅσον ἐφ' αὑτῷ'.

[74] For further discussion of the difficulties in rendering this passage, see Frede [1974, pp. 38–40]; Long and Sedley [1990, 2.204–05]; and Margaret Reesor [1989; pp. 46–48].

for he is reported by Diogenes Laërtius to have written a series of books on imperatives and questions under the general heading of *Logical Topics Concerning Lekta* (DL 7.191); moreover, in a damaged papyrus discovered at Herculaneum we have a discussion by Chrysippus on the relationship between predicates, statements, and imperatives.[75]Nevertheless, aside from these examples and some isolated entries in the texts of a few commentators "most of the attention was given to propositions" [Frede, 1987b, p. 345], and virtually all of the extant writing by the ancient commentators on the topic of the *lekton* is about the *axiōma*.

5.2 Lekta and rational presentations

In the passage from Sextus Empiricus quoted above (AM 8.70), we are told that "the *lekton* is that which subsists in accordance with a rational presentation" and that "a rational presentation is one in which it is possible that what is presented (τὸ φαντασθέν) be set forth in language." Another passage, this one from Diogenes Laërtius, informs us that "those presentations which are rational are processes of thought"[76](DL 7.51). If one compares these texts with the text of Passage A, where Sextus implies that the *pragma* is that which subsists coordinately with thought, this comparison would seem to indicate that there is some sort of correspondence between the terms '*pragma*' and '*lekton*' in these contexts. Confirmation for this correspondence is provided by a comparison of the discussion at the end of Passage A with what Sextus reports at AM 8.70 concerning complete and incomplete *lekta*. In Passage A, Sextus writes that the *pragmata* which are spoken or can be spoken (πράγματα λεκτά) differ inasmuch as some are complete, whereas others are incomplete. At AM 8.70, however, he writes that *lekta* are differentiated in that some are complete while others are incomplete. What would seem to be the case, then, is that *lekta* are *pragmata* which are spoken or can be spoken. If this judgment is correct, then it is evident that in the text at AM 8.70 one could substitute '*pragma* which is spoken or can be spoken' for '*lekton*' and thus interpret Sextus' remark as 'the *pragma* which is spoken or can be spoken is that which subsists coordinately with a rational presentation'. It also seems plausible that in the kinds of contexts under consideration the qualifying phrase 'spoken or capable of being spoken' could be dropped. This possibility seems to be realised, for example, in Passage A, where the *pragmata* referred to are *sēmainomena*, and hence, one would suppose, spoken. At any rate, by dropping the qualifying phrase, and substituting just the term '*pragma*' for the term '*lekton*' in the text at AM 8.70, one could simply say that 'the *pragma* is that which subsists coordinately with a rational presentation'. However, '*pragma*' should always be understood as '*pragma* spoken' or '*pragma* which can be spoken'.

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[75]This book, called *Logical Inquiries* (ΛΟΓΙΚΩΝ ΖΗΤΗΜΑΤΩΝ), is included in the collection by von Arnim as Fragment 2.298a. For an interesting view on the content of these writings and on the possible similarity of the Stoic theory to those of modern theorists in the logic of imperatives: see Inwood's *Ethics and Human Action in Early Stoicism*, [Inwood, 1985]

[76]αἱ μὲν οὖν λογικαὶ νοήσεις εἰσιν.

Possibly a relevant text here is that of Diogenes Laërtius in which he writes that under the general heading of dialectic, the doctrine of the *lekton* is arranged under the topic of *pragmata* and *sēmainomena* (7.63). These remarks seem to indicate that *lekta* belong to a *differentia* of *pragmata* and of *sēmainomena*. That *lekta* are a species of *sēmainomena* seems unproblematic, and we shall argue in the sequel that not all *sēmainomena* are *lekta*. It is not clear, on the other hand, what the other *differentiae* of pragmata would be. However, given the understanding of '*pragmata*' as 'states of affairs', it is plausible to suppose that the Stoics did not think that all states of affairs are spoken or capable of being spoken.

Another section of the text at AM 8.70 to consider is the remark that a rational presentation is one for which it is possible that what is presented (τὸ φαντασθέν) be exhibited by means of discourse. There is some controversy concerning the interpretation of 'τὸ φαντασθέν' in this passage. Mates writes that 'τὸ φαντασθέν' is "the objective content of the presentation," and equates it with 'τὸ λεκτόν' (1953, 22). In criticising this view, Long says that "'the presented object' (sc. τὸ φαντασθέν) is what a *phantasia* reveals, a 'thing' not a *lekton*. If I see Cato walking I am presented with an object which can be denoted in a complete *lekton*" [Long, 1971, 109m33]. We would reply that what I am presented with is not merely Cato, but Cato *walking*, which we take to be not merely an object, but a state of affairs. We would agree with Long, however, that τὸ λεκτόν should not be equated with τὸ φαντασθέν. It seems to us that 'τὸ φαντασθέν' should be understood simply as 'that which is presented', and the text should be taken as setting out a necessary and sufficient condition for a presentation to be rational. On this reading, we take it that the condition is fulfilled if and only if τὸ φαντασθέν is a πρᾶγμα λεκτόν , that is, a state of affairs which is spoken or can be spoken. On the other hand, the condition is not fulfilled if τὸ φαντασθέν is not a πρᾶγμα λεκτόν.[77] At any rate, we take it that Sextus' remark can be rephrased thus: "A rational presentation is one in which what is presented is a *pragma* which is spoken or can be spoken."

The following quotation from Diogenes Laërtius indicates that the Stoics appear to have drawn a distinction which is of relevance to the present discussion. They seem to have differentiated discourse from both mere utterance and speech, and to have referred to discourse as 'speaking *pragmata*'.

> Mere utterance (φωνή) and speech (λέξις) differ inasmuch as mere utterance is sometimes just noise, whereas speech is always articulate (ἔναρθρος). And speech also differs from discourse (λόγος) inasmuch as discourse is always significant (σημαντικός); hence, though speech (*lexis*) is sometimes meaningless, as for instance the word 'βλίτυρι', discourse is never so. And discourse or 'speaking' (τὸ λέγειν) also differs from mere utterance, for whereas sounds are uttered, states of affairs (πράγματα) are spoken: and such states of affairs, in fact, happen to be *lekta* (DL 7.57).

[77] The presentations of children, for example, are not rational until they have accumulated a certain stock of conceptions (e.g., see footnote 62, page 433), but since they apparently do have presentations, surely one can speak of 'that which is presented' in such presentations.

This notion of 'discourse' as 'speaking *pragmata*' is elaborated by Sextus Empiricus in a passage in which he is reports that "to speak (τὸ λέγειν), according to the Stoics themselves, is to utter sounds capable of signifying the state of affairs (πρᾶγμα) apprehended" (AM 8.80). Thus one might say that to engage in discourse, that is, to speak *pragmata*, is to utter articulate sounds which signify the state of affairs apprehended in a rational presentation.

Other passages which seem relevant are those recorded by Diogenes Laërtius at 7.66-67. Here he informs us that, according the Stoics, an *axiōma* "is a [state of affairs] which we assert to be the case when we speak it" (7.66).[78]we take it that what this means is that when one 'speaks a *pragma*'[79] by asserting it, the *lekton*, that is, what is said, is an *axiōma*: to put it another way, what is said when one asserts that some state of affairs holds or is the case is an *axiōma* or proposition. It might be instructive to compare Diogenes' account of the *lekton* called an imperative (προστακτικόν): "An imperative is a *pragma* which we command to be the case when we speak it" (DL 7.67).[80] Thus, when one speaks a *pragma* by commanding it, the *lekton*, or what is said, is a *prostaktikon*: in other words, what is said when one commands that some state of affairs be the case is a *prostaktikon* or imperative. Similarly, when one speaks a *pragma* by asking it, the *lekton* is a query or interrogation (ἐρώτημα). According to Diogenes, the sentence 'It is day' signifies an *axiōma* whereas the sentence 'Is it day?' signifies an interrogation. What this example would seem to indicate is that the same *pragma* or state of affairs—in this case, its being day—can function as the content of various types of *lekta*, depending on the speech act involved.

Brad Inwood plausibly suggests that this conception is comparable to the idea familiar in the speech act theories of Searle and Hare, that is, the idea of a "distinction between content and mode of assertion" [Inwood, 1985, p. 93].[81] It is similar, for example, to the distinction made by Searle "between the illocutionary act and the propositional content of the illocutionary act" (Searle, [1969, p. 30]). By 'illocutionary acts' he means acts of "stating, questioning, commanding, promising, etc." [Searle, 1969, p. 24], and by 'propositional content' he seems to mean (in the case of asserting or stating, for example) "what is asserted in the act of asserting, what is stated in the act of stating" [Searle, 1969, p. 29]. Thus according to Searle, uttering the sentence 'Sam smokes habitually' constitutes the performance of a different illocutionary act than uttering the sentence 'Does Sam

[78]ἀξιώμα μὲν γὰρ ἐστιν [πρᾶγμα] ὃ λέγοντες ἀποφαινόμεθα. My justification for inserting 'πρᾶγμα' into the text here is twofold: first, there is the passage at 7.65 where Diogenes describes the *axiōma* as a 'πρᾶγμα αὐτοτλὲς ἀποφαντὸν ὅσον ἐφ' ἑαυτῷ'; second, in the passages from 7.66-68, there are the instances of 'πρᾶγμα' occurring in similar grammatical constructions in the descriptions of the other types of *lekta*. For the translation in these contexts of 'πρᾶγμα' as 'state of affairs', see Long, ([1971], 107n10; Long and Sedley, [1990], 1.195, 202; Reesor, [1989, Ch. 3].

[79]See Margaret Reesor's comments on the Stoic notion of speaking as 'speaking a *pragma*' (state of affairs) [Reesor, 1989, pp. 33 34].

[80]προστακτικόν δέ ἐστι πρᾶγμα ὃ λέγοντες προστάσσομεν.

[81]As it is used in the phrase 'mode of assertion', the term 'assertion' should be understood as neutral among the various illocutionary acts.

smoke habitually?' or 'Sam, smoke habitually!', but although the 'mode of assertion' differs in each case, the propositional content, which might be represented by the complex {Sam, smokes habitually}, is the same. Inwood has reservations about this comparison, however, writing that "for the analogy to a speech act theory like Hare's or Searle's to be complete, it would have to be the case that the Stoics isolated a subject-predicate complex from its mode of assertion. And they appear not to have done this" ([Inwood, 1985], 95). We shall attempt to develop an interpretation to the contrary, an interpretation in which the complete *pragma* (τὸ αὐτοτελὲς πρᾶγμα) is just such a complex.

At 7.49, Diogenes Laërtius details an order of priority between presentation and discourse. He writes that according to the Stoics, "presentation is first, then thought, which is capable of speaking out, discloses by means of discourse that which is experienced through the presentation." At 7.57, Diogenes also writes that *pragmata* are spoken and that discourse is to speak *pragmata*. If one interprets 'τὸ λεκτόν' to mean 'that which is spoken or can be spoken', an interpretation we shall argue for in the sequel, then one can render the text at AM 8.70 as 'that which is spoken or can be spoken is that subsisting coordinately with a rational presentation'. Since 'that which is spoken' is the *pragma*, the foregoing interpretation becomes 'the *pragma* is that subsisting coordinately with a rational presentation'. Hence, the *pragma* will also be prior to discourse, since it subsists coordinately with the rational presentation. And of course, if the *pragma* is what is spoken, it would seem to be prior to discourse in any case. This priority, we take it, along with the passage at DL 7.66-67 in which Diogenes characterises each type of *lekton* as a *pragma* spoken in a certain mode, is a strong indication that the Stoics isolated the *pragma* from its 'mode of assertion'.

The question arises as to the nature of the *pragma*, or more particularly, of the complete *pragma*. The simplest procedure for setting out an account of this item will be to consider an example of a sensory presentation which is a presentation of a real feature of the world. A general characterisation of the *pragma* might be that it is an abstract structure which is the result of a mental process whereby the mind interprets the actual state of affairs apprehended in the presentation. This structure is assembled from the appropriate conceptions selected from those stored in the mind's stock of conceptions,[82] and it is this interpretation which allows what is perceived to be represented in language. *Pragmata*, then, are abstract structures which correspond, on the one hand, to the language used to represent them, and on the other hand, to the actual states of affairs or situations which engender the presentations of which they are the content. This latter correspondence, however, only holds in the case of veridical presentations.[83]

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[82] See page 433 for a discussion of how rationality is completed from our preconceptions (προλήψεις), and see page 434 for a discussion of how more complex conceptions might be produced from these primary conceptions.

[83] Contrast Graeser, [1978b, p. 8]: "[The Stoics] insisted that there holds no isomorphic correlation between thought on the one hand and things-that-are on the other. ... [They] implied that ontological analysis is bound to be subjective, or rather functional, in that it is man's mind that superimposes its concepts on reality." It seems to us, however, that if someone believed that

Consider an example. Suppose we see Dion walking. What there is, according to the Stoics, and hence, what is perceived, is the individually qualified substance of Dion in a certain state (πως ἔχον), that is, possessing the attribute 'walking' (τὸ περιπατεῖν).[84] The mind searches its stock of conceptions, and if the conception of Dion and of the attribute 'walking' are among them, then it possesses the necessary components for constructing the *pragma*. In general, for a simple example such as this, the components of the *pragma* could be thought of as an ordered pair of items, of which the first is either an individuating quality (ἴδια ποιότης) or a common quality (κοινὴ ποιότης), and second is a predicate (κατηγόρημα). For a particular *pragma*, the first of these components would be signified by either an individual name or a common name, and the second, by a nominalised infinitive verb. For convenience, we shall represent such a structure by first writing down a left brace, then the name signifying the individuating quality, then a comma, then the nominalised infinitive verb signifying the predicate (AM 9.211; Clement *strom.* 8.9.30.3), and last, a right brace. Hence, for the example under consideration, the *pragma* will be represented thus: {Δίων, τὸ περιπατεῖ}. In English this will be: {Dion, to walk}.[85]

We take those passages at AM 8.80 and DL 7.66-67 to indicate that to 'speak a *pragma*' is to perform what Searle defines as an 'illocutionary act' (24). According to these texts, the result of speaking a *pragma* in a certain mode is a certain type

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the same *logos* which structures reality is also immanent in our minds, then one would expect them also to believe in some sort of isomorphism between thought and reality. But then Graeser seems to take what Long and Sedley refer to as a "variant reading" [Long and Sedley, 1990, 1.274] of Diogenes Laërtius 7.134 as evidence that the *logos* itself is incorporeal [Graeser, 1978b, p. 99]. He mentions Posidonius as possibly holding such a view [Graeser, 1978b, p. 99]. This may be so. However, as Long and Sedley have argued [1990, 1.274], this cannot be the view of the Old Stoa.

[84]According to Stobaeus, Chrysippus held that even predicates are real, but only those which are actual attributes (συμβεβηκότα). He says that "'walking' (τὸ περιπατεῖν) is real (ὑπάρχειν) for me when I am walking, but it is not real when I have lain down or am sitting down" (*eclog.* 1.106.18–20).

Seneca records a dispute between Kleanthes and his pupil Chrysippus about the nature of walking. According to Kleanthes walking is breath *(spiritus = pneuma)* extending from the *hēgemonikon (principalis = hēgemonikon)* to the feet, whereas, according to Chrysippus, it is the *hēgemonikon* itself (*epist.* 113.23). Leaving aside the question of how the dispute turned out, it seems apparent that whatever else they meant by the term 'walking', both Kleanthes and Chrysippus thought that they were talking about something corporeal, for in Stoic doctrine both the *pneuma* and the *hēgemonikon* are bodies (SVF 2.879). But since there is no doubt that they conceived of predicates as being incorporeal, they clearly could not have been referring to the predicate {walking} by their use in this context of the term 'walking'.

[85]It seems apparent that the Stoics used the nominalised infinitive verb to signify a predicate. For example, at AM 9.211, Sextus reports that according to the Stoics, "the scalpel is corporeal, and the flesh is corporeal, but the predicate 'to be cut' is incorporeal" (σῶμα μὲν τὸ σμιλίον, σώματι δὲ τῇ σαρκί, ἀσωμάτου δὲ τοῦ τέμνεσθαι κατηγορήματος) (cf. Clement *strom.* 8.9.30.3). Although it may seem more natural to render the nominalised infinitive by a gerund, for example, 'walking' rather than 'to walk', it is not always the simplest representation, particularly in the case of complex predicates. Note that a finite verb seems to be the signification of an *incomplete* predicate. So at 7.63 Diogenes Laërtius says that the verb 'γράφει' (He/she writes) signifies an incomplete predicate (see the next section for further discussion of incomplete predicates).

of *lekton*. Obviously, '*lekton*' (or, more strictly, 'πρᾶγμα λεκτόν') will have the sense in these contexts of '*pragma* spoken'. An *axiōma*, on this account, is the result of speaking a *pragma* by asserting it, a *prostaktikon* is the result of speaking a pragma by commanding it, and a similar account can be given for the other types of *lekta*. Moreover, an *axiōma* is what is asserted in the act of asserting, a *prostaktikon* is what is commanded in the act of commanding, and so on. One can probably think of the *lekton* as an abstract structure which will include the elements of the associated *pragma*, but which will have a richer structure in that it will contain items not part of the *pragma*. For example, *lekta* will obviously have moods, and probably tenses as well. At any rate, as the Kneales point out, axiōmata will have tenses [Kneale and Kneale, 1962a, p. 153]. There may be items corresponding to various sentence operators, such as operators for negations and questions. In addition, there may be items corresponding to connectives and articles (cf. DL 7.58). Although We will need to look at *axiōmata* which involve items corresponding to connectives, we do not intend to give an analysis of the structure of *lekta* in general; hence, for the most part, we will simply represent a *lekton* by writing down its signifying sentence and enclosing it with a pair of braces. For example, one way in which one could speak the *pragma* Dion, walking would be to utter the sentence 'Dion is walking'. The *axiōma* associated with this utterance would be represented thus: {Dion is walking}.

At this point, there are some observations which should be made. First, it is evident that the *pragma* is what might be called the 'propositional content' of a rational presentation, but we would as soon avoid using this expression. Some commentators who speak of the 'propositional content' of a rational presentation seem to suppose that this content is a proposition (e.g., Frede [1987e, p. 154]). However, the only item which could be compared to a proposition in Stoic semantics is the *axiōma*, and we do not see why, supposing that the content of a rational presentation is a *lekton*, it should be an *axiōma* rather than some other type of *lekton*. Second, the formation or construction of the *pragma* would appear to be a constituent of the perceptual process. According to Chrysippus, a presentation reveals itself and that which caused it (AM 7.230; Aëtius *plac.* 4.12.1, DDG 401). Thus one is conscious of the mental process which is the presentation, as well as the external state of affairs (in the case of a sensory presentation) which caused the presentation. Sandbach writes that the presentation thus "gives information about the external object" [Sandbach, 1971b, p. 13]. But clearly, without the *pragma*, which we take to be the mind's interpretation of the external state of affairs, there can be no information received, and hence, no perception. Third, it was suggested above that a presentation is rational if and only if there is a *pragma* subsisting coordinately with it. This would be a *lekton* in the sense of a '*pragma* which can be spoken'. We do not think that there is necessarily a *lekton* subsisting coordinately with the presentation in the sense of a *pragma* spoken. This result would seem to be indicated by the priority of the presentation with respect to discourse (see page 428).

It is clear that this account of the *lekton* is fairly rudimentary at best. For example, we have said nothing of how this interpretation will function for non-sensory presentations. Although we will need to address this topic in particular and expand certain other aspects of the account as well (aspects such as *axiōmata* involving connectives, already mentioned above), we believe that what has been said so far will serve as a basis for developing a characterisation of the role of the *axiōma* in the theory of inference.

5.3 Incomplete lekta

Nouns and incomplete lekta

Another relevant point not brought out in Passage A but mentioned just after, is the distinction among *lekta* drawn by the Stoics between those *lekta* which are 'complete' (αὐτοτελές) and those which are 'incomplete' (ἐλλιπές). This distinction is confirmed by Sextus in another passage (AM 8.70-74) and also by Diogenes (7.65-68). Incomplete *lekta*, according to Diogenes, are those for which the signifying expression is also incomplete. For example, 'He writes' (Γράφει), although a grammatically complete expression, signifies an incomplete *lekton*, presumably because it lacks a definite subject, and hence, does not signify a complete state of affairs. A complete *lekton*, on the other hand, is one signified by a complete expression, for example, 'Socrates writes' (Γράφει Σωκράτης) (7.63). At 7.58 Diogenes reports that a verb (ῥῆμα) signifies an uncombined predicate, and at 7.64 he gives a characterisation of a predicate (κατηγόρημα) as "an incomplete *lekton* which has to be combined with a nominative case (ὀρθὸς πτῶσις) in order to form a complete *lekton*." Given that the expression 'Socrates writes' signifies a complete *lekton* these two passages would seem to suggest that the *significatum* of a noun such as 'Socrates' occurring in the subject position of a sentence such 'Socrates writes' is, according to the Stoics, a nominative case (ὀρθὸς πτῶσις) (DL 63-64). Moreover, no matter how odd or even obscure it might seem to us, what is further suggested is that for the Stoics the cases *(hai ptōseis)* are not understood primarily in a grammatical sense.

From what has been said above, it is clear that an isolated verb such as 'writes' can signify an incomplete *lekton*. An issue which arises is the question whether isolated nouns can also signify incomplete *lekta*. Many commentators seem to think that they can,[86] and they seem to think so for one or both of two closely connected reasons. One reason is the example given by Sextus Empiricus in Passage A. Recall that Sextus informs us in this passage that the Stoics distinguish among three things: that which signifies *(to sēmainon)*, i.e., the utterance *(hē phōnē)*; that which is signified *(to sēmainomenon)*, i.e., the *lekton*; and the subject of predication (*to tynchanon)*, i.e., the external existent *(to ektos hypokeimenon)*. As an instance of that which signifies he cites the utterance 'Dion'. Given this

[86]Mates [1953], 16-17; Kneale & Kneale [1962a], 144, 148; Watson [1966], 47-49; Graeser [1978b], 91; Sandbach [1975], 96.

example it seems natural to suppose that there is a *lekton* associated with this expression and that one may take this *lekton* to be something like its sense or meaning; one may suppose, moreover, that the referent of this meaning is the object picked out, i.e., Dion himself. All this seems to suggest a Fregean semantic analysis of the *lekton*, and this is the course which some authors appear to take.[87]

There is a difficulty with this approach, however, and it involves the fact that in Passage A Sextus is giving an account of a controversy over what it is that is true or false. According to him, the Stoics locate truth and falsity in 'that which is signified', which, as we have seen, is the *axiōma.* But we have also seen that an *axiōma* is signified by the utterance of a declarative sentence. Hence one would expect that Sextus would give a declarative sentence as an example of an utterance which is 'that which signifies'. Whatever else it might be, the utterance 'Dion' seems clearly not to be the utterance of a declarative sentence. Hence it is not the *significans* of an *axiōma*, and thus not the *significans* of anything either true or false. The inappropriateness of Sextus's example is emphasised by consideration of a passage from Seneca's *Epistulae Morales.* The content of this passage would seem to parallel that of Passage A.

> I see Cato walking. The sense (of sight) reveals this (state of affairs), the mind believes it. What I see is an object, toward which I direct both (my) sight and (my) mind. Then I say: "Cato is walking." What I say now, according to them, is not an object, but something declarative about an object: this (that I say) some call *'effatum'*, others *'enuntiatum'*, and others *'dictum'*.[88] Thus when we say 'wisdom' we understand something material; when we say 'He is wise', we say (something) about an object. It makes a great deal of difference, therefore, whether you indicate the object or say something about it (3: 117.13).

It is apparent that something like Seneca's example 'Cato is walking' is needed, and this requirement is all the more apparent when one considers the examples given by Chrysippus as quoted by Diogenes Laërtius, i.e., 'It is day', 'Dion is walking' (DL 7.65). One proposal for clearing up this problem is the suggestion that uttering 'Dion' may be taken as "equivalent to asserting the true proposition 'this man is Dion'" [Long, 1971, p. 77, 107n11]. Whatever are the merits of this particular suggestion, it seems that something of this sort must be posited, for we have another passage similar in context to Passage A in which Sextus also mentions the expression 'Dion' as being the *significans* of an "incorporeal *lekton*" (AM 8.75). Hence we cannot simply write off the example as an aberration in Sextus's account (cf. Frede [1987b, p. 349]). Be that as it may, we think that the infelicity of Sextus's example for the point it is meant to illustrate renders it questionable as evidence that Stoics viewed isolated nouns as *significantia* of *lekta.*

[87]For example Mates [1953, p. 19]; Gould [1970, 70n1].

[88]Note that 'effatum', 'enuntiatum', and 'dictum' are Latin translations of ἀξίωμα(cf. Cicero *acad.* 2.95 [*effatum*]; *de fato* 19.28 [*enuntiatum* = *enuntiatio*]).

Recall that, according to Passage A, the Stoics suppose that the sign (*to sēmainon*) and the object of reference (*to tynchanon*) are both corporeal (σωματικόν), whereas the *lekton* is incorporeal (ἀσώματος). We wish to discuss an assumption made by some authors which is based on the posited immaterial nature of the *lekton*. This assumption leads to the second reason for an affirmative reply to the question whether isolated nouns can signify an incomplete *lekton*. This is the assumption that since the *lekton* is incorporeal, whatever are its constituents must also be incorporeal. Hence, since the *significatum* of a noun such as 'Socrates' can be a constituent of a complete *lekton* -for example, the *lekton* signified by the expression 'Socrates walks'—it would seem to follow that the *significatum* of the expression 'Socrates' is incorporeal. But if it is, then it must be a *lekton* of some sort, since it could hardly be an incorporeal belonging to any one of the other classes of immaterial entities. Now it seems clear that the expression 'Socrates' will not signify a complete *lekton*; therefore, it seems natural to conclude that this expression signifies an incomplete *lekton*, and that, in general, isolated nouns can signify incomplete *lekta*.

Given the Stoic view cited above that nouns signify cases (πτώσεις), it would seem to follow from the argument in the last chapter that a case is an incomplete *lekton* and hence something incorporeal. On the other hand, if one assumes that "the *ptōsis* is definitely conceived of as something incorporeal" [Graeser, 1978b, p. 91], then it would seem to follow that a case is an incomplete *lekton*. Either way we get the conclusion that isolated nouns can signify incomplete *lekta*. One attractive feature of this argument is that it fits in rather well with the example 'Dion' presented by Sextus Empiricus in Passage A; indeed, some writers conclude that the supposed incorporeal nature of the cases provides confirmation of the legitimacy of Sextus's example (cf. Graeser [1978b, p. 91]), whereas others conclude that Sextus's example provides confirmation that the Stoics viewed the cases as incorporeal.[89]

We have already suggested that Sextus's example is suspect as evidence that isolated nouns signify *lekta*, and this would seem to count against the view that the Stoics thought of the cases as incorporeal. However, there are two other objections to these theses which would seem to be somewhat stronger. The first is based on the fact that in any discussion of this subject in the sources, only predicates are ever mentioned as being incomplete *lekta* (cf. Frede [1987b, p. 347]; Long [1971, pp. 104–05]; Graeser [1978b, p. 91]). The other is based on the report of Diogenes Laërtius (7.58) to the effect that the Stoics assumed that the *significata* of names and common nouns are, respectively, individual qualities (ἴδιαι ποιότητες) and common qualities (κοιναὶ ποιότητες). Now there is no doubt that the Stoics assumed that the qualities of material objects were themselves material;[90] hence, if proper nouns and common nouns signify qualities, and qualities are corporeal,

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[89]These conclusions are discussed both by Frede [1987b, p. 349] and by Long [1971, p. 105]; however, neither author agrees with them.

[90]cf. Rist [1969a], 159; Long [1971], 105; Frede [1987b], 347. For citations from the sources see SVF 2.449, 463; DL 7.134; Simplicius *in cat.* 209.10.

there would seem to be a difficulty for anyone wishing to maintain the view either that nouns signify incomplete *lekta*, or that cases are incorporeal.[91]

One might suppose that this should resolve the matter, but at least two more complications arise. The first complication involves two passages in Clement of Alexandria's *Stromateis* in which it is claimed that "a case is incorporeal ... and ... agreed to be incorporeal" (Frede [1987b], 350). As for the claim that a case is incorporeal, Frede has argued convincingly that non-Stoics of later periods in Greek philosophy would use the term 'case' with the conviction that cases are incorporeal "because they did not share the Stoic view that qualities are bodies" ([Frede, 1987b], 350). As for the claim that cases are agreed to be incorporeal, he argues that the examples cited by Clement are examples of things which would no doubt be agreed to be incorporeal by the Stoics, but which would not be agreed by them to be examples of cases. So much then for the difficulties raised by the passages of Clement.

The second complication involves interpreting a passage of Stobaeus (SVF 1.65) in such a way that common qualities, at least, are shown to be incorporeal (cf. Rist [1969a, p. 165]). This passage, which is described by Frede as "notoriously obscure and difficult" [Frede, 1987b, p. 348]), is as follows:

> Zeno <and his followers> say that concepts (ἐννοήματα) are neither somethings (τινα) nor qualified things (ποιά), but are mere images in the mind—only quasi-somethings or pseudo-qualified things (ὡσανεὶ δέ τινα καὶ ὡσανεὶ ποιά). These (sc. concepts) are called ideas by the ancients. For the ideas are (ideas) of the things falling under (ὑποπιπτόντων) the concepts, such as of men, or of horses, or, speaking more generally, of all living things, and of any other things which they say are ideas. The Stoic philosophers say that these (sc. concepts) are non-existents, and that whereas we participate (μετέχειν) in the concepts, the cases, which they call *prosēgoria*, we possess (τυγχάνειν).

Following Frede [1987b, pp. 348–49], we take it that the substance of this passage is claim that the Stoics from Zeno on refused to grant the Platonic Forms or Ideas, which they called 'concepts', any existential status at all—not even the existential status of the incorporeals such as *lekta*, void, place, and time. However, there are things which, because they 'fall under' (ὑποπίπτειν) the concepts, are called 'cases' (πτώσεις) by the Stoics, and which are contrasted with the concepts, the contrast

[91] Rist outlines this difficulty as follows:

> Our problem is why the Stoics put these common qualities into the category of quality, that is, of material objects ... rather than with other incorporeals like time, void, place and the *lekta*. The answer to this is not easy to find [Rist, 1969a, pp. 165–66].

It should be noted that the purported textual evidence adduced by Rist and others to show that common qualities are not corporeal, would not show, even if it were correct, that common qualities were classed with the incorporeals such as *lekta*. What would be shown, as we soon shall see, is that common qualities had no ontological status at all.

being that corporeal objects merely participate in the latter, whereas the cases they possess.

Now according to Frede [1987b, p. 348], the Platonists assumed that in addition to the transcendental forms or ideas, there are immanent forms which are embodied in concrete particulars. He suggests that the immanent forms of the Platonists correspond to the Aristotelian forms, and that both are qualities of some kind. Thus the Platonists differentiate between the transcendental form wisdom, which the Stoics would call the concept wisdom, and the embodiment of the form in Socrates himself, i.e., Socrates' wisdom. On the Stoic view, according to Stobaeus, Socrates' wisdom would be a case *(ptosis)*, because it falls under *(hypopiptein)* the concept wisdom. Moreover, Socrates would merely participate in the transcendental form wisdom, whereas he would possess the embodiment of that form. Hence the Stoic cases appear to correspond to the embodied forms of the Platonists and the Aristotelians, and like them, appear to be qualities of some kind. As such, they would be corporeal on the Stoic view, although they would be incorporeal on the Platonic or Aristotelian conception.

This interpretation of Stobaeus's passage seems to us to capture the substance of the Greek. Unfortunately, not all commentators agree. Rist, for example, thinks that we can deduce from this passage that it is the common qualities of the Stoics which correspond to the Platonic Forms, and hence, that such qualities must have been given the same ontological status as the Forms—which is to say, they were thought of as non-existents ([Rist, 1969a, p. 165]; cf. Reesor [1954, p. 52]). It is not difficult to see how one might arrive at this interpretation, for there would seem to be some confusion created inasmuch as the common nouns such as 'man' and 'horse', which, on the Stoic view, signify common qualities (DL 7.58), are used by the commentators to refer to the ideas or concepts. Thus, since the term 'horse' is used to talk about both the quality common to all horses and the concept 'horse', it should not be surprising that the concept and the common quality are taken to be identical. Now it may be that the Stoics themselves are to blame for at least some of this confusion, for it is easy enough to be careless about the distinctions one draws. On the other hand, given the view that the cases are qualities, and given the distinction between cases and concepts—both of which are integral to Frede's interpretation of Stobaeus's passage—there is no reason to suppose that the Stoics did not intend to maintain the distinction between concepts and common qualities. But if this distinction is observed, then there would seem to be no basis for maintaining that common qualities are not corporeal.

It should be noted that the distinction between concepts and cases is mentioned in other ancient texts. Simplicius, for example, using language similar to that of Stobaeus, reports that the Stoics called the concepts 'μεθεκτά' (which may be translated as *particibilia* (cf. Frede [1987b, p. 348]), because they are participated in (μετέχεσθαι) and the cases 'possessibles', because they are possessed (τυγχάνειν) (*in cat.* 209.12–14).

Predicates

At 7.63 Diogenes Laërtius comments that the class of incomplete *lekta* includes all predicates (κατηγορήματα) and if we are correct in rejecting the *significata* of isolated nouns from this class, then it includes only predicates. A predicate, according to Diogenes, is "that which is said about something, or a *pragma* constructed from one or more elements, or (as we have already noted above) an incomplete *lekton* which must be joined on to an nominative case (ὀρθὸς πτῶσις) in order to yield an *axiōma*" (DL 7.64). The first two of these characterisations are attributed to the Stoic Apollodorus and the passage is translated by Hicks as if they were in conflict with the last one. But it seems to us that the versions of Apollodorus are compatible with the third one and that the three are merely alternatives. Given what has been said above about complete *lekta*, we take the sense of the first of these alternatives to be the idea that in order to signify a complete *lekton* one must make some attribution to an object, and what is attributed is a property or attribute. Corresponding to this property at the level of *lekta*, is a predicate or incomplete *pragma*, and, at the level of language, a verb. In the example of the above paragraph, writing has been asserted about Socrates to form the *axiōma* 'Socrates writes'. But it would seem that one could also form the interrogative (ἐρώτημα) 'Is Socrates writing?' by asking of Socrates whether he is writing (cf. DL 7.66). In either case we take it that {γράφει} is 'that which is said' about Socrates.

The second alternative seems to reflect an ambiguity in the Stoic use of the term *katēgorēma* which we are rendering as 'predicate'. This ambiguity has been noted by Michael Frede in his article "The Origins of Traditional Grammar" [Frede, 1987b, pp. 338–59]. According to Frede, the Stoics made a distinction between those predicates which are simple and those which are complex [Frede, 1987b, p. 346]. There seems to be good reason to take the latter to be the result of combining a 'direct predicate' (τὸ ὀρθὸν κατηγόρημα) (DL 7.64) with an oblique case (ἡ ὀρθὴ πτῶσις). Such complex predicates, according to Diogenes, must be constructed in this way so as to be capable of combining with a nominative case to produce a complete *lekton*. The examples cited by Diogenes are signified by verbs such as 'hears', 'sees', and 'converses', and these are contrasted with those signified by such verbs as 'thinks' and 'walks' (DL 7.64). Now we know that for the Stoics a verb (ῥῆμα) is "a part of speech signifying an uncombined (ἀσύνθετον) predicate" (DL 7.58). Hence, it seems to be the case that some uncombined predicates (e.g., those cited as instances of direct predicates) cannot as they stand be joined with other elements to produce a complete *lekton*, the reason being that there is a sense in which the verbs signifying such predicates must first be combined with other parts of speech before they can be joined with a name or common noun in the nominative case to produce a complete thought. For example, in comparison with 'Dion is thinking' or 'Dion is walking', it seems plausible that the sentence 'Dion is seeing' or the sentence 'Dion is hearing' requires a complement in order to express a complete thought. To recapitulate, there are, it would seem, some uncombined

predicates which are required to be joined with other elements before they can partake in the production of a complete *lekton*. On the other hand, there are some which can partake in the production of a complete *lekton* just as they stand. Frede calls the former 'syntactic predicates' and the latter 'elementary predicates' ([Frede, 1987b], 346). We shall adopt this terminology in the sequel.

Given this ambiguity in the term *katēgorēma*, one can think of the third alternative characterisation of a predicate as a recipe for constructing an *axiōma* from either an elementary predicate or a syntactic predicate. On the one hand, an incomplete *lekton* which is an elementary predicate can be combined with a nominative case to produce an *axiōma* without further ado. On the other hand, an incomplete *lekton* which is a syntactic predicate must first be combined with other elements before it can be joined with a nominative case to form an *axiōma*.

It is perhaps appropriate at this point to make explicit what is suggested in the preceding discussion concerning the 'construction' of an *axiōma*: that is, the notion of a syntax of *lekta*.[92] A good way to introduce this task is to notice that the Greek noun from which the English word 'syntax' is derived, is itself derived from the Greek verb (σύνταξις) and that various forms of this verb are used by Diogenes in the passages from 7.63 to 7.74 to indicate the notion of 'putting together' various kinds of elements to form a *lekton* (cf. Frede [1987c, p. 323, 246]; Elgi, in Brunschwig [1978, p. 137]). Furthermore, we have evidence that Chrysippus, at least, was interested in such a notion, for among the more that seven hundred books he is reported to have written (DL 7.180), the following titles appear: (2) *On the Syntax of What is Said, four books* (Περὶ τῆς συντάξεως τῶν λεγομένων δ′), and (3) *Of the Syntax and Elements of What is Said, to Philip, three books* (Περὶ τῆς συντάξεως καὶ στοιχείων τῶν λεγομένων πρὸς Φίλιππον γ′) (DL 7.193).[93] Now according to A.A Long, the expression 'τὸ λεγόμενον', which may be rendered as 'what is said' or as 'that which is said', is extremely difficult to distinguish in sense from *to lekton* ([Long, 1971], 107n13); moreover, as Frede points out, the passage at DL 7.57 indicates that 'what is said' is in fact a *lekton*. These writings of Chrysippus seem to reinforce what is implicit in Diogenes' report of the Stoic characterisation of a predicate at DL 7.64 and in the whole discussion from DL 7.63 to DL 7.74: that is, that the Stoic theory of the *lekton* included the conception that *lekta* were analysable into various elements and that there was a set of syntactic principles whereby such elements were to be joined together to form a *lekton*.

[92]Much of what we write on the notion of a syntax of *lekta* is drawn from two papers by Michael Frede, both of which appear in his *Essays in Ancient Philosophy*. These are: "The Principles of Stoic Grammar" (especially 323-32) and "The Origins of Traditional Grammar" (especially 353-57).

[93]These titles have been numbered as they are since they are second and third in a sequence of four to which we shall refer.

5.4 Lekta and parts of speech

The Stoics seem to have thought that one constructs a *lekton*, and in particular an *axiōma*, by combining the elements of the corresponding declarative sentence in the right way. This amounts to putting together the elements of the sentence in such a manner that the structure of the corresponding *axiōma* is syntactically correct, its elements being combined in accordance with the syntax of *lekta* (cf. Frede [1987c, p. 324]). The supposition that the Stoics entertained some such notion of a relation between the elements of a sentence and the elements of the corresponding *lekton* is suggested by the remaining two titles in the sequence of four mentioned above. These are: (1) *On the Elements of Speech and on Things Said, five books* (Περὶ τῶν στοιχείων τοῦ λόγου καὶ τῶν λεγομένων ε′), and (4) *On the Elements of Speech, to Nicias, one book* (Περὶ τῶν στοιχείων τοῦ λόγου πρὸς Νικίαν α′) (DL 7.193). Note that (1) and (4) are concerned not only with the parts of speech but also with the elements of *lekta*, and that (2) and (3) are concerned not only with the elements of *lekta* but also with the syntax of *lekta*. The placement of these titles in this particular sequence seems to point to "a systematic connection between parts of speech, elements of *lekta*, and the syntax of *lekta*" [Frede, 1987c, p. 324]; moreover, such a connection would be explained by assuming that the Stoics envisaged the production of a *lekton* to take place in accordance with the theory outlined above.

As for the connection between the parts of speech and the elements of *lekta*, we have a text of Diogenes Laërtius which seems to suggest that this connection is a relation of signification. Both Chrysippus and Diogenes the Babylonian,[94] according to this text, stated that there are five parts of speech: these are individual name, common name, verb, conjunction, and article. Diogenes, in his treatise *On Language*, associates at least the first three of these with the corresponding elements of *lekta*. An individual name (ὄνομα), according to him, is a part of speech indicating (δηλοῦν) an individual quality (ἰδία ποιότης) (e.g., Diogenes, Socrates); whereas a common name is a part of speech signifying (σημαῖνον) a common quality (κοινὴ ποιότης) (e.g., man, horse). A verb (ῥῆμα) is a part of speech signifying an uncombined predicate (as we have already seen). A conjunction (σύνδεσμος) is an indeclinable part of speech, binding together the parts of a sentence, and an article (ἄρθρον) is a declinable element of a sentence, determining the genders and numbers of nouns (DL 7.58). The relation outlined in this passage between the elements of speech and the elements of *lekta* seems clear with respect to the first three parts of speech on the list. It also seems clear what the corresponding element at the level of *lekta* is for each of these parts of speech. If we take the participle 'δηλοῦν' to mean 'signified' in this context, then we can suppose that for the Stoics there is a relation of signification respectively between individual names, common names, and verbs at the level of parts of speech, and individual qualities, common qualities, and predicates at the level of *lekta*.

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[94] A Stoic also known as Diogenes of Seleucia, but called The Babylonian because Seleucia is near Babylon (cf. DL 6.81).

The passage is not so clear, however, about the elements of *lekta* which are supposed to correspond to conjunctions or articles. One assumes, given what has been said about the other three parts of speech, that whatever the nature of these elements of *lekta* might be, the connection between each of them and the corresponding part of speech should also be one of signification. But it is difficult to tell from the text, for conjunctions and articles are defined grammatically rather than by their signification at the level of *lekta*. In addition, as Frede points out [Frede, 1987c, p. 331], there is a difficulty inasmuch as the Stoics think that the class of conjunctions includes both conjunctions proper and prepositions, and that the class of articles includes both articles proper and pronouns. Thus it is not at all transparent how one is to envisage an element of a *lekton* which can be the significate both of conjunctions and of prepositions, or one which can be the significate both of articles and of pronouns.

5.5 *Ontological concerns*

In this section we intend to consider briefly some issues concerning the ontological status of the *lekton*. The first of these is the problem of how one ought to interpret the meaning of the term '*lekton*'. The question presents some difficulty inasmuch as it seems to be connected with ontological concerns. The other topic is the question whether the *lekton* was conceived by the Stoics as merely a construct of the mind, or as something having a more tangible status.

The interpretation of 'λεκτόν'

The substantive expression 'τὸ λεκτόν' is derived from the neuter nominative of the verbal adjective 'λεκτός', which in turn is derived from the verb 'λέγειν', *to say* or *to speak*. Since *lektos* is one of those adjectives having the sense both of the perfect passive participle and of the notion of possibility, *to lekton* is probably best understood either as 'that which is said' or as 'that which can be said'. Now it is true that one way in which the Stoics characterise the *lekton* is to say that it is 'that which is signified' (τὸ σημαινονένον) by a significant utterance (e.g. AM 8.11-12), and some writers take it that *to lekton* ought therefore to be rendered, either exclusively or primarily, as 'what is meant'.[95] This interpretation, which gains further credence from the fact that Liddell and Scott list 'to mean' as one of the senses of *legein*, is then taken to imply that a *lekton* was thought of by the Stoics as some kind of 'meaning' or 'sense', whatever such may be (e.g., Graeser [1978b]).

[95]Andreas Graeser, for instance, asserts that "in Stoic semantics [the verb *sēmainein*] stands exclusively for a relation that holds between the linguistic sign and its sense" [Graeser, 1978b, p. 81]. Since he also takes *sēmainomenon* to be synonymous with *lekton* [Graeser, 1978b, p. 87], it is apparent that he would give preference to this reading. The Kneales, who argue that the Stoics "deliberately identified *sēmainomena* with *lekta*," are of the opinion that "'what is meant' is probably the most literal translation of *lekton*" [Kneale and Kneale, 1962a, p. 140]. According to Bocheński, "the λεκτόν corresponded to the intension or connotation of the words" [Bocheński, 1963, p. 84].

There are at least two difficulties with this reading. One is the question whether the Stoics did in fact take 'σημαινομένον' and 'λεκτόν' to be synonymous. From the fact that they called *lekta sēmainomena*, it does not follow that all *significata* of significant utterances are *lekta*. Indeed, there is a passage in Diogenes Laërtius (7.58) which seems to make it clear that this is so. According to this passage, what is signified by a name is an 'individual quality' (ἰδία ποιότης), and by a common noun, a 'common quality' (κοινὴ ποιότης). Now since the qualities of corporeal bodies are, according to the Stoics, as much material as the bodies themselves (cf. Simplicius *in cat.* 217.32), and since *lekta* are not material entities, what seems to be suggested is that 'what is signified' by a name or common noun is not a *lekton*. In other words, 'τὸ σημαινομένον' is not coextensive with 'λεκτόν'.

The other problem with reading *to lekton* as 'what is meant' is that there is more than a little evidence supporting the idea that one species of *lekton*, the *axiōma*, had the role in Stoic semantic theory as that which is true or false. Hence, if one interprets the *lekton* as being in general a 'meaning' or 'sense', then one seems to commit the Stoics to saying that such things as 'meanings' or 'senses' are the sorts of things which can be true or false. We think that one would be hard pressed to find textual evidence for such a commitment. Thus it would appear that however else they may have thought of the *axiōma*, it is unlikely that the Stoics could have viewed it as such a thing as a meaning or sense. It seems to us rather that the Stoics would have agreed with Austin in his contention that "we never say 'The meaning (or sense) of this sentence (or of these words) is true'" ("Truth" in *Phil. Papers*, 87); hence, it seems unlikely that the Stoics could have viewed the *axiōma* as such a thing as a meaning or sense. But if it is improbable that the Stoics thought of the *axiōma* as a meaning or sense, then since the *axiōma* is a kind of *lekton*, it is not clear that one can legitimately promote an interpretation of the *lekton* as being in general a meaning or sense. From the point of view of the interpretation we are suggesting, rendering 'to *lekton*' as a meaning or sense cannot do justice to the various roles the concept plays in Stoic semantics (cf. [Long, 1971, p. 77]).

But even supposing we interpret '*to lekton*' as 'what is said', there is still some controversy whether we should also interpret it as 'what can be said'. The problem is summarised by Andreas Graeser as follows:

> For taking *lekton* to mean "that which can be said" may seem tantamount to committing oneself to the position that the *lekton* exists regardless of whether it is being expressed or not, whereas taking *lekton* to mean "what is said" seems rather to entail that the very entity in question exists only as long as the expression that asserts it [Graeser, 1978b, pp. 87–88].

This worry is reiterated later in Graeser's essay when he asks "Did the Stoics hold that the *axiōmata* or *lekta* respectively exist in some sense whether we think of them or not?" [Graeser, 1978b, p. 95], and it is echoed by A.A. Long when he wonders whether "*lekta* only persist as long as the sentences which express

them" [Long, 1971, p. 97]. In giving expression to this problem both Graeser and Long are concerned to reply to an assertion made by the Kneales to the effect that axiōmata "exist in some sense whether we think of them or not" [Kneale and Kneale, 1962a, p. 156]. The context in which this claim is made is an iteration of the various similarities and dissimilarities perceived by the Kneales between *axiōmata* and propositions. We shall not comment on the arguments adduced by the Kneales concerning this issue, nor on the counter-arguments presented by Graeser and Long. Indeed, we intend to develop an interpretation of the *lekton* which will require an understanding of the meaning of the term '*to lekton*' as being systematically ambiguous. On such a reading this controversy would seem to be of less concern. We do, however, wish to note that saying "lekta only persist as long as the sentences which express them" does not seem to render their existence any less mysterious or problematic than saying that they "exist in some sense whether we think of them or not."

The passage in Diogenes Laërtius at 7.66, discussed in the last section, would seem to suggest that the *axiōma* is the *significatum* of some actual utterance of a particular type, i.e., an assertion. Similarly, each of the other kinds of *lekta* is the *significatum* of the appropriate type of utterance (i.e., command, question, and so on). It seems apparent that on this account the question whether *axiōmata* "exist in some sense whether we think of them or not" should not arise, for the subsistence of the *axiōma* is clearly dependent upon the existence of an act of assertion. Obviously this dependent status will apply to the *lekta* corresponding to the various other types of illocutionary acts. We suggested in the last section that one ought to understand '*lekton*' in these contexts as a generic term denoting the content of a speech act; hence, it seems appropriate in such cases to take '*to lekton*' to mean 'what is said'.

On the other hand, there would seem to be room in the Stoic theory for *lekta* which subsist independently of any particular utterance. When one asserts, for example, that the state of affairs {Dion, walking} is a fact, or commands that it become a fact, or questions whether it is a fact and so on, what gets said, or exhibited (παραστῆσαι) in language, in such an utterance is the unarticulated objective content (τὸ φαντασθέν) of the rational presentation, that is, the πρᾶγμα {Dion, walking} (cf. AM 8.70). We have proposed that '*lekton*' is sometimes used to denote the *pragma* which is the unarticulated content of a rational presentation. In such contexts, it seems appropriate to understand '*to lekton*' to mean 'what can be said'.

The lekton and ontology

There is a tradition among ancient commentators that the Stoics posited a *summum genus* (γενικώτατον) which they called 'the something' (τό τι) (AM 8.32; PH 2.86; Seneca *epist.* 58.13-15), and under which they included not only material bodies or 'corporeals' (σώματα), but also a set of items 'without body' which they

called 'incorporeals' (ἀσώματα) (AM 10.218).[96] We are informed by Sextus Empiricus that under the class of incorporeals were included *lekta*, void, place, and time (κενόν, τόπον, χρόνον) (AM 10.218).[97] Now inasmuch as the Stoics thought that "bodies alone are existents,"[98] it is apparent that they did not take 'to be something' necessarily to mean the same as 'to exist' in the sense that material bodies exist.

In addition to material bodies and incorporeals. the class of somethings appears to have included a collection of items containing both fictional beings and theoretical constructs, particulars such as Centaurs (Seneca *epist.* 58.15) and limits (Proclus SVF 2.488; DL 7.135). Although it might seem natural to assume that these particulars ought to have been classified among the incorporeals, there is no evidence to support the view that the Stoics did so, for none of the texts providing a list of the incorporeals include such items in the list. The fact that the members of this class of 'mental constructs' are included among the 'somethings' but are never included among the incorporeals, would seem to indicate that the genus of 'the something' was differentiated into three subclasses: the class of material bodies or 'corporeals'; the class of 'incorporeals' which included *lekta*, void, place, and time; and the class of fictions or mental constructs (cf. Long and Sedley [1990, 1.163 66]). At least one respect in which such a tripartite differentiation would be significant is to give lie to the claim made by some modern commentators that the incorporeals were viewed by the Stoics as merely 'constructs of the mind' (e.g., Watson, [1966, pp. 38–39]). We believe that the commentary of Long and Sedley is sufficient to show that the Stoics did indeed propose this tripartite division of the genus of 'the something'. But granting this division as a component of Stoic ontology, the question occurs as to the basis for differentiating between fictional or theoretical constructs and the incorporeals. Since an adequate treatment of this problem is beyond the scope of this essay, we can only give a suggestion here of the reason.

It seems fairly clear, at least with respect to void, place, and time, that the Stoics needed these items in their ontology in order to develop their physical and cosmological theories. A consideration of the roles envisaged for these items makes it also seem clear that although these incorporeals fell short of having the real existence that substantial bodies have, it is unlikely that the Stoics viewed them merely as mental constructs. Similar reasons can be adduced on behalf of the *lekton*, supposing that Frede is correct in his suggestion that the concept of the *lekton* was first introduced in the ontology of Stoic causal theory [Frede, 1987a, p. 137]. On the other hand, the connection between the *lekton* and the immanent

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[96]For additional citations from the primary sources, see SVF 2.329-35; AM 10.234; AM 11.224; PH 2.223-25; Plutarch *adv. colot.* 1116b-c. For commentary on these notions by modern writers, see Long and Sedley [1990, pp. 162 66]; Long [1971, pp. 88 90]; Rist [1969a, pp. 152 54]; Watson [1966, pp. 92 96]; Sandbach [1975, p. 92].

[97]cf. Plutarch, *adv. colot.* 1116B.

[98]ὄντα γὰρ μόνα τὰ σώματα καλοῦσιν(Plutarch *comm. not.* 1073e). cf. Aëtius *plac.* 1.11.4, 4.20 (SVF 2.340, 387); Stobaeus *eclog.* 1.336, 338; Cicero *acad.* 1.39; Seneca *epist.* 117.2, 106.4; DL 7.56.

logos, a feature of its role as the *pragma* which is the content of a presentation, would seem to provide further reason why it is unlikely that the *lekton* was viewed merely as a mental construct.

Recalling our suggestion that for the Stoics 'to be something' did not seem to mean the same as 'to exist', the question naturally arises as to what 'to be something' did mean. It has been suggested that for the Stoics 'to be something' meant "to be a proper subject of thought and discourse" (Long and Sedley [1990, 1.164]). This idea is developed with the observation that since the Stoics thought that expressions such as 'Centaur' and 'limit' "are genuinely significant, they are taken to name *something*, even though that something has no actual or independent existence" [Long and Sedley, 1990, 1.164]. It is not obvious, however, what the force of the expression "genuinely significant" is supposed to be in this context. This shortcoming, however, can probably be filled out by a consideration of what is excluded from the genus of the something. We have a passage from Stobaeus *(eclog.* 1.136.21 = SVF 1.65) which would seem to indicate that the Stoics did not include what 'the ancients' (οἱ ἀρχαῖοι) called 'ideas' (ἰδέαι) in the class of somethings. Michael Frede plausibly suggests that these 'ideas' which the Stoics called 'concepts' (ἐννοήματα), are the transcendental Ideas or Forms of Plato ([Frede, 1987b], 348). According to Stobaeus, 'concepts' such as 'Man' or 'Horse' were referred to by Zeno and his followers as 'pseudo-somethings' (ὡσανεί τινα). A possible reason why these items might have been refused the ontological status of somethings can be gleaned from a passage of Sextus Empiricus. Clearly presenting Stoic doctrine, Sextus argues at AM 7.246 that the genera of which the particular instances may be of this kind or that kind cannot themselves be of either kind. Thus the generic 'Man' is neither Greek nor Barbarian, for if he were Greek, then all particular men would have been Greek, and, conversely, if he were Barbarian, then all particular men would have been Barbarian. We take the general point of this argument to be the idea that it is not possible to ascribe to the *ennoêmata* any of the attributes one may ascribe to the particulars which fall under them. But if one cannot say of the universal 'Man' that he is either Greek or Barbarian, young or old, tall or short, cowardly or brave, and so on, then the term 'man' would seem to lack 'genuine significance' when it is used in this way. Hence the force of the expression "genuinely significant" might be understood to specify a contrast between terms such as 'Centaur' and 'limit', which are taken to name items to which one can ascribe certain appropriate attributes, and terms such as 'Man', which are taken to name items to which one can not ascribe such attributes. Thus, although it makes sense to say 'A Centaur has four legs', it does not make sense to say '"Man" has two legs'. Hence, the Stoics might have thought that an item such as a Centaur or a limit could be said to be *something*, which is to say "the proper subject of thought and discourse," but it was evidently not part of their ontological commitment to think that an item such as the universal 'Man' could also be so.[99]

[99]Note that the Stoic use of the expression 'universal Man' is as a synonym for the expression 'the concept "Man"'. We should remind ourselves that such items belong to an ontological

The terms which the Stoics standardly used in their characterisations of the incorporeals were various forms of the verbs 'ὑφίστασθαι' (AM 8.70; DL 7.63),[100] and 'παρυφίστασθαι' (AM 8.12; Simplicius *in cat.* 361.10). These terms, which are both customarily translated as 'to subsist' (e.g., Long and Sedley [1990], 1.196, 162-66), are contrasted with the verb 'ὑπάρχειν' (e.g., Stobaeus *eclog.* 1.106.20) which, on at least one of its senses, can be translated as 'to exist'. This distinction, referred to by Galen as 'splitting hairs' (SVF 2.322), was, needless to say, the source of much critical commentary (cf. also Alexander *in topica* 301.19). We shall not attempt here to discuss this criticism, since it has, in any case, already been adequately addressed by A.A. Long [1971, pp. 84–90]. It seems clear, however, that the Stoics used this distinction to indicate the ontological status of the incorporeals as 'somethings', although not necessarily as existents. It has been suggested that the Stoic usage of the terms "seems to capture the mode of being that Meinong called *bestehen* and Russell rendered by 'subsist'" (Long and Sedley [1990, 1.165]). The parallel is perhaps even closer inasmuch as the Stoics also seemed to count 'fictions' or 'mental constructs' such as surfaces and limits as belonging to the class of 'somethings' and to use forms of these verbs to refer to them (Proclus SVF 2.488; DL 7.135). It would be wrong, though, to infer from this that they classed incorporeals as fictions, the views of some modern commentators notwithstanding.

6 *AXIŌMATA*

It is evident that in some respects *axiōmata* have a character similar to that of propositions. For one thing, several texts confirm the judgment that the Stoics attributed to *axiōmata* the property of being true or false.[101] There is some question, however, whether *axiōmata* were true or false in 'the basic sense'. This question arises because the Stoics assigned the terms 'true' and 'false' not only to *axiōmata*, but also to arguments (λόγοι) and to presentations (φαντασίαι). An argument was said to be true whenever it was conclusive (συνακτικόν) and had true premisses (PH 2.138; DL 7.79), and, according to Sextus, "a true presentation is one from which it is possible to produce a true predication (κατηγορία),[102] such as this in the present circumstances: 'It is day', or this: 'It is light'" (AM 7.244).[103] The

category different from that to which the common quality 'Man' belongs, the latter, according to the Stoics, being something corporeal.

[100] cf. Cleomedes SVF 2.541; Proclus SVF 2.521; Stobaeus *eclog.* 1.106.19.

[101] For example, Sextus Empiricus: AM 8.10; 12; 73; 74; Diogenes Laërtius: 7.65; 68.

[102] Martha Kneale observes that "if we take κατηγορία here as equivalent to ἀξίωμα then Sextus is defining the truth of presentations in terms of the truth of *axiōmata*" [Kneale and Kneale, 1962a, p. 150]. Although admitting that "this identification is plausible" [Kneale and Kneale, 1962a, p. 150], she is hesitant however to apply it, her reason being that the term '*katēgoria*' appears only this once in Sextus' writings, and its meaning is nowhere mentioned by him [Kneale and Kneale, 1962a, p. 150]. But it seems that many commentators think that this identification is more than plausible (e.g., Mates [1953, p. 34]; Long and Sedley [1990, 1.240]; Long [1971, p. 92]; Graeser [1978a, p. 201]).

[103] ἀληθεῖς μὲν οὖν εἰσὶν ὧν ἔστιν ἀληθῆ κατηγορίαν ποιήσασθαι, ὡς τοῦ 'ἡμέρα ἔστιν' ἐπὶ τοῦ παρόντος ἢ τοῦ 'φῶς ἔστιν.'

consensus among modern commentators, however, seems to be that "the basic application was probably to propositions" [Long, 1971, p. 92]. In addition to being the primary items to which the terms 'true' and 'false' are applied, *axiōmata* are like propositions in that they are signified by declarative sentences.[104] According to the Kneales, two further ways in which *axiōmata* are similar to propositions, is that "they are abstract, or, as the Stoics perhaps rather unhappily put it, incorporeal; and they exist in some sense whether we think of them or not" [Kneale and Kneale, 1962a, p. 156]. We have discussed the latter thesis in Subsection 5.5. As for the former, it may be that we can plausibly think of *axiōmata* as 'abstract'; it would seem, however, that we can criticise the Stoics for calling them 'incorporeal' instead of 'abstract' only if we are certain that they meant 'abstract' and not 'incorporeal'.

At any rate, however many of these characteristics of propositions one wants to apply to *axiōmata*, there are several differences which, according to the Kneales, indicate that *axiōmata* cannot simply be identified with propositions [Kneale and Kneale, 1962a, pp. 153–56]. For one thing, *axiōmata* appear to have certain 'grammatical' characteristics which we usually do not associate with propositions, but rather with the sentences which express them. For another thing, *lekta* obviously have moods. For another, *lekta* in general, and hence *axiōmata* in particular, have tenses. This is indicated by the titles of a series of four books written by Chrysippus and reported by Diogenes Laërtius. These titles are *Temporal Lekta*[105], τωο βοοκς (Περὶ τῶν κατὰ χρόνους λεγομένον α′ β′) and *Axiōmata in the Perfect Tense*, two books (Περὶ συντελικῶν ἀξιωμάτων β′) (DL 7.190). There are reports as well that predicates, which are major constituents of *axiōmata*, were distinguished according to voice (DL 7.64-65) and number (Chrysippus SVF 2:99.38-100.1). Michael Frede, in his discussion of the origins of traditional grammar, has suggested that for the Stoics the notion of syntax was applied primarily to *lekta* and only derivatively to parts of speech and sentences ([Frede, 1987b], 345-47). Hence, distinctions which we would expect to be made at the level of expressions are made by the Stoics at the level of *lekta*, and the features at the level of expressions which correspond to certain features distinguished at the level of *lekta*, take their names from these latter features ([Frede, 1987b], 345). If Frede is correct, then it should come as no surprise that *axiōmata* differ from propositions in these ways.

Another difference between *axiōmata* and propositions, is that *axiōmata* can change truth value. As the Kneales point out, this feature is what might be expected since *axiōmata* have tenses [Kneale and Kneale, 1962a, pp. 153–54]. Finally, *axiōmata* may cease to exist.[106] The evidence for this latter property

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[104]The examples of *axiōmata* which Sextus cites at AM 8.93-98 and which Diogenes Laërtius cites at 7.68-70 are all clearly signified by declarative sentences. The texts at AM 8.71 and DL 65-66 also indicate that *axiōmata* are signified by declarative sentences.

[105]Here we are translating 'τὸ λεγόμενον', which literally means 'that which is spoken' as '*lekton*'. A.A. Long has said that "in sense *lekton* can hardly be distinguished from τὸ λεγόμενον" ([Long, 1971], 107n13).

[106]cf. Graeser's remarks on these differences between *axiōmata* and propositions [Graeser, 1978b, pp. 94–95].

is a passage of Alexander of Aphrodisias (*in an. pr.* 177.25-178.1). He reports that according to Chrysippus, the *axiōma* 'This man is dead' (indicating Dion demonstratively) is impossible when Dion is alive but is 'destroyed' (φθείρεσθαι) when Dion has died (177.31). On the other hand, the *axiōma* 'Dion has died' which is possible when Dion is alive is apparently still possible when Dion has died (178.21-22). This result is what one would expect, given that a demonstrative must indicate the individually qualified substrate (ἰδίως ποιόν), that is, the qualified substance of Dion, whereas the name signifies the individuating quality (ἴδια ποιότης) (DL 7.58). When Dion has died, the individually qualified substrate ceases to exist as such, and thus can no longer be indicated demonstratively; the individuating quality, however, can still be referred to by the name.

According to Sextus Empiricus, the 'dialecticians' (i.e., the Stoics) declare that the first and most important distinction among *axiōmata* is that between those which are 'simple' (ἁπλᾶ) and those which are 'complex' (οὐχ ἁπλᾶ) (AM 8.93). Sextus reports that even though *axiōmata* are constructed of other elements, they are called 'simple' if they do not have *axiōmata* as constituents. Thus a simple *axiōma* is one which is neither constructed from a single *axioma* taken twice (δὶς λαμβανόμενον), nor from different *axiōmata* by means of one or more conjunctions (σύνδεσμος) (AM 8.94). The following, for example, are simple *axiōmata*, as is every *axiōma* of similar form: 'It is day', 'It is night', 'Socrates is conversing' (AM 8.93). Complex *axiōmata*, on the other hand, are those constructed from a single *axiōma* taken twice, for example, 'If it is day, it is day'; or those constructed from different *axiōmata* by means of a conjunction, for example, 'If it night, it is dark'. Further examples of complex *axiōmata* are such as the following: 'Both it is day and it is light', 'Either it is day or it is night' (AM 8.95). The content of these passages should be compared with the similar content of the text of Diogenes Laërtius at 7.68-69. Each author goes on to discuss the various kinds of simple and complex *axiōmata*, but we shall refer to Diogenes' text for this information.

Simple *axiōmata*, according to Diogenes, are classified as follows: 'negative' (τὸ ἀποφατικόν), 'negatively assertoric' (τὸ ἀρνητικόν), 'privative' (τὸ στερητικόν), 'assertoric' (τὸ κατηγορικόν), 'demonstrative' (τὸ καταγορευτικόν), and 'indefinite' (τὸ ἀόριστον) (7.69).[107] In the passage at 7.70, Diogenes provides some details about these various kinds of simple *axiomata*. A negative *axiōma* is constructed with a negative particle and an *axioma*, e.g., 'Not: it is day'. A negatively assertoric *axiōma* is produced from a negative constituent and a predicate, e.g., 'No one is walking'. A privative *axiōma* is constructed with a privative constituent and a possible *axiōma* (ἀξιώματος κατὰ δύναμιν), e.g., 'This man is unkind'. An assertoric *axioma* is constituted by a nominative case and a predicate, e.g., 'Dion is walking'. A demonstrative *axiōma* is constructed with a demonstrative nominative case and a predicate, e.g., 'This man is walking'. An indefinite *axiōma* consists of one or more indefinite constituents and a predicate, e.g., 'Someone is walking'. Diogenes makes a special note of the 'double-negative' *axiōma* (τὸ ὑπεραποφατικόν). This is a negative *axiōma* constructed with a negative particle

[107] We have followed the translations of Long and Sedley ([1990], 1.205) to render these terms.

and a negative *axiōma*, e.g., 'Not: not: it is day'. Such an *axiōma*, according to Diogenes, is assumed to have the same meaning as the *axiōma* 'It is day' (DL 7.69).

Of the complex *axiōmata* described by Diogenes, we will consider only those which have a role in the Stoic syllogistic system.[108] These are the 'conditional axiōma'(t'ο συνημμένον), the 'disjunctive *axiōma*' (τὸ διεζευγμένον), and the 'conjunctive *axiōma*' (τὸ συμπεπλεγμένον). The conditional *axiōma* is constructed by means of the conditional connective 'if' (εἰ).[109] This connective 'guarantees' (ἐπαγγέλλεται) that the second constituent of the conditional *axiōma* follows (ἀκολουθεῖν) from the first, as, for example, 'If it is day, it is light' (DL 7.71). A disjunctive *axiōma* is constructed by means of the connective 'or' (ἤτοι). This connective guarantees that one or the other of the constituent *axiōmata* is false, for example, 'Either it is day or it is night' (DL 7.72). A conjunctive *axiōma* is constructed by means of a 'conjunctive' connective, such as the particle 'καί' in this example: 'It is day and it is light' (καὶ ἡμέρα ἐστὶ καὶ φῶς ἐστι) (DL 7.72).

A topic of interest at this point might be that concerning the truth conditions for the various types of complex *axiōmata*. We shall discuss the truth conditions for the conditional in the section on inference, but for the moment we intend to consider some questions about *axiōmata* in relation to what has been written about presentations and *pragmata*. One of the distinctions among presentations recorded by Diogenes Laërtius is that between sensory (αἰσθητικαί) and non-sensory (οὐκ αἰσθητικαί) presentations (DL 7.51). He writes that "sensory presentations are those apprehended through one or more sense-organs, whereas non-sensory presentations are those perceived through the mind itself, such as those of the incorporeals and of other things apprehended by reason" (DL 7.51).[110] In Passage A Sextus implies that a Greek speaker, on hearing a significant utterance in Greek, will apprehend the *pragma* signified and subsisting coordinately with thought, whereas the non-Greek-speaker will not apprehend the *pragma* (AM 8.12). We interpreted this as a description of how a rational presentation would be induced in the mind of the Greek speaker by the utterance. The content of the presentation would be the *pragma* signified by the utterance.

We take it that such a presentation, although induced by a sound sensed through the hearing organs, or perhaps by the marks on a papyrus or a stone sensed through the organ of sight, would, nevertheless, be classified as a non-sensory presentation. For in order that a presentation be a sensory presentation, it seems evident that not only must it be apprehended through one or more of the sense organs, and hence have its cause in some portion of the qualified substrate, but also it must have

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[108]Sextus Empiricus' account of complex *axiōmata* is not so compact or concise as that of Diogenes, but it is perhaps more philosophically interesting. He talks about the conditional *axiōma* at AM 8.108-12, and about the conjunctive *axiōma* at AM 8.124-29, but he does not seem to have an account of the disjunctive *axiōma* which is comparable to that of Diogenes.

[109]This doxography is attested by Chrysippus in his *Dialectics* and by Diogenes the Babylonian in his *Art of Dialectic* (DL 7.71).

[110]αἰσθητικαὶ μὲν αἱ δι' αἰσθητηρίου ἢ αἰσθητηρίων λαμβανόμεναι, οὐκ αἰσθητικαὶ δ' αἱ διὰ τῆς διανοίας καθάπερ τῶν ἀσωμάτων κὰι τῶν ἄλλων τῶν λόγῳ λαμβανομένων.

a content in which either that portion of the qualified substrate is itself signified by a demonstrative, or the quality which individuates it is signified by a name. It is apparent that in the normal course of events, a presentation caused by an utterance may satisfy the first requirement, but it will not satisfy the second, for the content of such a presentation, as is indicated by the discussion in the preceding paragraph, is the *pragma* signified by the utterance, and not any feature of the utterance itself.

There is evidence that the Stoics viewed certain thought processes as some sort of 'internal discourse' (ἐνδιάθετος λόγος).[111] Such thoughts can no doubt be considered as 'utterances', and as such, will induce presentations in the mind. Clearly the presentations produced by such utterances will be non-sensory. Hence it would seem that a non-sensory presentation may be induced in one's mind either by someone (else) speaking a *pragma*, or by one speaking a *pragma* in thought. The *pragma* spoken (πρᾶγμα λεκτόν) might be an *axiōma* signified by an assertion, but it might also be *prostaktikon* signified by a command, or an *erōtēma* signified by a query, or some other type of *lekton*. No doubt we not only sometimes make assertions to ourselves in thought, but also sometimes ask ourselves questions or exhort ourselves to action. This latter notion of speaking imperatives to ourselves in thought seems to be a necessary feature of Brad Inwood's interpretation of the Stoic theory of action [Inwood, 1985, pp. 59-60 86–87]. Interesting as it might be to follow up on these other classes of *lekta*, *axiōmata*, however, are the *lekta* which are of interest in the present context.

We take it that the presentation induced by someone uttering a declarative sentence, or by someone uttering a declarative sentence in thought, will have a *pragma* as content which has a structure isomorphic to that of the *axiōma* signified. We are using 'isomorphic' here to suggest a structure preserving correspondence between the elements of the *pragma* and the elements of the *axiōma*. Thus, if we see Dion walking and so have a sensory presentation of Dion walking, the *pragma* accompanying this presentation can be represented by the simple structure {Dion, to walk}; however, if someone were to say to us 'Dion is walking' so that we have a non-sensory presentation of Dion walking at the present time, the attendant *pragma*, although representable on some level by the same structure, would seem to require a representation which includes an element to signify the present tense.

[111]In the *Theatetus* Socrates says that "when the mind is thinking, it is simply talking to itself, asking questions and answering them, and saying yes or no" (190a). In a similar vein in the *Sophist*, the Eliatic Stranger says that "thinking and discourse are the same thing, except that what we call thinking is precisely, the inward dialogue carried on by the mind with itself without spoken sound" (263a). No doubt there are problems with the view that all thinking is like internal discourse; it seems, however, that something of this tradition was carried on by the Stoics, for according to Sextus Empiricus, they held that "it is not with respect to uttered speech (προφορικὸς λόγος) that man differs from the irrational animals (for crows and jays and parrots utter articulate sounds), but with respect to internal discourse (ἐνδιάθετος λόγος)" (AM 8.275). And according to Galen, the Stoics define the mental process which provides the means of converting sensory data to knowledge, that is, the process by means of which "we understand consequence and conflict, in which separation, synthesis, analysis and demonstration are involved" (SVF 2.135), as 'internal discourse' (*endiathetos logos*).

One might, for example, portray this *pragma* as follows: {Dion, to walk: now}. However, since it is beyond the scope of this work to develop a detailed account of how such representations might be handled, we will resist the temptation. We have chosen to characterise the *axiōma* by merely enclosing its signifying sentence in braces, and although there are probably good reasons to develop distinct modes of representation for the *pragma* and the *axiōma*, we have chosen to depict the *pragma* by the same method.

To consider a more complex example, suppose that someone says to one 'Dion was walking about in Athens at noon yesterday'. It seems evident that the *pragma* which accompanies the presentation induced by this utterance will reflect the structure of the *axiōma*, and hence have as constituents not only the individuating quality signified by the name 'Dion' and the predicate signified by the verb 'walking', but also temporal components and the individuating quality signified by the name 'Athens'. We shall represent both the *pragma* and the *axiōma* as follows: {Dion was walking about in Athens at noon yesterday}. It is worth emphasising that we shall represent the *pragma* in this way only when it is the content of a presentation induced by an utterance.

7 THE CONDITIONAL *AXIŌMA*

The conditional *axiōma*, according to Chrysippus and Diogenes the Babylonian, is constructed from two *axiōmata* by means of the connective 'if' (εἰ) (DL 7.71). Of the two constituent *axiōmata*, the one signified by the sentence placed immediately after the connective is called the 'antecedent' and 'first' (ἡγούμενον), whereas the other is called the 'consequent' and 'second' (λῆγον) (AM 8.110).[112] The connective 'if' seems to 'promise' or 'guarantee' (ἐπαγγελλέλεται) that the consequent 'follows' (ἀκολουθεῖ) the antecedent (AM 8.111; DL 7.71); hence, the relationship of 'following' (ἀκολουθία) between its antecedent and consequent is evidently the characteristic property of the conditional *axiōma*. Since the conditional *axiōma* is, after all, an *axiōma*, it might be expected that for any particular conditional *axiōma*, one could give an account of the presentation to which it corresponds, and of the *pragma* which is the content of the presentation. A passage of Sextus Empiricus, which we intend to quote presently, provides a clue to the psychological aspects of this relationship. However, since the context in which this passage occurs is a discussion of the Stoic doctrine of signs, it might be useful to give a short summary of this teaching.

According to Sextus Empiricus, the Stoics define the sign (τὸ σημεῖον) as a true antecedent *axiōma* in a sound conditional, capable of revealing (ἐκκαλυπτικός) the consequent (AM 8.245; 250; PH 2.104).[113] Sextus reports that signs were distinguished between those which are 'indicative' (ἐνδεικτικόν), and those which

[112]Sextus notes that reversing the normal sentence order does not affect this rule. Thus, in each of the examples 'If it is day, it is light' and 'It is light, if it is day', the antecedent is the *axiōma* signified by the sentence 'It is day' (AM 8.110).

[113]At AM 8.104, Sextus implies that this definition was reserved only for the 'indicative sign'

are 'commemorative' (ὑπομνηστικόν) (AM 8.151). An indicative sign is said to indicate 'that which is naturally non-evident' (τὸ φύσει ἄδηλον), and is never observed in conjunction with the thing signified (8.154). The soul, for example, is naturally non-evident, and its existence is supposed to be indicated by bodily motions (8.155). A commemorative sign, on the other hand, signifies what is 'temporarily non-evident' (ἐπί τῶν πρὸς καιρὸν ἀδήλων), and is sometimes observed in conjunction with what is signified; hence, the perception of the sign brings to mind what is often perceived along with it but is momentarily unperceived. For example, since smoke is often observed in conjunction with fire, it is taken as a commemorative sign of fire even though the fire itself is unperceived (8.151-52).

7.1 *Akolouthia: psychological aspects*

The connection between the doctrine of signs and the notion of 'following' (ἀκολουθία) is spelled out by Sextus Empiricus in a passage which records the Stoic reply to several criticisms levelled at the theory of signs by the Skeptics.

> [The Stoics] say that it is not with respect to uttered speech (προφορικὸς λόγος) that man differs from the irrational animals—for crows and jays and parrots utter articulate sounds, but with respect to internal discourse (ἐνδιάθετος λόγος). Nor [does man differ from the irrational animals] with respect to the simple presentations (for they also form such presentations), but with respect to the 'inferential' (μεταβατική)[114] ανδ `ςομποσιτιοναλ' (συνθετική) presentations, because of which he immediately possesses the conception (ἔννοια) of 'following' (ἀκολουθία), and through the conception of following he apprehends the notion (νόησις) of sign (σημεῖον); for sign itself is such as this: 'If this, then this'. Therefore it follows that sign also exists in accordance with the nature and constitution of man (AM 8.275-77).

We interpret this passage as follows. The faculty of forming presentations from our conceptions and complex presentations from simpler ones is part of the nature and constitution of human beings. This faculty is itself founded on our capacity for 'internal discourse', which makes possible the 'inferential' and 'compositional' thought necessary for the production of such presentations. Thus, we differ from the irrational animals, for they do not possess this faculty for producing presentations, but must rely solely on their senses for the data from which presentations are formed. Moreover, because of this capacity for constructing inferential and compositional presentations, we also possess the conception of 'following', and since the relationship between the sign and what it reveals can be represented as a conditional *axiōma*, it is through the conception of following, that we understand the notion of sign.

[114]The adjective 'μεταβατικός' is derived from the noun 'μετάβασις', the basic meaning of which is 'a moving over' or 'a shifting' or 'a change of position' (Liddell and Scott). This etymology is reflected in the use of the adjective in this context which seems to suggest the *transition* from one conception to another by the process of inferential or discursive thought

Of immediate interest in this text is the statement that we possess the conception of *akolouthia* because of our ability to form inferential and compositional presentations. There is little indication in the passage as to how this capacity is supposed to produce the conception of following. It may be possible, however, to work out an interpretation by considering some clear examples of *akolouthia* from Sextus Empiricus' discussion of the theory of signs, and by keeping in mind the quotation of Aëtius concerning the acquisition of conceptions and preconceptions, as well as the texts of Diogenes Laërtius and Sextus Empiricus concerning the ways of producing complex conceptions.[115] Our conjecture is that the 'inferential' and 'compositional' presentations are those from which the conceptions of general conditionals are inscribed on the soul. Furthermore, we would suggest that it is the totality of conceptions of general conditionals from which the conception of following arises.

Consider, then, some examples of signs mentioned by Sextus Empiricus. The following are commemorative signs: smoke is a sign of fire and a scar a sign of a previous wound (AM 8.152-53; PH 2.102). A punctured heart is a sign of immanent death (AM 8.153). Lactation is a sign of conception (AM 8.252; PH 2.106) and a bronchial discharge is a sign of a lung wound (AM 8.252-53). In these examples, according to Sextus, the sign often appears together with what it indicates, and hence, when the latter is not evident, the sign is able to reveal it because we remember that they occur together, for instance, that a punctured heart results in death (AM 8.152-53). But it would seem to be implicit in the passage quoted above that a certain degree of prior conceptual development must take place before one acquires the conception of sign, and so understands one state of affairs as a sign of another states of affairs.

This development, as Sextus indicates, no doubt begins by one's noticing that certain types of presentations seem to occur together as a sequence. Thus a presentation of a man who has been wounded in the heart will be followed after some period of time by a presentation of the same man having died. The *pragmata* which are the contents of these presentations may be represented respectively by the complex {This man, to be wounded in the heart} and the complex {This man, to die}. They would be spoken respectively as the *axiōmata* This man is wounded in the heart and This man has died (cf. AM 8.254). Given some number of similar situations and supposing the capacity for 'inferential' and compositional' thought—perhaps along with certain conceptions and preconceptions already established, for example, general conceptions of causality—one might produce a non-sensory presentation of a causal connection between these types of events.[116] The content of this presentation would be a complex *pragma*, and the components of this complex *pragma* may be represented as follows: the first constituent will be

[115] See Section 4.2 for both of these references

[116] We are making two assumptions here which we have not made explicit but which seem plausible. These are (1) that a basic notion of causality would be recognised by the Stoics as a preconception (πρόληψις) acquired by most people, and (2) that the Stoics would recognise the examples of commemorative signs listed above as cases in which there is a causal connection between the sign and what it indicates

the *pragma* {Someone, to be wounded in the heart}, followed by, say, an arrow to represent the connection,[117] followed by the *pragma* {That one, to die}. Finally, the whole complex will be enclosed in braces. Thus the representation will be constructed as follows:

$$\{\{\text{Someone, to be wounded in the heart}\} \rightarrow \{\text{That one, to die}\}\}.$$

The *axiōma* which is this *pragma* spoken as an assertion might be represented in a similar manner, although it may contain certain constituents such as temporal elements not present in the representation of the *pragma*. The *axiōma*, then, might be represented as follows:

$$\{\{\text{Someone is wounded in the heart}\} \rightarrow \{\text{That one will die}\}\}.$$

Taking the particle 'if' (εἰ) as the connective which seems to provide the most natural way to signify the arrow, the sentence which signifies this *axiōma* will be the sentence 'If someone is wounded in the heart, that one will die'. This sentence which signifies a generalised conditional *axiōma* (τὸ καθολικόν) might be seen as expressing a law-like relationship between those situations in which someone is wounded in the heart and those situations in which that person dies of the wound. No doubt this could be viewed as a relationship of 'following' or 'consequence' in the causal sequence of events, and given a similar analysis of the causal relationships between other states of affairs, it seems probable that a general conception of following would be developed; moreover, one might plausibly assume that the relationship of following conceived to obtain between events in the causal nexus would be carried over to the *axiōmata*. In any case, there is no doubt that the Stoics viewed the relationship of *akolouthia* as one which holds between *axiōmata* as well as between events in the causal nexus.

A difficulty with this interpretation is that for most of the examples cited from Sextus Empiricus, the direction of the relationship between the sign and what it indicates, or between the antecedent and consequent of the parallel conditional *axiōma*, is not the same as the direction of the causal sequence. For example, a scar is said to be a sign of a previous wound. The general conditional *axiōma* might be expressed by the sentence 'If anyone has a scar, that one has had a wound'. According to the interpretation so far, having a wound would follow from having a scar, but clearly, the direction of *akolouthia* with respect to the causal sequence is from the occurrence of the wound to the formation of the scar. So it is obvious that some adjustment must be made in this account of the development of the conception of *akolouthia*.

Previously, we quoted A.A. Long ([1971, p. 46]: see page 425) to the effect that since the Stoics assumed that events occurred according to a strict causal nexus, they perhaps assigned to logic as its major function the task of making possible predictions about the future from considerations of what follows from

[117]The arrow will turn out to be the item at the level of *lekta* which is signified by the connective 'if' (εἰ)

present events. On this assumption, one might expect that they would have concentrated at first on examples in which the direction of *akolouthia* coincided for the causal sequence and the relationship of following in the conditional *axiōma*, and so stressed the development of the conception of following as we have interpreted it above. But of course, they would also have been interested in drawing out the present consequences of past actions or events, especially with respect to allocating responsibility in the sphere of ethics. Hence, the following relationship between the antecedent and consequent of a conditional *axiōma* need not always proceed from cause to effect as does the following relationship of the parallel causal sequence. And not the least consideration would be those instances of following between the parts of a conditional *axiōma* which do not correspond to instances of following in the causal nexus: in other words, conditional *axiōmata* matching logical connections. Nevertheless, it is not implausible to suppose that the conception of following as it relates to *axiōmata* had its basis in a preoccupation with the kinds of examples which involve reasoning from cause to effect.

There are not so many examples of indicative signs occurring in the text as there are of commemorative signs, but here are two: bodily movement is a sign of the presence of the soul (AM 8.155) and sweat flowing through the skin the sign of the existence of intelligible pores (AM 8.306). For the Stoics, the first example would be a straightforward instance of a causal relationship. The presence of the cause, however, must be inferred from the existence of the effect, hence acquiring the conception of the relationship between them will depend entirely on already established conceptions and preconceptions, and on the capacity for inferential and compositional thought. The relationship in the second example is not obviously a causal relationship, but seems to be a strictly logical. Yet acquiring the conception of the following relationship between these states of affairs would also seem to depend on previously acquired conceptions and preconceptions, such as theories about surfaces and the flow of fluids, as well as on inferential and compositional thought.

7.2 Pragmata spoken as conditionals

Suppose that someone utters the sentence 'If Dion is walking, Dion is moving'.[118] What is the nature of the *pragma* signified by this utterance and of the presentation which has this *pragma* as content? Apparently, the utterance of this sentence will induce a presentation in the mind of the hearer, the content of which will be a complex *pragma*. The components of this complex *pragma* may be represented as follows: the first constituent will be the *pragma* {Dion, to walk}, followed by, say, an arrow to represent the relation of *akolouthia*,[119] followed by the *pragma* {Dion, to move}. Finally, the whole complex will be enclosed in braces. Thus the representation will be constructed as follows:

[118]cf. DL 7.78 where this conditional is featured as the major premiss of the argument 'If Dion is walking, Dion is moving; but Dion is walking; therefore, Dion is moving'

[119]The arrow is the item at the level of *lekta* which is signified by the connective 'if' (εἰ)

$$\{\{\text{Dion, to walk}\} \rightarrow \{\text{Dion, to move}\}\}$$

As we indicated in the previous section, the *axiōma*, that is, the *pragma* spoken, will be represented by the construction

$$\{\{\text{Dion is walking}\} \rightarrow \{\text{Dion is moving}\}\}$$

The information conveyed by the *axiōma* and apprehended in the *pragma* is that any situation in which Dion is walking will be one in which Dion is moving; moreover, this information is communicated regardless of whether there ever is any real situation in which Dion is walking.

Similarly, the utterance of the sentence 'If Dion was walking about in Athens at noon yesterday, he was not on the Isthmus at noon yesterday' will induce a presentation which has as content the *pragma* which may be represented by the complex:

$$\{\{\text{Dion was walking about in Athens at noon yesterday}\} \rightarrow \{\text{Dion was not on the Isthmus at noon yesterday}\}\}.$$

What the *axiōma* conveys is that the existence of the situation in which Dion was walking about in Athens at noon yesterday, rules out the existence of the situation in which Dion was on the Isthmus at noon yesterday.[120] We shall see in the sequel that the standard criterion for the sound conditional *axiōma*, that is, one for which the consequent follows from the antecedent, is a reflection of the kind of relationship that holds between these situations. This relationship might also have been signified by a sentence constructed with the connective 'not both ... and ... ', but it may be that Chrysippus wished to reserve this connective to signify contingent relationships between states of affairs.[121]

Each of the conditional *axiōmata* in these examples will be sound (ὑγιές) or true (ἀληθές) if and only if the consequent *axiōma* follows from the antecedent *axiōma*, which is to say, if and only if it is the case that the *pragma* signified by the consequent sentence follows from the *pragma* signified by the antecedent sentence. How would someone know that this relation holds? In the first example, if one were to have the conception of the general conditional *axiōma* If anything is walking, that thing is moving, and were to know that the relation holds for the general conditional, then it seems clear that one would know that it holds for the particular conditional. It is probably important that this conception could, in principle, be acquired by experience, although it is probably more likely that it would be taught. It could be acquired by experience because in Stoicism something walking possesses an attribute which is identifiable as a certain configuration or mixture of the

[120] The Stoics characterise the relationship between these situations by saying that they 'conflict' (μάχεται) with one another

[121] cf. Mueller [1978, p. 20]; Kneale and Kneale [1962a, p. 161]; Long and Sedley [1990, 1.211]. Note that the relationship between these situations could not be signified by the Stoic disjunction, since that connective indicates that exactly one of the disjuncts is true. But evidently it need not be the case that Dion be either at Athens or on the Isthmus.

pneuma with the individually qualified substrate; moreover, something moving also has an attribute identifiable in the same manner. Thus one could find out through direct experience that anything which possessed one attribute also possessed the other, and in this way come to have the conception of the general conditional. In the other example, one would rely on the conception of the general conditional

$$\{\{\text{Something is at place A at time t}\} \to \{\text{that thing cannot be at place B at time t}\}\}.$$

It is unclear how the acquisition of this conception would be explained. It may be that it would be classified as a preconception (πρόληψις). In any event, knowing that the relation holds for the particular conditional would seem to require knowing that it holds for the general conditional.

8 SEMANTICS AND INFERENCE

The foundation of the Stoic system of inference was the so-called 'indemonstrables' (οἱ ἀναποδείκτοι), a set of five basic argument types which are attributed to Chrysippus by several sources (AM 8.223; DL 7.79; Galen *inst. log.* 34-35), although there is some controversy whether they actually originated with him. Their origin has been ascribed to the Peripatetic philosopher Theophrastus by both Prantl and Zeller,[122] but it has been argued by Bocheński[123] and others[124] that this claim is doubtful. On the other hand, there is some indication that arguments of this sort were discussed by Aristotle and his followers. In the *Prior Analytics* at 50^a 16-50^b 4, Aristotle discusses arguments from agreement (ἐξ ὁμολογίας) and arguments by reduction to the impossible (διὰ τῆς εἰς ἀδύνατον ἀπαγωγῆς).[125] He says that many other arguments are concluded from hypothesis, and these he promises to consider and distinguish in the sequel. But, as Alexander of Aphrodisias points out (*in an. pr.* 390.1), Aristotle never did fulfil this promise. Alexander conjectures, however, that perhaps Aristotle was speaking of certain arguments from hypothesis mentioned by Theophrastus and Eudemus and some others of his followers (390.2-3). These include 'arguments by connection' (τούς τε διὰ συνεξοῦς), which are also called 'conditional' (συνημμένον) or 'hypothetical by an additional premiss' (τοὺς διὰ τῆς προσλήψεως ὑποθετικούς), 'arguments by

[122] Zeller [1962, 119n2]; Mates [1953, p. 69] attests that both Prantl and Zeller held this view, and Kieffer [1964, p. 66] confirms that Prantl held it. He cites Volume 1, page 386 of Prantl's *Geschichte der Logik im Abendlande* (Graz, 1955). (Photographic reprint of Leipzig, 1855, edition).

[123] Both Mates [1953, p. 69] and Kieffer [1964, p. 8] cite Bocheński's *La Logique de Thu)opraste (Collectanea Friburgensia*, N.S. fasc. xxxvii) (Fribourg: 1947). Mates cites pages 116–17 and chapter 7, note 5; while Kieffer cites page 103 ff.

[124] Kieffer writes that Prantl "by a strained interpretation of certain passages in the later commentators on Aristotle ... reached the unjustified conclusion that the traditional hypothetical syllogisms were the discovery or invention of Theoprastus" ([Kieffer, 1964], 66). It seems clear that Mates ([1953], 69) also rejects Prantl's conclusion.

[125] This is the wording of Alexander of Aphrodisias (*in an. pr.* 389.31).

separation' (τοῦ διὰ τοῦ διαιρετικοῦ), also called 'disjunctive' (διεζευγμένον), and 'arguments from a negated conjunction' (τοὺς διὰ ἀποφατικῆς συμπλοκῆς) (390.3-6). In another place, Alexander attributes this alternative terminology to 'the younger philosophers' or *neōteroi* (νεώτεροι), and there is no doubt that this is Stoic terminology. The terms 'συνημμένον', 'διεζευγμένον', and 'συμπλοκὴ ἀποφατική' are attested as Stoic in numerous places. The term 'τροπικόν' is attested by Sextus Empiricus (PH 2.202) and by Galen (*inst. log.* 17.1), and the term 'προσλήψις' by Sextus (AM 8.413) and by Diogenes Laërtius (7.76). According to Kieffer [1964, p. 66], the terms 'argument by connection' (διὰ τοῦ συνεχοῦς) and 'argument by separation' (διὰ τοῦ διαιρετικοῦ) are not found in Aristotle's works. Kieffer [1964, p. 67] cites Bocheński's argument (*La Logique de Thu)ophraste* 108) that since these terms are Peripatetic but not Aristotelian, they likely were coined by Theophrastus and Eudemus. All in all it would seem to be rather unclear to what extent these philosophers advanced the study of hypothetical arguments. If indeed they were responsible for this terminology, then perhaps, as the Kneales suggest [Kneale and Kneale, 1962a, p. 105], its existence is an indication that they made some headway in the analysis of such arguments. On the other hand, we have the testimony of Boethius that neither Theophrastus nor Eudemus carried the investigation into the hypothetical syllogisms much further than where Aristotle left off (Graeser *Die logischen Fragmente des Theophrast, fr. 29*).

A general description of the indemonstrables would be that they are arguments with two premisses of which the 'major', called '*tropikon*' (τροπικόν), is either a conditional, a disjunction, or a negated conjunction, and the minor, called '*proslēpsis*' (πρόσληψις), is a categorical. If the major is a conditional, then the minor is either its antecedent, in which case the conclusion is its consequent, or it is the negation of the consequent, in which case the conclusion is the negation of the antecedent. If the major is a disjunction, then either the minor is one of the disjuncts, in which case the conclusion is the negation of the other disjunct, or it is the negation of one of the disjuncts, in which case the conclusion is the other disjunct itself. If the major is a negated conjunction, then the minor is one of the conjuncts and the conclusion is the negation of the other conjunct. The indemonstrables were often represented by the Stoics as they are below, that is, as *schemata* having ordinal numbers as variables:[126]

[126]See Mates [1953, p. 68] (Table II) for an extensive documentation of the evidence for the indemonstrables. Strictly speaking, I should be talking about the 'conditional *axiōma*', the 'disjunctive *axiōma*', and the 'conjunctive *axioma*', but for the most part we shall refer to 'conditionals', 'disjunctions,' and 'conjunctions' in the sequel.

(I) If the first, the second;
but the first;
therefore, the second.

(II) If the first, the second;
but not the second;
therefore, not the first.

(III) Not both the first and the second;
but the first;
therefore, not the second.

(IV) Either the first or the second;
but the first;
therefore, not the second.

(V) Either the first or the second;
but not the first;
therefore, the second.

In what follows we intend to consider mainly those arguments having a conditional *axiōma* as the major premiss, its antecedent as minor premiss, and its consequent as conclusion, which is to say, those arguments exemplifying *schema* (I) in the above list.

8.1 The sound conditional

In this section we consider several topics concerning the character of the Stoic conditional. These questions would seem to have a bearing on one's view of the role of the conditional in the Stoic system of inference and on one's view of Stoic logic in general. These topics are: (1) the debate over the criterion for a sound conditional, (2) the question whether there was a standard criterion for the Old Stoa, and (3) an account of the notion of conflict (μάχη) which appears in the criterion attributed to Chrysippus.

The controversy

It is well known to students of ancient logic that in the fourth century B.C. a controversy prevailed among various 'dialecticians'[127] as to the proper criterion for a sound conditional (τὸ ὑγιὲς συνημμένον)[128] Although this dispute is mentioned briefly by Cicero in *Academica* 2.143, our information comes mainly from the writings of Sextus Empiricus. At 8.108 in *Adversus Mathematicos*, Sextus outlines the Stoic characterisation of the conditional *axiōma.*[129] The conditional, according

[127]This reference to the 'dialecticians' would seem to be one in which the term should be taken in the general sense of 'one who practices dialectic' or 'logician', rather than in the sense of denoting a member of the Dialectical School (cf. footnote 20, page 408).

[128]In his translation of Sextus Empiricus, R.G. Bury renders 'συνημμένον' as 'hypothetical syllogism'; however, as Mates points out ([Mates, 1953], 43), this term always denotes an 'if ... , then ... ' proposition in the examples given by ancient commentators, never an argument or inference-schema.

The Stoics seem to use 'ὑγιές' (sound) and 'ἀληθές' (true) interchangeably in these contexts. We shall argue in the sequel that we should understand 'true' in the sense of 'sound' when it so occurs, rather than the other way around.

[129]The Greek is 'ἀξίωμα'. This term is often translated as 'proposition' but we have not committed ourselves to so translate it (see page 423). It would seem better for us to transliterate

to the Stoics, is a non-simple *axiōma* whose parts are joined by the connective 'if' (εἰ). The part preceding this connective is called the 'antecedent' or 'first' (ἡγούμενον), whereas the other is called 'consequent' or 'second' (λῆγον). Such an *axiōma* "seems to promise that its second component follows consequently on its first" (AM 8.111), and if this promise is carried through so that the consequent does indeed 'follow' the antecedent, then the conditional is true (ἀληθές), but if not, then it is false (ψεῦδος) (AM 8.112).

Note that in these passages where he is discussing the controversy on conditionals (AM 8.112-17; PH 2.110-12), Sextus appears to use 'ὑγιές' (sound) and 'ἀληθές' (true) as though they were synonymous. In the glossary of *Stoic Logic*, Mates claims that, according to Stoic usage, these terms are interchangeable ([Mates, 1953], 132). Jonathan Barnes states that the conditional "'If p then q' is ὑγιής iff it is true." On the other hand, he also writes that "of course, 'ὑγιής' does not mean 'true'" ([Barnes, 1980], 169n11). It seems possible that there is an ambiguity in Sextus' use of the adjective 'ἀληθές', which, given that he is reporting on Stoic doctrine, one might assume to be a reflection of an ambiguity in Stoic usage. Sextus seems for the most part to represent the Stoics as using this term to describe a statement expressing an *axiōma* the content of which correctly represents 'the way things are'. For example, he reports in one place that the definite *axiōma* 'This man is sitting' or 'This man is walking' is true (ἀληθές) whenever the person indicated by the demonstrative is actually sitting or walking (8.100).[130] On the other hand, there is some evidence to suggest that in the passages cited above (AM 8.112-17; PH 2.110-12), he may also use the term (ἀληθές) in the sense of 'genuine' or 'real'.

First, there is the evident fact that he seems to use 'ὑγιές' and 'ἀληθές' interchangeably. But, as Barnes has pointed out, 'ὑγιές' does not mean 'true' in the sense of a correct representation of the 'way things are'. On the other hand, according to other sources on Stoic logical theory, it does appear in certain contexts to mean 'genuine' or 'proper'. Consider, for example, the following dilemma set out by Aulus Gellius (2.7.6-10):

> A father's command is either honourable or base;
> if his command is honourable, it is not to be obeyed merely because it is his order, but because it is right that it be done;
> if his command is base, it is not to be obeyed because what is wrong ought not to be done;
> therefore, a father's command ought never to be obeyed.

Gellius rejects this argument on the basis that the leading premiss "cannot be considered what the Greeks call a sound and proper (ὑγιές *et* νόμιμον) disjunctive proposition." (2.7.21). He claims that it requires the additional disjunct "or are neither honourable nor base" in order to be considered a genuine Stoic disjunction.

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the term, since at least part of our thesis involves the question of its meaning.

[130] Similarly, Diogenes Laërtius relates that on the Stoics account, the axiōma 'It is day' is true (ἀληθές) just in case it really is day, otherwise, it is false (ψεῦδος) (DL 7.65).

Gellius' motivation here seems to be a reflection of his claim at 16.8.12-14 that the disjuncts of a Stoic disjunction must exhaust the alternatives.

Sextus himself makes the same claim at AM 8.434 where he includes an argument invalid because of deficiency in his classification of invalid Stoic arguments.[131] Here Sextus writes that in the following argument the disjunctive premiss is deficient:

> Either wealth is an evil or wealth is good;
> but wealth is not an evil;
> therefore wealth is good.

In order to be ὑγιές , according to Sextus, the disjunctive premiss ought to read as follows: 'Wealth is either a good or an evil or indifferent'. In both this example and in Gellius' example, the etymological derivation from the sense of 'ὑγιές' as 'sound' to its sense as 'complete' or 'having the required characteristics' would seem to be clear. Moreover, since that which is incomplete could not be a proper exemplar of its kind, the derivation to its sense as 'genuine' also seems clear.

At AM 8.111-112, Sextus writes that a conditional proposition "seems to promise" that its consequent follows (ἀκολουθεῖ) from its antecedent. Moreover, he adds that such a proposition is ἀληθές just in case this promise is fulfilled. Since a conditional proposition which lacked this characteristic could not be a proper Stoic conditional, one might suppose that a conditional is also therefore ὑγιές when this promise is fulfilled. Now inasmuch as 'ἀληθές' can be used in the sense of 'genuine' or 'real', it is possible, therefore, that Sextus uses both 'ὑγιές' and 'ἀληθές' in that sense. This would explain why he uses them interchangeably, even though 'ὑγιές' does not mean 'true' in the sense of a correct representation. However, a conditional proposition which was 'sound' or 'true' in the sense of 'genuine'—which is to say that its consequent follows from its antecedent -could not help but be 'true' in the derivative sense that it would correctly represent the real connections between the states of affairs represented in the antecedent and consequent.

To return to Sextus' account, it would seem to be agreed among the dialecticians that a conditional *axiōma* is sound whenever the consequent 'follows' from the antecedent. The disagreement arises, however, over the question of a criterion to determine when this relation of following holds. At AM 8.112, Sextus sets up the dispute as follows:

> Now on the one hand all the dialecticians assert in common that the conditional proposition is sound whenever its consequent follows its antecedent. On the other hand, concerning when and how it follows they are at odds with one another, and set forth conflicting criteria for the notion of 'following'.[132]

[131] According to the classification of invalid arguments reported by Sextus at PH 2.146 and AM 8.429, Gellius' argument in its unaltered form is also an example of an argument invalid because of deficiency.

[132] κοινῶς μὲν γάρ φασιν ἅπαντες οἱ διαλεκτικοὶ ὑγιὲς εἶναι συνημμένον ὅταν ἀκολουθῇ τῷ ἐν αὐτῷ ἡγουμένῳ τὸ ἐν αὐτῷ λήγονη)= περὶ δὲ τοῦ πότε ἀκολουθεῖ καὶ πῶς στασιάζουσι πρὸς ἀλλήλους. καὶ μαχόμενα τῆς ἀκολουθίας ἐκτίθενται κριτήρια.

The most valuable discussion of the controversy over the criterion for a sound conditional is presented by Sextus in the passage at PH 2.110-12, wherein he outlines the four distinct and competing accounts. In this presentation Sextus apparently orders these definitions from the weakest to the strongest, in each case citing an example which is allowed by the next weaker interpretation, but which is rejected by the one under discussion.

Sextus begins by summarising the position of Diodorus Cronus of the Dialectical School and the conflicting position of his pupil, Philo the Dialectician.[133] He attributes the first account to Philo, and states that according to this version, a conditional is sound whenever it is not the case that the antecedent is true and the consequent false (PH 2.110, cf. AM 8.113). In the passage at AM 8.113, Sextus presents what is in effect a truth table for the Philonian conditional. According to this summary, there are three combinations of truth values for the components of the conditional which make it sound and one which makes it false. These assignments correspond to the assignments in the truth table for the material conditional (cf. AM 8.245); consequently, there is general agreement among modern logicians that Philo's definition amounts to a definition of the material conditional (cf. Mates [1953, p. 44]; Bocheński [Bocheński, 1963, p. 89]). The second definition cited by Sextus is ascribed to the Diodorus Cronus. According to Diodorus, a sound conditional is one which neither was capable nor is capable of having a true antecedent and a false consequent (PH 2.110; cf. AM 8.115). Mates has argued cogently that a sound Diodorian conditional is an always true Philonian conditional ([Mates, 1953], 44-46).

Sextus attributes the third version of the correct criterion to those who advance the view that there must be a 'connexion' or 'coherence' (συνάρτησις) between the antecedent and consequent of a sound conditional. According to this view, a conditional proposition is sound whenever the contradictory (ἀντικείμενον)[134] οφ ιτς ςονσεχυεντ ςονφλιςτς (μάχηται) with its antecedent (PH 2.111). Unlike the first two cases, this definition is not linked by Sextus to the name of any particular philosopher. Recently, however, several authors (e.g., Kneale and Kneale [1962a], 129; Gould [1970], 76; Mueller [1978], 20) have cited a passage in Cicero (*De Fato*, 12) as evidence that the 'connection' view is that of Chrysippus. We shall refer to this definition of a sound conditional as the 'connexivist view', in accordance with its attribution by Sextus to "those who introduce connexion."[135]

The fourth definition, according to Sextus, is advocated by "those who introduce 'implication' (ἐμφάσις)." It states that in a sound conditional the consequent

[133]In the passage AM 8.112-17 Sextus gives a more detailed account of the differences between Diodorus and Philo, but he does not include any mention of the other competing views.

[134]In his glossary, Mates points out the distinction between 'ἀντικείμενον', which means the contradictory of a proposition, and 'ἀποφατικόν', which means a proposition with 'not' prefixed to it. His example makes the distinction clear: "The propositions ἡμέρα ἐστίν [It is day] and οὐκ ἡμέρα ἐστίν [It is not day] are both ἀντικείμενον with respect to one another, but only the latter is ἀποφατικόν' ([Ματες, 1953], 133).

[135]The name has been adopted in certain modern interpretations such as those of Storrs McCall (e.g., in Anderson and Belnap [c1975-, pp. 434-52] and in McCall [1966].

must be 'potentially contained' (περιέχεται δυνάμει) in the antecedent (PH 2.112). According to Mates ([1953], 49), this fourth definition cited by Sextus is not discussed by any other ancient sources, nor has its ancestry been attributed to any particular philosopher. In addition, with such a dearth of information, it has been little discussed by modern commentators. Martha Kneale has suggested that this may even be a Peripatetic view (Kneale and Kneale [1962a], 29). Long and Sedley, on the other hand, think that it may not be significantly different from the connexion account ([Long and Sedley, 1990], 1.211). In any event, this version does not bear the name of any ancient philosopher, nor has a name been adopted as a consequence of its modern interpretations, as in the case of the connexivist thesis. More recently, however, Michael J. White has speculated that the motivation for this fourth type of conditional is somewhat akin to the ideas put forward in modern relevant logics (White [1986, pp. 9–14]; hence we might call this fourth view the 'relevantist' view. However, since White's speculations are somewhat tenuous, and since the philosophers who propose the definition invoke the notion of the virtual 'inclusion' or 'containment' of the consequent in the antecedent, it would seem better to call this fourth view the 'inclusion' or 'containment' criterion.

As an example of a conditional which is sound according to Philo's criterion, Sextus cites the following: 'If it is day, I converse'. This conditional is sound, he says, when in fact it is day and the subject is conversing (PH 2.110). And indeed, if Philo's definition is the analogue of the material conditional, it would also be sound whenever either it is not day or the subject is conversing. But Sextus tells us that according to Diodorus this conditional is false (ψεῦδος),[136] since it is obviously capable of having a true antecedent and false consequent whenever it is in fact day, but the subject remains silent (PH 2.110). As is the case with each of the critiques offered by Sextus, one has to consider the possibility that this objection was not in fact put forward by Diodorus but was contrived by Sextus himself for exegetical reasons. It was noted above that in presenting these definitions of a sound conditional Sextus' intention seems to have been to order them from the weakest to the strongest, one definition being stronger than another just in case an example can be found which is rejected as being a sound by the former, but which is accepted by the latter. Martha Kneale [1962a, p. 129]) has pointed out that if Sextus did so arrange them, then it cannot be assumed that these criteria were actually conceived in the order presented. But even if one cannot make this assumption, it seems to us that one can put forward an account of the development of the controversy which is at least partially along the lines of Sextus' arrangement.

For one thing, it is unclear why chronological priority should be a factor in the debate between Diodorus and Philo. Since these philosophers were teacher and pupil, then regardless of which definition was put forward first, it seems plausible

[136]In his discussions of conditionals, Sextus seems for the most part to use 'ψεῦδος' to contrast with both 'ὑγιές' and 'ἀληθές' (cf. AM 8.112-17 and PH 2.110-12). On the other hand, he also uses 'μοχθηρός' for this purpose (cf. PH 2.105, 111), but this latter term seems to be used more extensively to mean 'invalid' or 'faulty' in connection with arguments (PH 2.150; AM 8.433) or argument *schemata* (PH 2.146, 147; AM 8.429, 432).

to suppose that it was Diodorus himself who articulated the objections to Philo's account and put forward the counter-example. In addition, since several modern commentators agree that the connexion view can be attributed to Chrysippus, it seems feasible that this criterion was formulated later than both the Diodorian and the Philonian definitions; moreover, it seems quite reasonable to suppose that it was Chrysippus who raised the objections to the Diodorian view. On the other hand, it would be somewhat more difficult to substantiate Sextus' ordering of the connexion and inclusion accounts, the reason being that there is no confirmation other than Sextus' own testimony to support the hypothesis that the inclusion criterion was formulated after the connexion account. Nor is there any other evidence to support his version of the inclusionist objections to the connexivist criterion. Hence, in contrast to Kneale's assumption that "we can take it that the objections mentioned by Sextus were in fact put forward at some time" [Kneale and Kneale, 1962a, p. 129], we would urge that one not take his account of the debate between the inclusionists and the connexivists for granted.

The conditional presented by Sextus as being sound according to Diodorus' criterion but not sound according to the connexion criterion is the following: 'If it is not the case that atomic elements of existents are without parts, then atomic elements of existents are without parts'[137] (ΠΗ 2.111). Τηις *αξιόμα* ωουλδ βε τρυε ον Διοδορυσ' αςςουντ βεςαυσε ιτ αλωαψς (ἀεί)[138] begins with the false clause 'It is not the case that atomic elements of existents are without parts' and ends with the true clause 'atomic elements of existents are without parts'; hence, it never was capable, nor is it capable of beginning with a true antecedent and ending with a false consequent (PH 2.111). It seems clear that the *axiōma* 'Atomic elements of existents are without parts' is conceptually or analytically true, and hence necessary. What is more relevant, however, is that it would count as a necessary proposition according to the versions of necessity of both Diodorus and Chrysippus. The definition of Diodorus is worded as follows: "The necessary is that which being true, will not be false" (*necessarium, quod cum verum sit non*

[137] εἰ οὐχ ἔστιν ἀμερῆ τῶν ὄντων στοιχεῖα, ἔστιν ἀμερῆ τῶν ὄντωνστοιχεῖα.

Bury translates this as: 'If atomic elements of things do not exist, atomic elements exist', whereas Martha Hurst has a reading similar to ours [Hurst, 1935, p. 489]. Mates appears to agree with Bury's translation, and his argument for this reading is as follows: "[W]e are explicitly told that the denial of the consequent is not incompatible with the antecedent. Since the denial of the consequent *is* the antecedent, this implies that the antecedent is not incompatible with itself. But if the antecedent were the negation of an analytic statement, it would be incompatible with itself" [Mates, 1953, p. 50]. The problem with this argument is that Mates is assuming a 'non-connexivist' interpretation of 'incompatible'. According to this view, any necessary proposition is incompatible with itself. But this is just the assumption that the connexivists wish to deny (cf. page 482).

It is perhaps worth mentioning that Hurst cites this example as evidence in her argument against a temporal reading of Diodorus' definition of a sound conditional [Hurst, 1935, p. 489], the temporal interpretation being the one favoured by Mates. In doing so, however, we believe she errs in not taking seriously the possibility that this is a counter-example brought against the Diodorean criterion by the connexivists, and not necessarily an example which Diodorus would have put forward himself.

[138] Bury fails to translate the Greek word for 'always', thereby missing the point of the example.

erit falsum) (Boethius *in de interp.* 234); whereas that of Chrysippus is worded thus: "The necessary is that which being true does not admit of being false, or admits of being false but is prevented by external factors from being false" (DL 7.75).[139] It is evident that on either account of necessity, Diodorus' criterion for a sound conditional will make the counter-example sound merely by the fact that the consequent is necessary or that the antecedent is impossible, since either circumstance is sufficient to insure that the conditional never was capable, nor is capable, of having a true antecedent and a false consequent. Thus it seems plain why Diodorus' definition would be rejected by someone who thinks that a sound conditional requires a connexion or coherence between the antecedent and consequent, for clearly his criterion would permit a conditional to be sound even though there is no connection whatever between its parts.

Note that the rejection of the counter-example cited in the previous section can be generalised by stating that the connexivist criterion renders false any conditional in which the antecedent and consequent are contradictories. This characteristic property of the connexivist view of implication is stated by Storrs McCall as follows: "[N]o proposition connexively implies or is implied by its own negation, since it is never incompatible with its own double negation, nor is its own negation incompatible with itself" [McCall, 1966, p. 415]. According to McCall, "this connexive property of propositions was known to Aristotle" [McCall, 1966, p. 415]. In the *Prior Analytics* Aristotle argues that "it is impossible that the same thing should be necessitated by the being and by the not-being of the same thing" (57^{b3}). If it is supposed, for example, that if A is white, then necessarily B is great, and if A is not white, then necessarily B is great, then "it follows of necessity that if B is not great, then B itself is great; but this is impossible" (συμβαίνει ἐξ ἀνάγκης τοῦ B μεγάλου μὴ ὄντος αὐτὸ τὸ B εἶναι μέγα τοῦ δ' ἀδύνατον) (57^{b13}). Consequently, McCall dubs this property, which he represents in Polish notation as *NCNpp*, 'Aristotle's Thesis' ([McCall, 1966], 415).[140]

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[139]ἀναγκαῖον δὲ ἐστιν ὅπερ ἀλθὲς ὄν οὐκ ἔστιν ἐπιδεκτικὸν τοῦ ψεῦδος εἶναι, ἢ ἐπιδεκτικὸν μὲν ἐστι, τὰ δ' ἐκτὸς αὐτῷ ἐναντιοῦται πρὸς τὸ ψεῦδος εἶναι. This account of the necessary is not specifically attributed to Chrysippus by Diogenes; however, as Mates points out, a passage of Plutarch (*de Stoic repugn.* 1055d-e) would seem to indicate that this view cited by Diogenes is that of Chrysippus.

Compare the view called 'Stoic' by Boethius (*in de interp.* 235): *necessarium autem, quod cum verum sit falsam praedicationem nulla ratione suscipiat* (The necessary is that which when it is true, by no account will admit of a false affirmation). According to Martha Kneale [1962a, p 123], this version "can safely be attributed to Chrysippus" since the context in which it occurs in Boethius is similar to that in which Cicero (*de fato* 12-20) contrasts the views on modality of Chrysippus and Diodorus.

[140]Another characteristic connexive property mentioned by McCall is 'Boethius' Thesis', represented as: *CCqrNCqNr* ([McCall, 1966], 416). In *De syllogismo hypothetico*, Boethius presents an inference *schema* which McCall takes to require the assumption of the connexivist principle for a sound conditional. The *schema* is this: If p, then if q then r; if q then not-r; therefore, not-p [McCall, 1966, p. 415]. If we take *Cpq* and *CpNq* as connexivist conditionals, then p is in conflict with both *Nq* and *NNq*. Hence, it seems reasonable on connexivist grounds to say that *Cpq* and *CpNq* are in conflict, and by the connexivist definition for a sound conditional, we get *CCqrNCqNr*, i.e., Boethius' Thesis. Presumably, then, Boethius' argument proceeded as follows

As an instance of a conditional sound according to the connexivist criterion, Sextus puts forward the example 'If it is day, it is day' (PH 2.111). This conditional is connexively sound since obviously every proposition must be in conflict with its own contradictory. Sextus claims that on the inclusion view this proposition and every such 'duplicated' (διαφορούμενον) conditional would be false, the reason being that "it is not feasible that any object should be included in itself" (PH 2.112). Sextus does not give an example of a conditional sound according to the inclusion criterion, and the reason may be, as is suggested by Michael J. White, that "it would ill accord with his purpose of producing suspension of belief ... with respect to *all* accounts to leave the reader with the impression that [this] last account ... is correct" [White, 1986, p. 10]. White also suggests that Sextus "gives the impression of having to strain a bit" [White, 1986, p. 10] in his attempt to show that the aforementioned connexivist paradigm would be false according to the inclusion view. As he expresses the point, this rather "literal-minded" interpretation of 'περιέχεται δυνάμει' does nothing to convince one that these conditionals were indeed rejected by the inclusion view.[141]

It is of interest in this regard to note some comments which have been recorded concerning the relationship between the connexion and inclusion conditionals. The remark of Long and Sedley to the effect that they may not differ significantly from one another has already been mentioned above. Add to this the comments of Martha Kneale that "the difference between them was small" and that "the objection which the partisans of implication brought against the theory of connexion is not of a fundamental kind" (Kneale and Kneale [1962a, p. 134]), as well as the observation of Mates that "the fourth type of implication seems to be a restricted type of Chrysippean implication" [Mates, 1953, p. 49], and there seems to be reason enough to concur with White's doubts about the accuracy of Sextus' report concerning the relationship between these two accounts of implication. Given these doubts, one is tempted to speculate that the order of appearance between the connexion and inclusion definitions may have been the reverse of Sextus' arrangement. If so, then it may be that Chrysippus saw a need for more precision than that afforded by the inclusion definition, and thus formulated the connexion account in response to that perception.

(cf. McCall [1966, p. 416]):

(1)	$p \rightarrow (q \rightarrow r)$	Assumption
(2)	$q \rightarrow \neg r$	Assumption
(3)	$(q \rightarrow r) \rightarrow \neg(q \rightarrow \neg r)$	Boethius
(4)	$\neg\neg(q \rightarrow \neg r)$	2 Double Negation
(5)	$\neg(q \rightarrow r)$	3,4 Modus Tollens
(6)	$\neg p$	1,5 Modus Tollens

[141] Commenting on Sextus' statement that the inclusion definition would reject a conditional of the form 'If p, then p', Michael Frede says that "Sextus Empiricus himself makes it very clear his comment is merely his own interpretation of the definition" [Frede, 1974, p. 90].

The standard Stoic conditional

There are several reasons to suppose that from Chrysippus on, the connexivist account was the standard doctrine of the Old Stoa concerning the criterion for a sound conditional. First, there is some textual evidence in Diogenes Laërtius. At 7.71, in recording the Stoic account of non-simple *axiōmata*, Diogenes reports that according to Chrysippus in his *Dialectics* and Diogenes of Babylon in his *Art of Dialectic*, a conditional is a non-simple *axiōma* constructed by means of the connective 'if' (εἰ), such that this connective promises that the consequent follows (ἀκολουθεῖν) the antecedent. Later, at 7.73, he attests that according to the Stoic criterion, a true (ἀληθές) conditional is one in which the contradictory of the consequent conflicts (μάχεται) with the antecedent. Now although this version of a sound conditional is not actually attributed to Chrysippus and Diogenes of Babylon, the juxtaposition of these passages would seem to indicate a connection. In any case, there is further indirect textual support afforded by the passage of Cicero at *De fato* 12. We have already mentioned that this passage has been cited by several modern commentators as providing strong evidence that the connection or coherence criterion was the account accepted by Chrysippus.[142]

Although the testimony of Cicero and Diogenes Laërtius would appear to be sufficient to establish that Chrysippus propounded the connection doctrine for a sound implication, it does not focus any light on the question as to what criterion Zeno and Kleanthes supported. Indeed, the information available in the texts would seem to be inadequate to establish any certainty in this regard. However, there are some passages in which Sextus Empiricus attributes the Philonian criterion to 'the Stoics' or to 'the Dogmatists', and these remarks are taken by at least one writer to indicate that Zeno adopted Philo's definition of a sound conditional [Rist, 1978, p. 391]. Since Zeno and Philo were contemporaries and both students of Diodorus Cronus, this is a plausible conjecture. On the other hand, there is nothing specific in these passages to link the Philonian criterion to Zeno, and in fact, there is room for some doubt concerning Sextus' attribution of this definition to the Stoics.

As a sceptic Sextus was out to discredit the views of all the so-called 'dogmatic philosophers'. With respect to the Stoics this would have involved, among other things, showing their logical system to be useless as a means of making inferences or providing demonstrations. Thus it is significant that in those passages where he attributes the Philonian definition to the Stoics, Sextus then invokes this alleged characteristic in an attack on some aspect of the Stoic theory of inference. For example, consider the passages at PH 2.104 and at AM 8.244 where he assails the Stoic doctrine of signs.[143] According to Sextus, the Stoics define the sign as an

[142] See pages 407 and 479 for references. Long and Sedley take it that the 'cohesion' criterion is the standard Stoic doctrine for a sound conditional; moreover, they think that "it probably had the approval of Chrysippus" [Long and Sedley, 1990, 1.211].

[143] Sextus claims not to reject altogether the existence of signs, but only those signs which the 'dogmatic philosophers' and 'logical physicians' call 'indicative' (ἐνδεικτικόν) (AM 8.156). On the other hand, those signs called 'commemorative' (ὑπομνηστικόν) are accepted by Sextus, since

antecedent *axiōma* in a sound conditional, capable of revealing (ἐκκαλυπτικός) the consequent (AM 8.245; PH 2.104). Since there are three possible combinations of truth values for a sound conditional,[144] the Stoics further stipulate that the sign will be the antecedent of a sound conditional which begins with a true *axiōma* and ends with a true *axiōma* (AM 8.248-50; PH 2.106). Clearly, nothing in this definition of a sign commits the Stoics to the Philonian criterion for a sound conditional; however, Sextus claims that this is the criterion they accept (AM 8.247; PH 2.105). In proceeding with his criticism, Sextus points out that, according to the Stoics, the thing signified (σημειωτόν) is either 'pre-evident' (πρόδηλον) or 'non-evident' (ἄδηλον) (AM 8.265; PH 2.116). But if pre-evident, then "it will not admit of being signified, nor will it be signified by anything, but will be perceived of itself" (AM 8.267). On the other hand, if it is non-evident, then it cannot be known that it is true, since if it were known, it would then be pre-evident (AM 8.267). Hence, although the truth-value of the antecedent is known to be true, that of the consequent is necessarily uncertain. Therefore, the truth-value of the conditional is uncertain, since the truth-value of the consequent must be known in order to determine the truth value of the conditional (AM 8.268). Hence the Stoic account of a sign is useless, since the soundness of a conditional with a non-evident consequent is indeterminate (AM 8.268).

At AM 8.449, using a similar strategy, Sextus attacks the Stoic argument *schemata*, and in particular, the first indemonstrable.[145] He intends to show that an argument having this *schema* cannot in fact be a demonstrative argument, and hence, is of no use in demonstrating a conclusion. According to the Stoics, a demonstrative argument (ἀποδεικτικός λόγος) is an argument which is conclusive (συνακτικός), has true premisses and a true conclusion, and deduces a non-evident conclusion from pre-evident premisses (PH 2.140; AM 8.422). Starting with this definition, Sextus proceeds roughly as follows (AM 8.449-52). Given any argument for which the premisses are pre-evident, the conclusion is either pre-evident and known, or it is non-evident and unknown. If the conclusion is pre-evident and known, then according to the definition of a demonstrative argument, such an argument is not demonstrative. On the other hand, if the conclusion if non-evident and unknown, then the truth value of the conditional premiss is indeterminate. For the antecedent of this conditional is the minor premiss of the argument, which is pre-evident and known to be true, while the consequent is the conclusion of the argument, which we are assuming to be non-evident and unknown. Hence, since it cannot be determined whether the premisses are true, it cannot be determined that the argument is demonstrative; therefore, in either case, the argument is not

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he takes them to be among the 'common preconceptions of mankind' (ταῖς κοιναῖς τῶν ἀνθρώπων πρλήψεσιν) (8.157). Thus, Sextus is apparently attacking the conception as well as the Stoic theory of indicative signs in these passages, but not the Stoic theory of commemorative signs.

[144] Note that on each of the four accounts, not just on the Philonian definition, a conditional with a true antecedent and false consequent will not be sound; moreover, on each account a sound conditional *may* have either a true antecedent and true consequent, or a false antecedent and false consequent, or a false antecedent and true consequent.

[145] i.e., an argument with the *schema* 'If p, then q; p; therefore q.

demonstrative. But if it is not demonstrative, then it is of no practical use as a means of inference.

As Ian Mueller points out [Mueller, 1978, p. 23], if the truth-functional interpretation of the conditional is taken as the Stoic criterion, then no defence against Sextus' argument can be mustered on behalf of the first indemonstrable. On the other hand, if the connexion interpretation (or the inclusion interpretation) were taken as the Stoic definition, then Sextus' argument would fail. It should be obvious that the ascription of either the connexion criterion or the inclusion criterion to the Stoics would, in addition to blocking Sextus' criticism of the indemonstrables, also nullify his objection, discussed above, to the Stoic doctrine of signs. With respect to the criticism of the indemonstrables, Mueller expresses the point thus:

> There is no way out of this situation, a fact that strongly suggests that Sextus' insistence on applying the truth-functional interpretation to the conditional represents an argumentative device rather than an accurate reflection of standard Stoic doctrine. If the first premise of an undemonstrable argument expresses a stronger than truth-functional connection between its component propositions, there is no reason why the first premise can not be established independently of the conclusion [Mueller, 1978, p. 23].

Mueller goes on to point out that the ascription of a strong interpretation to the Stoic conditional "means that philosophically a great deal of weight must be placed on the knowledge of necessary connections between propositions" [Mueller, 1978, p. 23]. It seems to us, however, that although Mueller's point is correct, his putting the matter in this way is somewhat misleading. Although we will not argue for the point here, we would suggest that it is because their view places a great deal of philosophical weight on a knowledge of necessary connections between *axiōmata* that one ought to ascribe a strong interpretation of the conditional to the Stoics, and not the reverse.

A further point mentioned by Mueller is that many of the criticisms put forward by Sextus in the course of his writings are directed against the possibility of there being such knowledge of necessary connections as the Stoics suppose [Mueller, 1978, p. 23]. However, since it is not our intention here to defend Stoic doctrines from the objections of Sextus Empiricus and other critics, these criticisms are not a concern. What is of concern is to minimise the effects of Sextus' claim that the Stoics adopted the Philonian criterion for a sound conditional. One of these effects, as has already been noted above, is Rist's conjecture that it was Zeno himself who opted for the Philonian view. Clearly this conjecture is at odds with the interpretation we intend to put forward; we take it, however, that what has been said about Sextus' motives in ascribing the Philonian conditional to the Stoics is sufficient to cast some doubt on his claim, and hence, to render Rist's conjecture doubtful as well.

There is yet another passage which would seem to indicate that the Stoics chose the Philonian interpretation for the conditional. This passage occurs in Diogenes

Laërtius, and hence, since Diogenes does not write in a polemical tone, one cannot in this case invoke the sort of argument used against Sextus' attribution. There are, however, some doubts which can be raised against this ascription as well. The passage in question is at 7.81. Here Diogenes says that according to the Stoics:

> The true follows from the true, as, for example, 'It is light' from 'It is day'; and the false, from the false, for example, 'It is dark' from the false 'It is night'; and the true from the false, for example, 'The earth exists' from 'The earth flies'; but the false does not follow from the true, for from 'The earth exists', 'The earth flies' does not follow.

The difficulty with taking this passage as an indication the Stoics adopted the Philonian conditional (e.g. Mates [1953, 44n14]) is that one is immediately confronted with an inconsistency in Diogenes' account of Stoic logic. For at DL 7.73, he reports that, according to Stoic theory, a true conditional is one in which the contradictory of the consequent conflicts with the antecedent. This criterion is precisely the definition of a sound conditional which Sextus described as the one put forward by "those who introduce 'connexion' (PH 2.111).[146] So if one takes the passage at DL 7.81 to indicate that the Stoics adopted the Philonian conditional, then it appears that the account from which Diogenes got his information is inconsistent, since it attests to the adoption by the Stoics of two incompatible definitions of a sound or true conditional. One possibility, of course, is that these different views were predominant at different periods in the history of the Stoic school. However, according to Hicks (DL 7.38nb), the source for the entire doxography on the Stoics from DL 7.49 to DL 7.83 is Diocles of Magnesia, a scholar of the first century B.C. considered by Mates "to have a fair knowledge of Stoic logic" [Mates, 1953, p. 9]. Consequently, one would expect that the account given in these passages would be fairly unified; moreover, one would also expect that if criteria from different periods were included in this doxography, some mention would have been made of the fact. Given these considerations, we would suggest that rather than suppose an inconsistency in Diogenes' source, one take the connexivist definition as the standard Stoic criterion for the period covered by this doxography and look for some other interpretation to explain the passage at DL 7.81.

Such an interpretation might be suggested by considering the examples introduced by Diogenes in this passage. It seems to us that the examples of a sound conditional which he cites are all conditionals which would be sound according to the connexivist criterion (and also, perhaps, according to the inclusion criterion). For instance, he illustrates the true-true case with the conditional 'If it is day, it is light', and the false-false case with 'If it is night, it is dark'. It seems plausible that the Stoics might have taken the contradictory of the consequent in these conditionals to be in conflict with the antecedent on the grounds that these

[146]We are not unaware that Diogenes' definition refers to a true (ἀληθές) conditional, not a sound (ὑγιές) conditional. We have already argued, however, that in these contexts both 'ὑγιές' and 'ἀληθές' should be understood as 'proper' or 'genuine' (See page 477).

states of affairs are related by a necessary causal sequence. Diogenes exemplifies the false-true case with 'If the earth flies, the earth exists', and for this example it is feasible that the Stoics might have invoked conceptual grounds to argue that the contradictory of the consequent would conflict with the antecedent. What we have in mind, then, is that the passage quoted above can be interpreted as a demonstration of the point that it is *possible* for a sound conditional to have either a true antecedent and true consequent, or a false antecedent and false consequent, or a false antecedent and true consequent. And in order to facilitate this demonstration, the Stoics naturally presented examples of conditionals which they took to be sound according to the connexivist criterion, since they adopted the connexivist definition as their criterion for a sound conditional, as we are informed at DL 7.73. For the case in which the antecedent is true and the consequent false, which is to say, for the case where the conditional is not sound, Diogenes gives the example 'If the earth exists, the earth flies'. It would seem that the Stoics might again appeal to conceptual considerations in order to say that in this example the contradictory of the consequent *does not* conflict with the antecedent; moreover, since this criterion determines an unsound conditional (DL 7.73), they could also say on connexivist grounds that this conditional is not sound.

We have already suggested that, given the Stoic definition of a sign, Zeno's interest in the doctrine of signs would afford him reason to take a stance with respect to the criterion for a sound conditional (see page 407). And we have also put forward the view that the purpose of logic for Zeno's wise man is to allow him to make correct judgements about the connections between particular states of affairs on the basis of his knowledge of the general causal principles governing such connections (see page 422). Hence, we take it that Zeno's interest in signs is a manifestation of his general concern to draw out the implications of one's actions in accordance with the natural sequence of events. And since Zeno identified the natural sequence of causation, which he called *heimarmene* with the *logos* or rational principle of the universe (DL 7.149), and since he also identified the *logos* itself with 'necessity' (Lactanius, Tertullianis SVF 1.160), our understanding is that he saw these causal connections as necessary in the sense implied by these identifications. Moreover, we propose that he chose the conditional construction as the syntactical representation of these connections because such representation is suggestive of the causal sequence of events. Hence, the Stoic use of the particle 'εἰ' is technical and already implies a strong interpretation of the conditional since it presupposes a necessary connection between the antecedent and consequent (see page 423).

Therefore, it seems unlikely to us that Zeno would have adopted the view of either Philo or Diodorus, for on either of these conceptions a conditional may be sound even though there is no connection between its parts. Moreover, if the invention of the connexion view can be attributed to Chrysippus, then it seems evident that Zeno could not have opted for this definition since it would not have been available to him. One might conjecture that he proposed an account of his own, and if this were the case, then plausibly he introduced the inclusion criterion.

On the other hand, it is possible that he put forward a version which is completely unrecorded, although it seems to us that this alternative is not so plausible as the first one. In any event, although we shall pursue the matter no further in this work, it would be of interest to explore the possibility that the inclusion definition of a sound conditional was the criterion with which Zeno worked.

Consequence and conflict

The view of Mates and others notwithstanding, it would appear to be an open question how one is to understand the use of the verb 'μάχεται' in the passages where it is used in the definition of a sound conditional by Sextus Empiricus (PH 2.111) and Diogenes Laërtius (7.73; 77). The consensus among these commentators is that it should be understood as 'is incompatible' where 'incompatible "is used in its ordinary sense," which is to say, in the sense that two "incompatible propositions *cannot* both be true, i.e., their conjunction is logically false" [Mates, 1953, p. 48]. On this view, then, a valid argument, according to the connexivists, is such that it is not logically possible for both the contradictory of the conclusion and the conjunction of the premisses to be simultaneously true. But even leaving aside the difficulty of determining the Stoics' understanding of 'logical possibility',[147] there remain some etymological questions as to whether one ought to accept this account as accurately reflecting the intension of 'μάχεται' in Stoic terminology.

To forestall possible objections to putting an etymological cast on the problem, it would be useful to consider Mates' criticism of Philip De Lacy for the latter's use of "weaving together" as a translation of the Greek term 'συμπλοκή', which is standardly translated as 'conjunction'.

> συμπλοκή, the technical term for conjunction, should not be translated as "weaving together." There is no virtue in employing etymological translations for technical terms, since a term becomes technical precisely by being *dissociated* from its etymological and other connotations and associated unambiguously with its denotation [Mates, 1953, 92n24].

Doubtless one can agree that it is never virtuous and perhaps always somewhat fanciful to *translate* a technical term by summoning forth its etymological origins. But it does not follow that in attempting to *understand* a term, the technical meaning of which is either unclear or controversial, one ought to ignore its semantic history.

In the present case, there are those who believe that 'incompatibility' designates a non-truth-functional relation which exists between propositions.[148] They would

[147] Consider, for example, Gould's comment that "it may be the case that [the] distinction [between empirical impossibility and logical impossibility] had not, as a matter of fact, been discerned in the Hellenistic age" [Gould, 1970, p. 81].

[148] See, for example, E.J. Nelson [1930, esp. pp. 440–43]; and R.M. Stopper, [1983, pp. 281–86].

criticise Mates' and those who agree with him on the grounds that, according to his characterisation, it would turn out that an impossible proposition would be incompatible with any proposition, even itself. This result is not in accord with their logical intuitions. The relation which Mates describes is, on their view, more aptly designated as *incompossibility*.[149] What is relevant to this controversy, however, is the question of how the Stoics understood the meaning of 'μάχεται', which, as has been noted, is standardly translated as 'incompatible'. It is here that one can look to etymology for assistance.

The primary meaning for μάχεσθαι, the infinitive form of the verb, is *to fight* or *to battle* or *to war*. Now, one would hardly want to translate the term 'μάχεται' in a logical context as 'fights' or 'battles' or 'wars'. To the modern logician, not only do they seem somewhat fatuous as a description of a relation between propositions, but also they seem rather out of place in a logic treatise. Nevertheless, these renderings would seem to reflect more faithfully the etymology of 'μάχεται' than does the translation 'is incompatible', at least where the latter is understood as Mates understands it. Probably 'conflicts' is just the right compromise. It is bloodless enough for a logic book, yet it remains faithful to the etymological origins of the Greek term, more so, it would seem, than 'is incompatible'.

Now with this translation in mind, consider the thesis that the Stoics understood this notion of conflict in terms of 'incompossibility', where this term is taken in the sense that two propositions are incompossible just in case it is not possible that they both be true. It has been pointed out that, according to this characterisation, it is a sufficient condition for two propositions to be incompossible if one of them is necessarily false. Hence, the propositions 'All triangles have four sides' and 'Chrysippus is the greatest of Stoic logicians' would be incompossible. Would the Stoics have considered these propositions to be in any sense 'in conflict'? It is difficult to see how anyone would suppose them so. On the other hand, consider the propositions 'All triangles have four sides' and 'All triangles have five sides'. These propositions are clearly incompossible because both are impossible, but it is also clear that they are related in such a way that if one is affirmed, then the other must be denied. This relation, moreover, is independent of the truth values or the modal status of the individual propositions. It is this sort of relation which the critics of Mates' view appear to have in mind as the proper meaning of the term 'incompatibility'. And, one might assume, it is also what they would expect that the Stoics had in mind when the latter spoke of propositions or states of affairs being 'in conflict'.

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[149] M.R. Stopper quoting from a paper by Mauro Nasti de Vincenti ("Logica scettica e implicazione stoica," in *Lo scetticismo antico*, ed. G. Giannantoni, Naples, 1981.), writes that "'P' conflicts with 'Q' just in case 'P' and 'Q' are not compossible," and he symbolises this definition thus [Stopper, 1983, p. 284]:

(A3) $C(P,Q) \leftrightarrow L\neg(P \wedge Q)$

He goes on to say that (A3) has "some strange consequences." For example, "any impossible proposition is incompossible with any other proposition whatsoever" [Stopper, 1983, p. 285].

8.2 *The conditional and inference*

Validity and conditionalisation

The so-called *principle of conditionalisation* is presented in several places by Sextus Empiricus as a Stoic criterion for a valid argument.[150] As it is framed by the Stoics, this canon states that an argument is conclusive[151] ωηενεερ ιτς ςορρεσπονδινγ ςονδιτιοναλ ις σουνδ (ὑγιές : PH 2.137) or true (ἀληθές: AM 8.417): that is, the conditional which has the conjunction of the premisses as antecedent and the conclusion of the argument as consequent. As an example, Sextus presents the following case at PH 2.137. The argument

(1) If it is day, it is light;
it is day;
therefore it is light

has as its corresponding conditional the following:

(2) If (it is day, and if it is day, it is light), it is light.

The application of the principle here is the assertion that since the corresponding conditional is sound, the argument is valid. According to Sextus' account at AM 8.111-12, it was agreed among the 'dialecticians' that a conditional *axiōma* is sound whenever its consequent *axiōma* 'follows' its antecedent *axiōma*. Hence, to say that (2) is sound is just to say that its consequent, which corresponds to the conclusion of (1), indeed follows its antecedent, which corresponds to the conjunction of the premisses of (1). It seems evident that if the Stoics wished to attribute such a property to a valid argument, then they must have assumed that the relation of 'following' (ἀκολουθία), which they took to be the relation holding between the antecedent and consequent in a sound conditional, was the same relation holding between the premisses and conclusion of a valid argument (cf. PH 2.113).

Taking into account the debate over the sound conditional discussed in Section 8.1, as well as the principle of conditionalisation, one would expect that there would have been recorded as many distinct conceptions of a valid argument as there were accounts of a sound conditional. This does not, however, seem to be the case. Other than the conditionalisation principle itself, there appears to be no mention in the fragments of a criterion for a valid argument except the one implied by Diogenes Laërtius at 7.77. In this passage Diogenes presents the following characterisation of an *invalid* argument.

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[150] e.g., AM 8.415; PH 2.113, 137. See Mates ([1953], 74-77) for a discussion of this principle. As Mates points out, this principle need not be taken as defining the Stoic conditional, but merely as a characterising a property common to all valid arguments.

[151] In some places (e.g., PH 2.137, 146) Sextus uses 'συνακτικός' and 'ἀσυνακτός' for 'conclusive' and 'inconclusive' (or 'valid' and 'invalid'), whereas at other places (e.g., AM 8.429) he uses 'περαντικός' and 'ἀπεραντός'. Hence, as Mates indicates in his glossary [Mates, 1953, pp. 132–36], these terms appear to be interchangeable.

> And of arguments some are conclusive (valid) and some inconclusive (invalid). Inconclusive are those in which the contradictory of the conclusion does not conflict with the conjunction of the premisses.[152]

Although it is not explicitly stated, this characterisation would seem to imply that a *valid* argument is one in which the contradictory of the conclusion *is* in conflict with the conjunction of the premisses.

In addition to the above account which implies a criterion for a valid argument, Diogenes also reports the following Stoic criterion for a sound conditional.

> So, then, the true conditional axiōma is one in which the contradictory of the consequent conflicts with the antecedent, as in this example: 'If it is day, it is light' (DL 7.73).[153]

It is evident that the criterion for a sound conditional described in the passage at DL 7.77 is identical to the one which Sextus Empiricus reports at PH 2.111. This is the criterion proposed by "those who introduce 'connexion' or 'coherence' (συνάρτησις)" as a condition on the relation of following between the antecedent and consequent of a sound conditional. It was mentioned earlier that this standard has been ascribed by several modern commentators to Chrysippus himself (see page 479). In light of his influence on the development of Stoic logic, it is probable that if this 'connexivist' view was indeed the one he advocated, then it would have been the one accepted by the Stoa.

The formulation of the connexivist criterion leaves no doubt that its adoption would commit the Stoics to a strong interpretation of the criterion for a sound conditional *axiōma*. Thus it seems plausible that for the Stoics the term 'ἀκολουθεῖν' expressed a real connection or coherence between the antecedent and consequent, and, in some sense, a necessary relation between them. Since the conditionalisation principle implies that the same relationship holds between the premisses and conclusion of a valid argument, we can infer that such a connexion obtained between them as well.

Now in accordance with the conditionalisation principle, the ubiquitous Stoic example

(3) If it is day, it is light;
it is day;
therefore it is light

would be valid just in case the following conditional were sound:

[152] τῶν δὲ λόγον οἱ μέν εἰσιν ἀπέραντοι, οἱ δέ περαντικοί. ἀπέραντοι μὲν ὧν τὸ ἀντικείμενον τῆς ἐπιφορᾶς οὐ μάχεται τῇ δια τῶν λημμάτων συμπλοκῇ.

[153] συνημμένον οὖν ἀληθές ἐστιν οὗ τὸ ἀντιχείμενον τοῦ λήγοντος μάχεται τῷ ἡγουμένῳ, οἷον 'εἰ ἡμέρα ἐστί, φῶς ἐστι.'

The question concerning the interpretation of the Greek term 'μάχεται', which we have rendered as 'conflicts', has already been discussed (see page 489). We have argued that the notion of conflict which the Stoics had in mind requires some degree of common content between the *axiōmata* in this relationship.

(4) If (it is day, and if it is day, it is light), it is light.

And in accordance with the description of a sound conditional given by Diogenes Laërtius at 7.73, (4) would be sound just in case the contradictory of its consequent were in conflict with its antecedent. Thus, in conformity with the conditionalisation principle and the description of a sound conditional presented by Diogenes, (3) would be valid just in case the contradictory of its conclusion were in conflict with the conjunction of its premisses. This would seem to suggest that the characterisation of a valid argument given by Diogenes at 7.77 is derived from an application of the connexivist notion of a sound conditional to the principle of conditionalisation.

There are, however, difficulties with this proposal. The first objection is that there are the passages in Sextus Empiricus (PH 2.104; AM 8.245) which seem to indicate that the Stoics adopted the Philonian account of a sound conditional (cf. [Mates, 1953, p. 43]). A further objection is that both Mates [1953, p. 60, 75] and Bocheński [1963, p. 97] cite passages at PH 2.137 and AM 8.415 to support the thesis that the conditionalisation principle required a 'Diodorean-true' conditional. The views of the Dialecticians Philo and Diodorus have been discussed earlier (see page 479), however, a brief summary of their views might be in order for the present. According to Philo, then, a conditional is sound whenever it does not have a true antecedent and false consequent (PH 2.110; AM 8.113). According to Diodorus, on the other hand, a conditional is sound if it neither was capable nor is capable of having a true antecedent and false consequent (PH 2.110; AM 8.115).

In replying to the first objection one probably cannot deny that the texts appear to support the view that the Philonian account gained some measure of acceptance among the Stoics. One might point out that acceptance of this account was by no means unanimous, as the passage at AM 8.245 indicates. And even if this was the view chosen by many Stoics, the debate continued.[154] If it were the case that they did opt for the Philonian criterion, then one would expect that applications of the conditionalisation principle would reflect that fact. But we believe that a more telling reply would be to point out the inconsistencies in Sextus' various reports. First, the adoption of the Philonian truth conditions would seem to be in conflict with the reported wide acceptance of the doctrine that an argument is valid when and only when its corresponding conditional is sound. Since there is no necessity in the relation between the antecedent and consequent of a sound Philonian conditional, it is hard to see how such a conditional could underwrite the validity of its corresponding argument. Against this reply, one might propose, as Josiah B. Gould does [Gould, 1974, p. 160], that the advocates of the Philonian view perhaps invoked the Diodorian truth conditions in applications of the conditionalisation principle. One might point to those passages cited by Mates and Bocheński wherein it appears that the Stoics had the Diodorean conditional in mind when

[154] χρίσεις δὲ τοῦ ὑγιοῦς συνημμένον πολλὰς μὲν καὶ ἄλλας εἶναι φασιν, μίαν δ' ἐξ ἁπασῶν ὑπάρχειν, καὶ ταύτην οὐχ ὁμόλογον, τὴν ἀποδοθησομένην (AM 8. 245).

they framed this principle. This approach, however, has its own problems. This interpretation of the texts mentioned above would seem to be inconsistent with the account of the criteria for invalidity referred to by Sextus at PH 2.146-51 and AM 8.429-34.

In this account Sextus reports that the Stoics deemed an argument invalid according to a list of four criteria. These are: having premisses and conclusion which are incoherent with one another, having redundant premisses, being propounded in an invalid form, and having a deficient premiss. A problem arises when one attempts to square the first criterion on this list with the proposal that the principle of conditionalisation required a Diodorean-sound conditional. As a consequence of this proposal, an argument such as the following would appear to be valid:

(5) If Dion is walking, he is moving;
but wheat is being sold in the market;
therefore, the elements of existents are without parts.

One would be committed to judge (5) as valid if the following, which is its corresponding conditional, were Diodorean-sound:

(6) If (wheat is being sold in the market, and if Dion is walking, then he is moving), then the elements of existents are without parts.

But (5) could not be considerd valid according to the criterion which prohibits incoherent (διάρτησις) arguments from being valid. The problem, therefore, is that if the principle of conditionalisation requires a Diodorean-sound conditional and if (6) were Diodorean-sound, then (5) would be valid, contrary to the criterion for invalidity mentioned above. On the other hand, if this criterion for invalidity were to prevail, then (5) could not be valid and the conditionalisation principle could not require a Diodorean-sound conditional, provided that (6) is Diodorean-sound. Thus, if (6) is Diodorean-sound and (5) is not valid, then it is not clear that one ought to accept the thesis that the principle of conditionalisation requires a Diodorean-sound conditional.

Now it is apparent that Diodorus would have been committed to the soundness of (6) merely because of the modal status of its consequent, for according to Sextus Empiricus (PH 2.111), Diodorus would deem the following conditional to be sound.

(7) If it is not the case that the elements of existents are without parts, then the elements of existents are without parts.

It was determined earlier (see page 481) that the consequent of this conditional would have been considered necessarily true according to the Diodorean view of necessity. Hence, the corresponding conditional of (5) would be Diodorean-sound merely because, according to Diodorus, its consequent could have been neither false nor false. That is, (6) neither was nor is capable of having a true antecedent and a false consequent, since it neither was nor is capable of having a false consequent. It

would seem apparent, then, that (6) is Diodorean-sound. It is not clear, therefore, that one need accept the contention that the principle of conditionalisation requires a Diodorean-sound conditional.

In view of the foregoing arguments, neither of the objections considered is decisive against the proposal that the connexivist standard was the criterion for a sound conditional which prevailed in the early Stoa. The formulation of this account, which was put forward by "those who introduce 'connexion' or 'coherence'" as a condition on the relation of following between the components of a sound conditional, would seem clearly to have committed the Stoics to a strong interpretation of the relationship between the antecedent and consequent of a conditional *axiōma*. Thus the Stoics would seem to have understood the term 'to follow' (ἀκολουθεῖν) as expressing a necessary relation, in the appropriate sense of necessary, not only between the antecedent and consequent of a sound conditional, but also, as a result of the connection between a valid argument and its corresponding conditional, between the premisses and conclusion of a valid argument.

General conditionals

According to Josiah Gould, as we have seen, Chrysippus thought that one could generalise on the observed relations between different types of states of affairs or events and express these generalisations as conditional statements [Gould, 1970, pp. 200–201]: see page 426). What is required, then, is an account of such general conditionals, and clearly the relationship between singular and general conditionals will need to be sorted out. Unfortunately, there are very few examples of such general conditionals in the extant texts; however, the few that there are would seem to be sufficient to indicate the pattern. An example occurring in Cicero is as follows: "If anyone (*quis*) was born at the rising of the dogstar, he will not die at sea" (*De fato* 12). Another example occurs in Sextus Empiricus where he informs us that, according to the writers on logic, "the definition 'Man is a rational, mortal animal', although differing in its construction, is the same in meaning as the universal (καθολικόν) 'If something (τί) is a man, that thing (ἐκεῖνο) is a rational, mortal animal'" (AM 11.8). Other examples are available, but the pattern for the general conditional seems apparent. Evidently, the subject of the antecedent clause is expressed by an indefinite pronoun, and though it is not clear in the Latin example, the Greek example would seem to indicate that the subject of the consequent clause having anaphoric reference to the subject of the antecedent clause is also an indefinite pronoun.[155]

Now consider the example of a singular conditional from AM 8.305 and what we might call its 'associated' general conditional, the latter being constructed on the pattern determined above. The singular conditional is If Dion is walking, Dion is moving, and the associated general conditional would be If someone is walking,

[155] It should be noted here that although 'ἐκεῖνο' would normally be classed as a demonstrative pronoun, it seems evident that in constructions such as this where it serves as a relative pronoun with anaphoric reference to an indefinite pronoun, it must be taken as an indefinite relative pronoun.

he (or that one) is moving. In his paper "Stoic Use of Logic," William H. Hay has suggested that what we have here is, in effect, a universally quantified conditional and an instantiation of it [Hay, 1969, 151n22]. If this assessment is correct, then it would evidently imply not only that the Stoics used general conditionals in place of statements using 'all',[156] as well as employing a rule of instantiation for deriving singular conditionals from general ones, but also that their logic cannot be viewed on this account as simply a logic of propositions. The suggestion expressed by Hay raises a difficulty which is communicated by Charles Kahn in the following dilemma:

> Either Stoic logic is based solely on the propositional connectives, and then it is epistemically sterile ... Or else it involves generalized conditionals and a rule of instantiation, but then it is defective as logic since we are left without any account of the quantified conditional [Kahn, 1969, p. 164].

Now we believe that what Khan has in mind here in setting out the first horn of this dilemma is a propositional logic with a classical truth-functional interpretation of the propositional connectives. It is worth noting that the classical interpretation of the connectives is only one of many possible interpretations which might be assigned to them; hence, given an appropriate interpretation, a propositional logic need not be so barren as Kahn envisages. In any case, it seems evident that Stoic logic was not a classical propositional logic, and could not, therefore, be viewed as 'epistemically sterile' on the assumption that it was; moreover, it also seems clear that the Stoics themselves did not consider their logic to be so. Thus, we would reject the first horn of the dilemma. As for the other horn, we find it difficult to agree that the Stoic system was 'defective' as logic because it lacks an account of the quantified conditional. Kahn writes that "it is time to return to a more adequate view of Stoic logic within the context of their theory of language, their epistemology, their ethical psychology, and the general theory of nature" [Kahn, 1969, p. 159]. This suggestion would seem to imply that Stoic logic be assessed on it own terms and not as an attempt at constructing a modern formal system. In putting forth his criticism, Kahn seems to be ignoring his own reproach. At any rate, it may be that one can give an account of general conditionals which justifies the inference from general conditionals to singular or particular conditionals, and do so without invoking universal quantifiers and a rule of instantiation.

A general conditional, as has been noted above, seems to be signified by a conditional sentence having an indefinite pronoun in the subject position of the antecedent and an indefinite pronoun having anaphoric reference to the subject of the antecedent. It seems evident that the general conditional would be true just in case every associated particular conditional which has either a demonstrative pronoun or a name in the subject position, is true. Thus the general conditional

[156] Mates has pointed out that "nowhere in the rather elaborate classification [of propositions] is any provision made for universal affirmative propositions, that is, for propositions beginning with 'all'" [Mates, 1953, p. 32].

may have been viewed as the conjunction of its associated particular conditionals. If the Stoics were to have allowed the inference of the conjuncts of a conjunction without an explicit rule of conjunction elimination, then this might explain why they seem to have supposed that one could infer the particular conditional from the general conditional without a rule of universal elimination.

9 FORM IN STOIC LOGIC

Over the long history of what is referred to as Stoicism, there was no doubt much unrecorded even unnoticed variability in metalogical doctrine. No doubt many distinctions which we now take for granted were 'beneath the level of specificity of their intentions'. Nor is there any reason to suppose that the Stoics surpassed twentieth-century philosophers in their awareness of the degree of indeterminacy of their adopted theoretical language, or of their prospects for success. And no doubt, their approach consisted, to some extent, in talking in order to find out what they were talking about. So there might be little point in looking for a mathematically precise account of their doctrines, even if the historical records were much more complete than they are. In fact their intellectual environment was so different from our own as to have long since rendered their semantic space largely inaccessible to us. We simply cannot reconstruct, let alone reproduce, the effects that their theoretical vocabulary could have been counted upon to have. The best we can hope for is an illusion of precise positive understanding. We can, however, take some precautions against particular misunderstandings of their project. More specifically, and for all likely purposes, most usefully, we can take account of ways in which their logical culture and methods differed from our own, and take due note of the superficiality of apparent similarities between their approach and ours. Positively, we can give more reliable shape to Stoic logical theory by using our own richer notational resources to approximate their conceptions and engage their subject matter. And we can try to triangulate their position by considering what theoretical resources lay nearly within their reach.

In this section we illustrate the difficulties by a detailed consideration of the Stoic notion of (διεζευγμένον) (disjunction) in relation to the question as to whether Stoic logic can be regarded as formal in the twentieth-century use of the word.

9.1 First blush

The superficial similarities of the indemonstrables to a set of natural deductive rules may tempt the unwary to a reconstruction in the language of twentieth-century formal systems, to define the elements of the language, the atoms, the connectives, the well-formed formulae, and then to introduce the rules for extending proofs. This would be to suppose that the Stoics viewed the connective vocabulary of the indemonstrables as having uniform logical status. A closer examination would reveal that the supposition was unwarranted. They seem to have been interested in vocabulary whose correspondents had, for them, some degree

of physical éclat. So they were more interested in disjunctions than conjunctions, and more interested in conditionals, than in negations. In fact, although there seems to have been some unclarity on this score, their focus was primariy upon relationships, conflict and consequence, for example, and only secondarily upon the vocabulary that was used to distinguish them.

A related temptation would be to suppose that because a connective would admit an introduction or an elimination rule that coincides with an indemonstrable, that must have been the connective that the Stoics had in mind. Consider first the accepted doctrine that indemonstrables [IV] and [V] rely upon the exclusive disjunction of

α	$\veebar$	β
1	0	1
1	1	0
0	1	1
0	0	0

I. M. Bocheński:

> ... out of the fourth and fifth indemonstrables which were fundamental in Stoic logic, we see that exclusive disjunction (matrix '0110') was meant. ([Bocheński, 1963], 91)

W. and M. Kneale (on Galen's remark that 'Either it is day or it is night' is equivalent to 'If it is not day it is night'):

> Possibly his expression is loose and he means to say that the disjunctive statement is equivalent to the biconditional 'It is not day, if and only if, it is night'. For the assertion of such an equivalence would indeed be in keeping with the Stoic doctrine of disjunction, provided always that the conditional is understood to convey necessary connection. ([Kneale and Kneale, 1962b], 162)

Benson Mates:

> Two basic types of disjunction were recognized by the Stoics: exclusive and inclusive. Exclusive disjunction (διεζευγμένον) was most used, and is the only type of disjunction which occurs in the five fundamental inference-schemas of Stoic propositional logic. ([Mates, 1953], 51)

Łukasiewicz:

> It is evident from the fourth syllogism that disjunction is conceived of as an exclusive 'either-or' connective. ([Łukasiewicz, 1967], 74)

Ian Mueller:

> 'The first or the second' is true if and only if exactly one of the first and the second is true. (In modern logic it is customary to use 'or' inclusively, and hence to substitute 'at least' for 'exactly' in the truth conditions for disjunction. The fourth indemonstrable argument shows that disjunction is exclusive in the Stoic system.) ([Mueller, 1978], 16)

9.2 Some evidence

All of these authors cite ancient sources for this account, among them, Cicero, Gellius, Galen, Sextus Empiricus and Diogenes Laërtius. Their accounts are the following:
Cicero:

> There are several other methods used by the logicians, which consist of propositions disjunctively connected: Either this or that is true; but this *is* true, therefore that is not. Similarly either this or that is true; but this is not, therefore that *is* true. These conclusions are valid because in a disjunctive statement not more than one [disjunct] can be true.[157]

Gellius:

> There is another form which the Greeks call διεζευγμένον ἄξιωμα and we call *disiunctum*. For example: 'Pleasure is good or evil or it is neither good nor evil.' Now all statements which are contrasted ought to be opposed to each other, and their opposites, which the Greeks call ἄντικείμενα, ought also to be opposed. Of all statements which are contrasted, one ought to be true and the rest false.[158]

Galen:

> ... the disjunctives have one member only true, whether they be composed of two simple propositions or of more than two.[159]

[157] *Topica*, 14.56-7. *Reliqui dialecticorum modi plures sunt, qui ex disiunctionibus constant: aut hoc aut illud; hoc autem; non igitur illud. Itemque: aut hoc aut illud; non autem hoc; illud igitur. Quae conclusiones idcirco ratae sunt quod in disiunctione plusuno verum esse non potest.*
[158] *Noctes Atticae*, 16.8. *Est item aliud, quod Graeci* διεζευγμένον ἄξίωμα *nos 'disiunctum' dicimus. Id huiuscemodi est: 'aut malum est voluptas aut bonum neque malum est'. Omnia autem, quae disiunguntur, pugnantia esse inter sese oportet, eorumque opposita, quae* ἄντιχείμενα *Graeci dicunt, ea quoque ipsa inter se adversa esse. Ex omnibus, quae disiunguntur, unum esse verum debet, falsa cetera.*
[159] *Inst. Log.*, 5.1. ... τῶν διεζευγμένων εν μόνον ἔξόντων ἄληθές, ἄν τ᾽ ἔχ δυοῖν ἄξίωματων ἅπλῶν ῎άν τ᾽ ἔχ πλειόνων συγχέηται. (The translation is that of Kieffer [1964].)

Sextus Empiricus:

> ... for the true disjunctive announces that one of its clauses is true, but the other or others false or false and contradictory.[160]

Diogenes Laërtius:

> A disjunction is [a proposition] conjoined by means of the disjunctive conjunction 'either' (ἤτοι) . For example, 'Either it is day or it is night.' This conjunction declares that one or the other of the propositions is false.[161]

9.3 The question of arity

The first point to attend to is that three of the five authors admit disjunctions of more than two disjuncts, while two illustrate the construction with two-member disjunctions. No great importance is attached to this by the commentators, and it is unclear whether none of them thinks it significant. There need, of course, be no great importance in the fact that the earliest and the latest of the sources quoted above define disjunction specifically with reference to two-termed disjunctions. In Diogenes' example, it may only be because the illustration is two-termed that the last comment is framed as it is. It is reasonable to surmise that neither Cicero nor Diogenes Laërtius would have precluded three-term or four-term disjunctions, and that their account would coincide with those of Gellius, Galen and Sextus Empiricus, according to which, in the three-term case, the disjunction is true if and only if exactly one of its disjuncts is true. Since none of the modern commentators explicitly addresses the issue of arity, one might have assumed that that is their view of the matter as well. Bocheński [1970, p. 91] mentions the greater generality of Stoic conjunction 'the [conjunctive] functor was defined by the truth-table '1110' [sic] as our logical product (only an indeterminate number of arguments was meant)', and one may assume that his omission of the corresponding remark about ἤτοι is an oversight. But some explain three-member disjunctions as though they nested a two-member disjunction. Commenting on the form:

Either the first or the second or the third; but not the first; and not the second; therefore the third

which Sextus attributes to Chrysippus, the Kneales [1962a, p. 167] surmise:

> Here, it seems, we must think of the words 'the second or the third' as bracketed together in the disjunctive premiss; for the conclusion

[160] *PH* 2.191. τὸ γὰρ ὑγιὲς διεζευγμένον ἐπαγγέλλεται ἓν τῶν ἔν αὐτῶ ὑγιές εἶναι, τὸ δὲ λοιπὸν ἢ τὰ λοιπὰ ψεῦδος ἢ ψευδῆ μετὰ μάχης.

[161] *DL* 7.72. διεζευγμένον δέ ἐστιν ὃ ὑπο τοῦ «ἤτοι» διαζευχτικοῦ συνδέσμου διέζευκται, οἷον «ἤτοι ἡμέρα ἐστῖ ἢ νύξ ἐστιν.» ἐπαγγέλλεται δ' ὅ σύνδεσμος οὗτος τὸ ἕτερον τῶν ἀξιωμάτων ψεῦδος εἶναι.

can then be obtained by two applications of indemonstrable 5. If this procedure is correct, the disjunction may be as long as we please, since the conclusion can always be proved by a number of applications of the same indemonstrable.

But though bracketing will have the required effect in the case of the fifth indemonstrable, its effect will be quite other in the case of the fourth; for correctly inferring from the truth of the first disjunct the falsity of the *disjunction* of the second and third will not then let us infer the falsity of the third from the truth of the second: the disjunction of the second and the third may be false because both the second and the third disjuncts are true. The conclusion must be that although we can in isolated instances treat three-term disjunctions as nestings, nevertheless if we are to give a unified account of Stoic disjunction, we may *never* understand three-term disjunctions as understanding the second and third to be implicitly bracketed. Brackets are simply not permitted. If this *arity*-free account is the correct and most general account of the Stoic notion of disjunction, several observations may be made: first that were we to symbolise such a connective it would be unambiguous and natural to do so in prefix notation as:

$$\vee_{\sigma}(\alpha_1, \ldots, \alpha_n)$$

where the subscript σ serves to make the Stoic connection explicit. For the (ἤτοι) of Greek, like the 'or' of English, is not specifically a binary connective, and the Stoic practice of representing sentences by nominals (τὸ πρότερον, τὸ δέυτερον, τὸ τρίτον, the first, the second, the third) tends to mask the distinction which, when in a philosophical set of mind, we implicitly make in English between, say, a list of three nominals composed with 'or' and a three-term disjunctive sentence. In the former case, we do not, indeed cannot think of the *or*-list of two of the names as forming a new name and that disjoined to the third. In ordinary English we are not required to think of the or-composition of three sentences in this way either. No rules of well-formedness force us to parse a three-clause sentence composed with 'or' into a two-clause sentence one of whose clauses is a disjunction. Except for the exclusivity, the Stoic construction

ἤτοι τὸ πρότερον ἢ τὸ δεύτερον ἢ τὸ τρίτον
(Either the first or the second or the third)

alternatively,

ἤτοι τὸ ᾱ ἢ τὸ β̄ ἢ τὸ γ̄

is more like the syntax of ordinary Greek than the modern symbolization

$$\alpha_1 \vee (\alpha_2 \vee \alpha_3)$$

is like the syntax of ordinary English, since to repeat the 'either' to express the inner parenthesis would be stilted and unidiomatic. Now, to be sure, we could abbreviate a modern n-term exclusive disjunction analogously by:

$$\veebar(\alpha_1, \ldots, \alpha_n)$$

since exclusive disjunction is an associative operation. But although the ambiguity is not vicious, we would normally understand such a formula as associated to the left or to the right, since $\veebar$ is a binary connective, and well-formedness requires it. That modern exclusive disjunction is a binary truth-function and that the Stoic notion had no fixed arity should not be lost sight of when comparing the two. It will serve to remind us that the truth conditions of the two constructions are not in general the same, a fact upon which none of the modern commentators seems to have remarked. Consider as example the exclusive disjunction:

$$\veebar((2+2=4),(2+3=5),(2+4=6)).$$

When it is disambiguated into, say:

$$(2+2=4) \veebar ((2+3=5) \veebar (2+4=6)),$$

it becomes evident that since the second disjunction is false (since both of its disjuncts are true) and the first disjunct is true, the whole disjunction is true in spite of (or rather *because* of) the fact that all its disjuncts are true. The Stoic disjunction:

$$\vee_\sigma((2+2=4),(2+3=5),(2+4=6))$$

is false, since more than one of its disjuncts are true. Since Stoic disjunction has no fixed arity, it would be suitable to regard it as a kind of restricted propositional quantifier, having, in prefix notation, the reading

Exactly one of the following is true:

Since exclusive disjunction is commutative and associative, a quantifier reading would be suitable for it as well. But as a simple induction will demonstrate, its quantificational rendering would be:

An odd number of the following are true:

The two kinds of disjunction will, of course, coincide on the two-clause case, but will coincide for no n-clause case for $n < 2$. A three-clause exclusive disjunction, for example, will be true if and only if either exactly one or exactly three clauses are true, as will a four-clause exclusive disjunction. A five- or six-clause exclusive disjunction will be true if and only if either exactly one or exactly three or exactly five disjuncts are true, and so on.

A valid Stoic disjunction of two terms would disjoin a sentence α with a sentence equivalent to the negation of α. A true n-term Stoic disjunction would disjoin n

finite state descriptions. As an example, imagine the formulation of a row of a truth table, that is, the effect of conjoining propositional variables or negations of propositional variables accordingly as 1's or 0's appear under them in that row. A valid Stoic disjunction in m independent variables would be equivalent to the 2^m-term disjunction of the formulations of the rows of a table displaying all possible combinations of their truth values. Particularly if, as some of the early sources suggest, the Stoic notion of disjunction was that of an intensional operation, a sentence of the form

ἤτοι τὸ πρότερον ἢ τὸ δεύτερον ἢ τὸ τρίτον ἢ τὸ τέταρτον
(Either the first or the second or the third or the fourth)

given such a technical use of 'ἢ' would assert that the four sentences bore to one another a relationship akin to the relationship of the formulations of rows of a two-variable truth table:

$$\text{ἤτοι } (p \wedge q) \text{ ἢ } (p \wedge \neg q) \text{ ἢ } (\neg p \wedge q) \text{ ἢ } (\neg p \wedge \neg q).$$

As a consequence of this, if we seriously adopt the view that the disjunction that Chrysippus had in mind in the fourth indemonstrable is the present day 0110 disjunction, then the Stoics really had at least two different kinds of disjunction represented by the same piece of notation in their logical system. And having come this far, we could admit no grounds for regarding the disjunction of the fifth indemonstrable as anything but a third sort, namely 1110 disjunction. The more plausible account would be that they had one sort of disjunction in mind, namely, the disjunction of no fixed arity which happens to resemble 0110 disjunction in the two-term case.

9.4 The consequences for their idea of form

The standard notion of form as applied to propositional argument schemata follows these lines: let F be the set of sentences of a language L and S an argument schema expressed with constants of the language L and metalogical variables of the metalanguage ranging over F. Then the *argument form* F_S associated with S is the set of arguments which can be generated from S by uniform substitution of sentences of F for metalogical variables in S. This notion of form depends upon a fixed meaning for the constants of L. In the propositional case, for example, we assume that $\neg$, $\wedge$, $\vee$, $\rightarrow$ and so on do not change their meanings as we uniformly substitute sentences for the metalogical variables flanking them. We do not account 'α ; therefore α or β ' an invalid form because from the sentence 'You may go or you may stay' it follows that you may stay. We say rather that 'You may go; therefore you may go or you may stay' is not of the form 'α; therefore α or β'. We might retreat, if pressed, to the claim that they share *grammatical* but not *logical* form. Or we might admit the argument to the form but insist that the conclusion must then be understood as a disjunction, from which 'You may stay' does not

follow. It seems certain that the Stoics never articulated a notion of argument form in these or equivalent terms. But, if we are to take into account the totality of evidence from early sources, according to which διεζευγμένον was understood in something like the quantificational reading outlined earlier, and the generally held view that they gave to the particle ἤτοι a technical meaning accordingly, then the Stoic schema [IV]:

ἤτοι τὸ πρότερον ἢ τὸ δεύτερον·
τὸ δε πρότερον·
οὐκ ἄρα τὸ δεύτερον).

Either the first or the second;
But the first;
Therefore not the second.

is insufficiently general to capture the inferential force of the connective ἢ in their technical sense. The three-term σ-disjunction is not obtainable from the two-term σ-disjunction by substitution of a two-term σ-disjunction for one of the original disjuncts. Some such schema as:

$$\begin{array}{l} \vee_\sigma(\alpha_1, \ldots, \alpha_i, \ldots, \alpha_n) \\ \alpha_j \ (1 \leq j \leq n); \\ \therefore \neg\alpha_k \ (1 \leq k \leq n)(k \neq j) \end{array}$$

would be required. So if the Stoic notion of disjunction was as general as the early commentators suggest, and we are to judge their conjectured position by standards of any rigour, then we must conclude that their understanding of the role of the fourth indemonstrable schema was something other than that of specifying a form in the substitutional sense of the word. Notably it is only the fourth indemonstrable that straightforwardly gives rise to such a problem of reinterpretation, since the other logical connections exhibited in the earlier indemonstrable schemata, viz. *if* ... *then* ... and *not both* ... *and* ... represent specifically binary connections, at least for the Stoics, and at least so far as the evidence tells us. Of these, only *not both* ... *and* ... readily admits of generalization to the n-ary case, and there is nothing in the sources to guide us in choosing between the generalization to *At most one of the following is true* and the generalization to *Not all of the following are true*, interpretations which again coincide only in the two-term case. If we suppose that they took conjunction seriously as a logical connection, perhaps the second is the more natural; for there is nothing to require the translation of the initial καί as *both* except in the two-term case. Even here a slightly dissimilar case would arise if we tried to construct the generalized schema. For in the two-term case, the connective *not both* ... *and* ... coincides in sense with the Sheffer stroke, which, since it is not an associative operation, cannot, in the n-term case, be straightforwardly thought of quantificationally. The sentence:

$$\alpha \mid (\beta \mid \gamma)$$

would mean something like:

> Either all of α, β, γ are true, or α is false.

There would, however, remain the problem that the third indemonstrable schema:

> Not both the first and the second; The first; Therefore, not the second

is insufficiently general in form to define the class of arguments which the general account of conjunction would license.

Now it is unfortunately convenient to treat Stoic logic, however fragmentary and indirect our understanding of it, as a product of the same general understanding of the issues that we ourselves are able to bring to bear. In this frame of mind, we are apt to see our scholarly task as one of rational reconstruction in the light of that general understanding. In such a frame of mind, we might well agree with Josiah Gould [1970, p. 83] that

> it is clear in each of our fragments that the author intends the adjective 'undemonstrated' to qualify what we would today call 'argument forms.'

and that the examples given are

> what we would today call substitution instances. [Gould, 1974, p. 84]

Better to ask of the fragmentary information available to us, what stage the Stoics' general understanding might have reached, allowing the relics of their doctrines a reasonable degree of tentativeness without assuming that their approach, had it only succeeded, would have been our own. This is, admittedly, a delicate task, not least because we cannot know whether we have succeeded in it. But the approach permits us, as need arises, to say 'They did not foresee this difficulty' rather than 'This view creates a difficulty on the modern understanding and must therefore not be attributed to them.' As an exercise, one might ask whether, on the evidence, the Stoics had hit upon something like our notion of logical form. If they had, well and good, but if they had not, then we ought not to suppose that all of the indemonstrables were regarded as *formally* valid or correct schemata in any single sufficiently well-defined sense of 'formal' to be of use. We would not be compelled, as we are by the contrary assumption,to assert of them that their use of ἤτοι ... ἢ ... in the fourth and fifth indemonstrables was a technical one according to which it meant what is meant by 0110 disjunction. As we have seen, unless they meant different things by ἤτοι ... ἢ ... in different contexts, 0110 disjunction is not what they meant anyway, even if there is something, in the relevant respects determinate, that they did mean. In spite of what we have said about the notion of form, there is no harm in applying the word formal to the Stoics' work. By some standards, it is not *informal* and by those standards we may therefore call it formal where that is the contrast intended; and we may therefore distinguish their uses of ἤτοι and ἢ in formal contexts from their uses of them in merely expository ones, where by this we mean just to distinguish the ceremonial from the everyday.

10 THE LINGUISTIC EVIDENCE

Was there something that ἤτοι and ἤ meant? What is the evidence? Quite apart from the remarks of the early commentators, there is the evidence provided by the Greek and Latin languages themselves. It is an urban myth that there is an exclusive sense of *or* in English, and a suburban myth that Latin lexically marked the distinction between 1110 and 0110 disjunctions by *vel* and *aut*. It is unclear when these myths first arose. We have been unable to find them in any sources earlier than the twentieth century. It is true that the Latin commentators used *aut* ... *aut* ... to convey the Stoic use of ἤτοι ... ἤ ... , but we must not place too much weight upon this. It was the best choice on grounds quite separate from the fictional one that *aut* corresponded to exclusive 'or'. We should recall that in the course of explaining the truth conditions of what he takes to be the Stoic notion of διεζευγμένον, Gellius uses *aut* in a long disjunctive antecedent clause of a conditional which is transparently intended to abbreviate a conjunction of conditionals:

> ... *si aut nihil omnium verum aut omnia plurave quam unum vera erunt, aut quae disjuncta sunt non pugnabunt, aut quae opposita eorum sunt contraria inter sese no erunt, tunc id disjunctum mendacium est* ... *Noctes Atticae* 16.8.14.
>
> *(... if none of them is true, or all or more than one are true, or the contrasted things do not conflict, or those opposed are not contrary, then it is a false disjunction ...)*

Evidently the choice of Latin vocabulary in which to cast the connective of the fourth and fifth indemonstrables was not dictated by the need to convey exclusivity formally. Had no Megarian or Stoic ever dreamt of the *fourth* indemonstrable, the most suitable Latin translation of the *fifth* indemonstrable and for the regimentation of ordinary language arguments of the corresponding grammatical cast, would nevertheless have used *aut*. There is no reason to suppose that the mere use of *aut*, independently of ancillary discussion and explanation of what it was intended to convey, would have made the *formal* correctness of the fourth indemonstrable, or of particular instances of it, transparent to Roman commentators.

Greek, like Latin, possessed no special connective by which 0110 disjunction was distinguished from 1110 disjunction. The 'logical' sense of ἤτοι ... ἤ ... and its variants was essentially that of *either* ... *or* ...; like *either* ... *or* ..., its use was indifferent as to the number of terms joined and as between exclusive and non-exclusive fillings; any additional imposition of an exclusive reading was through emphasis and intonation. In particular the use of ἤτοι as an auxiliary had no special role as an indicator of exclusivity, that particle being a compound of ἤ meaning variously *or* or *than*, and the enclitic τοι an etymological cousin of the second person singular pronoun. Its ordinary use was emphatic, akin to the use in English of *now surely* or in Welsh English of *Look you*. Galen reports ἤ as an alternative to ἤτοι in Stoic usage, although he himself uses the latter exclusively

in the context of the indemonstrables [Frede, 1974, pp. 93 4], Certainly the use of ἤτοι ... ἤ... in ordinary non-philosophical written Greek was uncommon by contrast with some philosophical writing and there is evidence that the philosophers have pressed into use a construction normally reserved as a spoken form. Thus Denniston [1954, p. 553]:

> ἤτοι ... ἤ ... (often ἤτοι ... γε ... ἤ) is common in Plato and Aristotle. It is difficult to say what degree of vividness τοι retains here. On the one hand, Thucydides confines ἤτοι, like simple τοι, to speeches ... this suggests that he felt τοι as vivid in the combination. On the other, the frequency of ἤτοι in the matter-of-fact style of Aristotle suggests that for him τοι did nothing more than emphasize the disjunction.

Βυτ ονε ουγητ νοτ το ινφερ φρομ τηις τηατ ἤτοι ... ἤ ... is more common than ἤ ... ἤ ... in Aristotle and Plato, or that either of them set aside the former for uses which prefigured the Stoic use. Neither is by any means true. In particular, Aristotle uses ἤ ... ἤ ... in the overwhelming majority of cases, and in many which would have provided excellent examples of disjunction for the Stoics:

> Πρῶτασις μὲν οὖν ἔτὶ λόγος καταφατικὸς ἢ αποφατικὸς τινὸς κατα τινος. οὗτος δὲ ἢ καθόλου ἢ ἔν μέρει ἢ ἀδιόριστος (*Prior Analytic* 24^a 16).
>
> *(A premiss then is a sentence affirming or denying something of something. This is either universal or particular or indefinite.)*
>
> ... συλλελόγισται ὅτι ἄσυμμετρος ἢ συμμετρος ἢ διαμετρος (46^b 31).
>
> *(... he has proved that the diagonal is either commensurate or incommensurate)*

and others where, if his understanding of the meaning of ἤτοι ... ἤ ... anticipated the Stoic use of it, we would expect ἤτοι ... ἤ ... :

> ... ἅπαν γὰρ ζῷον θνητὸν ἢ ὑπόπου ἢ ἄπουν ἔστι (46^b 15).
>
> *(... every mortal animal is either footed or footless)*

On the other hand, Aristotle's uses of ἤτοι ... ἤ ... , either give no evidence that he was after a distinction that anticipated the Stoics', or else provide evidence that he had no such intention. Thus, when in the course of explaining kinds of contrariety he denies that everybody must be black or white, he uses ἤτοι ... ἤ ... :

> ... ουθ γα·ρ πα΄ν ἤτοι λευκὸν ἢ μέλαν ἐστίν (*Categories* 12^a 13).
>
> *(... not everyone is either white or black)*

but the reason for denying this is that there are intermediates between white and black, namely all the other colours. It is in any case used here between predicates and not between whole sentences.

The relative scarcity of ἤ ... ἤ ... as opposed to ἤτοι ... ἤ ... in the logical setting does not of course indicate that the Stoics gave the word ἤτοι any special technical *sense* as distinct from a technical *use*. The thesis that it has a special sense is forced upon us only if we also adopt the view that their enterprise was a formal one in the substitutional sense. We have already seen that on any straightforward interpretation, it was not. A more plausible guess would be that that combination gradually gained favour in general philosophical practice and presented itself to the philosophical innovator, perhaps Theophrastus, casting about for suitably perspicuous notation as a construction already set apart for special philosophical applications. Compare the current use of *It is not the case that* .... Again, it need hardly be said that there was no special *intensional* sense of ἤτοι or ἤ or ἤτοι ... ἤ ... in ordinary Greek, the necessity or contingency of a disjunction being entirely determined by its disjuncts. But insofar as intended exclusivity can be conveyed by emphasis, the intention is conveyed more easily with more syllables than with fewer. And on that score ἤτοι is more emphatic than ἤ. Greek, like Latin and, come to that, like English, had a great variety of connectives all of which could receive translation as *or*, but whose significance in discourse is best understood by immersion in the literature in which they occur. Like *sive* ... *sive* ... of Latin, Greek had εἴτε ... εἴτε which was common as a conjunctively distributive connective in the antecedents of conditional constructions. Homeric Greek sometimes has τε in place of a second ἤ, and ἤ καί, in place of a second εἴτε. Aeschylus sometimes answers εἴτε with εἴτε καί. But Greek was in general more fluid in its use of particles than Latin. There are recorded instances of ἤ ... καί ... where ἤ ... ἤ ... would be expected; and there are idioms in which καί occurs with the sense of ἤ, as in

ἀνθρωπίνη σοφία ὀλίγου τινος ἀξία ἐστίν καί οὐδενός

Human wisdom is worth little *or* nothing

χθές καί πρωην

yesterday *or* the day before.

In general, the use of particular particles in the Greek of the last several centuries of the old era varied, not only over time, but from author to author, even from work to work, and particularly from genre to genre.[162] As a symptom of this greater fluidity, there is evident a larger freedom in the use of particles in abbreviative constructions, especially favouring the use of constructions relying upon superficial grammatical ellipsis over those requiring (or rather, as our thesis demands, *at least capable of receiving*) a truth-functional logical transformation. The use of *or* constructions in *if*-clauses to force a conjunctive reading is reinforced

[162] For a detailed authoritative discussion, see Denniston [1954].

by the availability of a non-elliptical reading for an antecedent in *and*, particularly in a language which, like Latin, is less subtly variable than Greek in its use of particles,. In Greek, for whatever reason, this tension between grammatical ellipsis and logical transformation was less insistent than in Latin, so that when the context demands it, an *if*-clause occurrence of καί more readily accedes to a conjunctively distributive reading. And one finds καί sometimes following εἴπερ and grammatically absorbed by it, producing something akin to *if even* ... , as one finds them in the opposite order having the sense of *even if* .... The *logical* particles whose English counterparts we have been taught to think of as, to extend Ryle's colourful metaphor, importantly combat-ready, lived altogether more easily in one another's company.

Any attempt to construct a useful formal system that still retains a connection with the inferential practices that have inspired it cannot but sacrifice non-logical distinctions, and the logic of the Stoics, arising as it did out of a language so fluid in its particulate usages as the Greek of their period, was not to be excepted. The abstraction of the logically essential into a simplified vocabulary was part of the task, but refining their very conception of the task and what was essential to it was all a part of the same continuing academic enterprise. As modern logic has no distinct notation for *whether* ... *or* ... , letting *if* ... *or* ... do the work, and as Roman logicians did not retain *sive* ... *sive* ... or *tum* ... *tum*, so Greek logicians shed εἴτε ... εἴτε as they did the distinction between the suppositive negating adverb μή and the absolute οὐχ (οὐς , οὐκ) since the retention of μή in negated *if*-clauses would complicate conditionalization of arguments of the form of the second indemonstrable. In any case, however much greater fluidity there may have been, and however much simplified the account of logical connection, it remains true that the role of καί in Greek was *preponderantly* aggregative or agglomerative. And the role of ἤτοι ... ἤ ... was *preponderantly* separative, as that of *either* ... *or* ... is in English. Its ordinary understanding was certainly such as to support an inference schema such as the fifth indemonstrable. But the use of ἤτοι ... ἤ ... in the fourth indemonstrable goes beyond simplification. For, as we have noted, there was, in Greek as in Latin and English, no *or* word that indicated exclusive disjunction. If the Stoics intended that the fourth indemonstrable should be understood formally in the substitutional sense, they could not have counted upon that *formal* correctness being evident from the ordinary understanding of its logical vocabulary, as they could have in the case of the first, second, third and fifth. Consider, for example, a remark of Sextus:

> ... τὸ δὲ διεζευγμένον ἓν ἔχει τῶν ἓν αὐτῷ ἀληθέ, ὢς ἐαν ἀμφότερα ᾖ ἀληθῆ ἢ ἀμφότερα ψευδῆ, ψεῦδος ἔσται τὸ ὅλον
>
> ... the disjunction has one of its clauses true, since if both are true *or* both are false, the whole will be false. (*SE AM* 8.283)

Ειδεντλψ, τηε λαστ οςςυρρενςε οφ ἤ is not to be understood 'in an exclusive sense', in spite of the exclusiveness of its disjuncts, but rather in the ordinary sense which makes the conditional in whose antecedent it occurs elliptical for, or at least

equivalent to, a conjunction of conditionals. The inequivalence of the conditional having an exclusive disjunctive antecedent (in modern notation),

$$(\alpha \veebar \beta) \rightarrow \gamma$$

and the corresponding conjunction of conditionals

$$(\alpha \rightarrow \gamma) \wedge (\beta \rightarrow \gamma)$$

has consequences elsewhere. On a purely syntactic/semantic understanding of διεζευγμένον by which it would mean just *declarative sentences joined by ἤ understood in the technical 0110 sense*, that inequivalence would make it difficult to square the fondness for the dilemma, which was ubiquitous from pre-Socratic through Hellenistic writings, with the general acceptance of the principle of conditionalisation, which was generally accepted by the Stoics. Now the ordinary application of conditionalisation as a test of validity would conjoin the premisses in the *if*-clause of a conditional, having the conclusion as *then*-clause. Presumably, in the particular application, an argument involving dilemma would yield a pair of conditionals whose validity would then be considered. The difficulty lies in the fact that the conditional

$$(\alpha \vee_{\sigma} \beta) \rightarrow \gamma$$

might be necessarily true because both α and β were necessarily true and $\alpha \vee_{\sigma} \beta$ therefore necessarily false, but both conjuncts of

$$(\alpha \rightarrow \gamma) \wedge (\beta \rightarrow \gamma)$$

false because γ was false. On those grounds alone, it is unlikely that διεζευγμένον was a simple syntactic/semantic item in the Stoic conception of logic.

10.1 Cicero's clanger

There is further evidence of this tension between the normal use of ἤτοι ... ἤ ... and the Stoic use of it in the fact that there is a greater confusion sown in the accounts of διεζευγμένον than there is in the accounts of the other connections. If the fourth indemonstrable was intended as a *formally* admitted schema in the substitutional sense, the difficulty can only have been one of understanding a new technical sense being lent to the grammatical form ἤτοι ... ἤ ... It cannot be confidently rejected that Cicero, whose *faux pas* in his *Topica* still costs him invitations, was among the victims of the confusion.

At least it must be said that a formal reading by which the fourth indemonstrable does represent a new technical use of ἤ ought to dispose us more charitably toward Cicero's curious augmentation, in *top.* 13.57 of the standard five indemonstrables. Cicero claims there, so far as we know erroneously, that there was a Stoic indemonstrable the Latin form of which would have been:

Non et hoc et illud; non hoc; illud igitur.

Not both this and that; not this; therefore that.

When the indemonstrables are understood formally, this would seem on first hearing to represent a truly resounding logical clanger. Since the indemonstrables are almost universally regarded as formal, this estimation has been the conventional view.[163] There is no independent evidence that any Stoic logician ever included this kind of argument in his list of indemonstrables. On this point, we take it that Cicero was merely wrong. But *could* there have been such a kind of argument? If we are right about what inferences could be justified by reference to the meaning of ἤτοι ... ἤ ... in Greek, and if the indemonstrables are formal, then the use of ἤτοι ... ἤ ... in the fourth indemonstrable forces an exclusive reading which did not exist in the natural language. For there, the nearest we have to an exclusive ἤτοι ... ἤ ... is the use of ἤτοι ... ἤ ... with exclusive alternatives. But then the analogous *technical* use of *not both* ... *and* ... would force a reading according to which from the falsity of one element the truth of the other would follow. Indeed, anyone whose understanding of Stoic logic was indirect and conjectural, and whose knowledge of Greek was not, might well have considered that given the eccentric character of the fourth indemonstrable, the Stoics could be expected to have a corresponding dual eccentricity of the sort embodied in Cicero's argument. It is true that the use of *not both* ... *and* ... never implies by itself that both sentences cannot be false, but neither is there a use of *or* that implies by itself that both sentences cannot be true. However, there are uses of *not both* ... *and* ... with sentences which *cannot* both be false, just as there are uses of *or* with sentences which cannot both be true. Understood as a formal theory, there is nothing more eccentric about Cicero's supplement than there is about the undoubtedly Stoic fourth indemonstrable. But suppose for the sake of argument that Cicero's addition were to be found extensively in Stoic logical writings and attributed, say, to Chrysippus. Any historian of Stoic logic finding himself unwilling to accept that indemonstrable as merely representing a technical usage, ought to feel no more willingness to accept, on those terms, the Stoics' eccentric use of 'or' in the fourth.

Make the parallel more explicit. The formalist historian claims that the Stoics used ἤτοι ... ἤ ... technically to mean *Either* ... *or* ... *and not both* ... *and* ... In ordinary Greek, its meaning comprehended the former conjunct but not the latter. If Cicero were right, there would be a second pill to swallow: that the Stoics used *Not both A and B* technically to mean the same thing. In ordinary Greek, its meaning comprehended the latter but not the former. Even on a formalist rendering, the mistake ought to seem on reflection no great logical howler. But when we consider, as we shall, the notion that the indemonstrables were not *formally* intended, we may also entertain among others, the possibility that Cicero's

[163]The Kneales are a notable exception, and offer a plausible and detailed alternative account. Bocheński has remarked (in conversation) that to ask Cicero about logic is about as sensible as to enquire of Sartre about the writings of Carnap. Calvin Normore has offered that the error may be imputed to Cicero's well known insomnia. Both may well have some bearing.

error represents at worst a merely historical error or a badly worded description, a wrong but not unreasonable reconstruction from memory of something he had read or heard from Diodotus or Philo.[164] But again, on a non-formalist construal, especially given our more less comprehensive ignorance of the teachings of minor Stoic teachers, it could well be an accurate recollection of something taught him (however erroneously) by Diodotus. It would not have been an impossible kind of argument, on a non-formalist view, for a Stoic to have noted. Consider what textbooks of this age say about *aut.*

10.2 The question of form

So we return to the question whether the Stoics regarded the indemonstrables as *formally* correct schemata in anything like the modern understanding of formal correctness. The evidence is clouded and there are many imponderables. We do not know with certainty to what extent the technical vocabulary, ὑγιής , *validus*, διεζευγμένον and so on had been freed from its etymological roots for Galen or Sextus, or with certainty what points of terminology and doctrine remained a matter of controversy into the Christian era. We do not know with what exactitude the logical vocabulary was defined by Chrysippus or others. But it would not be too pessimistic at least to lower our estimations of their capacity for logical description. As we have seen, the standard *substitutional* notion of valid form does not adequately account for the Stoic account of disjunction in inference, since it does not accommodate connectives of no fixed arity. A relaxed, *descriptive* notion of valid form might come closer to theirs. The difference is this: a *substitution* account presents a schema and (perhaps implicitly) a rule of uniform substitution, or asserts that for every pair of sentences a and b, such and such a conclusion may be inferred from such and such premisses. Arguments of the same form retain the syncategorematic vocabulary and repeat sentences in the same pattern as the repetition of metalogical variables in the schema. One might say that the substitutional account stands for an abstract syntactic description applicable to any argument of the form. What we shall call a *descriptive* account would give an explicit description, saying what belongs in each premiss, and what in the conclusion, perhaps illustrating by a schema, or an example. Of the fourth indemonstrable it might say: 'An argument of the fourth type has a *diezeugmenonic* major premiss etc.' and mention that a *diezeugmenonic* sentence is of the grammatical form:

ἤτοι τὸ $\overline{\alpha}$ ἢ τὸ $\overline{\beta}$

(not 'a *diezeugmenonic* sentence is *any* sentence of the grammatical form

ἤτοι τὸ $\overline{\alpha}$ ἢ τὸ $\overline{\beta}$'.)

The class of valid arguments of that descriptive form would be the class of arguments satisfying the description, which might but might not coincide with the

[164] Particularly bearing in mind that the account in the *Topica* is a reconstruction written, not in a library, but during a journey.

class of arguments obtained by uniform substitution in the illustrating schema. In the case of arguments with a *diezeugmenonic* major premiss, presumably the two notions would not coincide if the understood arity of disjunction were variable. If the distinction between the two notions of form were never explicitly stated, it is credible that discussions would sometimes vaguely have assumed the one and sometimes vaguely the other.

In the case of the fourth indemonstrable, a substitution account would offer the schema:

> Either α or β; but α; therefore, not β.

A rule of substitution would license any argument obtained by substituting an occurrence of some declarative sentence A for every occurrence of α and an occurrence of some declarative sentence B for every occurrence of β as an argument of the form of the schema. Alternatively, a substitution account would say:

> For every sentence α and every sentence β, from α or β, and α, not-β may be inferred.

A descriptive account of [IV] (for the general case) would be this: 'From a disjunction together with one of its disjuncts, the negation of any distinct disjunct may be inferred.' What constitutes an argument of this description depends upon what is meant by *disjunction*, but we may say that the simplest argument of this kind would of be the form:

> Either the first or the second; the first; therefore, not the second.

Now if, in addition, our notion of disjunction had as its foundation the notion of a relationship between states of affairs or situations such that exactly one of them must obtain (and only derivatively of a string of sentences alleging such states, separated by *or*), rather than *simply* any string of sentences separated by *or*, the puzzle about the technical meaning of ἤτοι ... ἢ ... would be less perplexing. Indeed there would be no puzzle. Both the fourth and fifth indemonstrables would represent valid *kinds* of argument, and the schemata presented would represent the forms of the simplest arguments of this kind. Why *Either* ... *or* ...? It is the obvious connective, since it permits the construction of a true sentence out of contradictories and, in any case, the *Either* ... *or* ... construction is the one in which these contradictory alternatives are naturally contemplated. That the fifth indemonstrable is justifiable solely on the basis of the meaning of *or* and the fourth only on the basis of the particularities of its arguments, on such an account, does not matter. It is relevant only in the presence of convincing evidence that they had in mind a substitutional notion of form. That is precisely what is lacking.

The evidence suggesting the less finely tuned notion of validity is by no means unequivocal. The clearest case of a descriptive presentation of the indemonstrables is that of Ioannes Philoponous in his Scholia to Ammonius:

> The disjunctive syllogism proceeds on the basis of complete incompatibles.[165]

But all of the early sources give, more or less, a descriptive account of the fourth and fifth indemonstrables. Cicero, who gives barely more than schemata, feels it necessary to add the comment that 'these conclusions are valid because in a disjunctive statement not more than one (disjunct) can be true', a remark more significant for having seemed necessary than for what it says. Much that is otherwise puzzling is less so on the view that their notion of validity had, at least not yet, become fixed upon a substitution account. If the notion of disjunction was the descriptive one, meaning essentially sentences in a certain relation, every disjunction which was ὑγιής or *validus* in the more etymological sense of 'proper' or 'sound' would also be ὑγιής or *validus* in the derivative sense of true, even in the further derivative sense of valid. This would explain Gellius' rejection of the premiss

> *Aut honesta sunt, quae imperat pater, aut turpia*
>
> *A father's commands are either honourable or base*

on the grounds that it is not what the Greeks call 'a sound and regular disjunction' (ὑγιής *et* νόμιμον διεζευγμένον). (Gellius *Noctes Atticae* 2.7.21) It would justify Favorinus' response to Bias' dilemma, that its major premiss (You will marry either a beautiful or an ugly woman) was not a proper disjunction (*iustum disiunctivum*), since it was not inevitable that one of the two opposites be true, which must be the case in a disjunctive proposition. (*Noctes Atticae* 5.11.8) On the substitution account, the truth or falsity of a premiss ought not to affect validity. On the descriptive account, particularly in the case of a disjunctive premiss, its falsity cannot but affect at least the question whether it is of the particular valid kind, since if it is false, it is not a genuine disjunction. There are other, similar instances, as for example Sextus' rejection of the argument:

> Wealth is either good or bad; but wealth is not bad; therefore, wealth is good

on the grounds that the first premiss does not state an exhaustive disjunction of the possibilities.[166]. These 'extra-logical considerations' [Gould, 1974, pp. 165-66] and this 'serious confusion between a disjunction and a true disjunction' [Mates, 1953, pp. 52 53] have puzzled earlier modern commentators. But if the specification of form was thought of as being given descriptively rather than substitutionally, so that the distinction between disjunction and true disjunction was not present, then the inexhaustiveness of the major premiss would debar justification by reference to [V] as the incompleteness of conflict would debar justification by reference to [IV]. And notice the restricted claim Sextus Empiricus is, on one occasion, content to allow himself about the nature of disjunction:

[165] Ammonius *in an. pr. Praefatio* xi. The translation is Mates's [1953, p. 131].

[166] SE AM 8.434

τὸ γὰρ ὑγιὲς διεζευγμένον ἐπαγγέλλεται ἓν τῶν ἐν αὐτῷ ὑγιὲς εἶναι, τὸ δὲ λοιπὸν ἢ τὰ λοιπὰ ψεῦδος ἢ ψευδῆ μετὰ μάχης (*SE PH* 8.191).

The true disjunction declares that one of its clauses is true, but the other or others false or false and contradictory.

It is a curious restriction if the distinction between a disjunction and a true disjunction is an important one.

Again, Galen's discussion of the distinction between διεζευγμένον and παραδιεζευγμέ makes it clear that he at least does not understand the claims of the fourth and fifth indemonstrables according to a substitution sense of validity. For he recognised what could be called paradisjunctive syllogism as a distinct type of syllogism, while evidently not regarding it as exhibiting a distinct form. Having given an account of Chrysippus' classification of the indemonstrables, he remarks:

In syllogisms of this sort *i.e.*, disjunctive and hypothetical, the major premisses determine the minor; for neither in the disjunctive do more than two additional premisses occur nor in the conditional, while in the case of incomplete conflict (ἐλλιπῆ μάχη) it is possible to make one additional assumption only. (*Institutio Logica* 7.1)

But when earlier he discusses the distinction between complete and incomplete conflict, a single multi-termed sentence does duty for both.

For 'Dion is walking' is one simple proposition, and also 'Dion is sitting'; and 'Dion is lying down' is one proposition, and so, too, 'He is running,' and 'He is standing still,' but out of all of them is made a disjunctive proposition, as follows: 'Dion is either walking or is sitting or is lying down or is running or is standing still'; whenever a proposition is formed in this way any one member is in incomplete conflict with another, but taken all together they are in complete conflict with one another, since it is necessary that one of them must be true and the others not. (*Institutio Logica* 5.2)

Notice that as an example of conflict, Galen's is a good one in its listing states that cannot simultaneously obtain, but a bad one in providing a list that is not, as Galen suggests, genuinely exhaustive. (Dion might be crouching or signalling.) From an inferential point of view, its inexhaustiveness is unimportant, since given the truth of the disjunction it follows, solely in consequence of the meaning of the particle ἤ , that if one of the disjuncts is false, then one of the others is true.

Ian Mueller, in his discussion of the possible non-truth-functional status of the Stoic sentential combinations says

We cannot be sure about 'or,' but I suspect that a disjunction was taken to be true only if the disjuncts were mutually exclusive and exhaustive of the alternatives. [Mueller, 1978, p. 20]

It is a more plausible conjecture that what was meant by 'disjunction' was what would be called 'true disjunction' on a substitution interpretation, that in the case of disjunction, the etymological sense of 'sound' suggesting the correct internal relationships among parts, was not absent from the understanding of ὑγιής . In an application to a very simple object, this sense would be tantamount to 'true' in the sense of 'genuine'. (For in a sufficiently simple kind of object, little in the way of internal relationships can fail before the object is not merely defective of the kind, but no longer an instance of the kind.) No disjunction that was true in the sense of 'genuine' could fail to be true in the sense of 'representative of how things are'. Now to say this is not to say that they were confused between form and content as we imagine we understand the distinction. It is to say that the boundary between the two had not yet been clearly drawn, let alone drawn where, at least in propositional logic, we now draw it. In fact, we can go further. For one need only read De Morgan's Cambridge lectures to see how far much of the philosophical establishment as recently as the nineteenth century was from grasping our present understanding of form. The resistance from Sir William Hamilton and his followers to the liberation of the idea of logical form from the shackles of Kantianism was one of the most serious academic obstacles that De Morgan had to surmount in getting his logical ideas accepted. One could argue that that struggle for liberation was one of the major contributions of nineteenth century logic. And the battle will not be certifiably won until the day when logic textbooks no longer call 1110 disjunctions exclusive on the grounds that their disjuncts are incompatible. It is therefore a serious matter to suppose that the Stoics were in full possession of the notion, particularly when the historical evidence indicates so clearly that their conception of the nature and place of logic were fundamentally different from that of twentieth century theorists.

Finally, when the matter is viewed in this light, one is tempted to speculate that it was precisely their preoccupation with the dilemma both as a form of argument and as a paradigm of moral predicament which fixed the attention of the later Peripatetics and the Stoics upon inference patterns such as [IV], as having a fundamental place in a codification of academic inferential practice. If this were true it would not be surprising that they would wish to exclude as improper those disjunctions of which both disjuncts could be true, for these are the just the instances which would defeat conditionalisation of dilemmas. Since the substitution account of validity would not rule out such disjunctions and a descriptive account would, the descriptive account seems on that score to be the more likely candidate for the Stoic conception of validity.

11 WHAT STOIC DISJUNCTION MAY HAVE BEEN

We have concentrated upon disarming the prejudices which might suggest that they introduced a technical sense of ἤτοι ... ἢ ... and *aut* ... *aut* ... or relied upon senses of those constructions already present in Greek and Latin. But we have allowed, without comment, anachronistic terminology such as 'proposition'

to creep into our account (albeit only descriptively) which might itself create the false impression that the Stoics had some such notion in common with us. It would in particular be a serious misunderstanding to suppose that the Stoics had in their notion of a *lekton* a notion corresponding to the Fregean proposition. It is precisely in the nature of the *lekton* that most recent commentators have found grounds for denying the earlier assumption that the nature of Stoic logic could be well enough understood by comparison with modern calculi. Their arguments, to which may be added the arguments given here against the application to their work of a substitutional notion of logical form, have drawn their premisses from quite a different source, namely the nature of the relationship between Stoic logic and Stoic epistemology and physics.

One account which forcefully presents Stoic logic in a non-formalist interpretation, is due to Claude Imbert [1980]. She takes as her point of departure the Stoic notion of φαντασία , usually translated as 'presentation', taken up as an alternative to Aristotle's theory of imitation and applied to the art of Alexandria. It is through an understanding of the nature of φαντασίαι and their relationship to the major premisses of the indemonstrables that we understand why Stoic logic is conceptually incomparable with modern calculi.

> The conclusion of a Stoic syllogism is inferred from other sentences which translate natural signs apprehended in presentations, and which never presuppose the existence of transcendent forms or universals ... Every logical structure rests on the possibility of translating presentations into discursive sequences, and each sequence must exhaust the scientific content latent in its presentation. Inference thus depends on a rhetorical function which maps utterances (*lekta*) on to contents of presentations (phantasia). [Imbert, 1980, pp. 187–88]

The transition from impression, which all animals have, to a presentation characteristic of human apprehension, depends upon the capacity to grasp connections among the contents of experience. Complex utterances, hypothetical, conjunctive and disjunctive, represent three ways of grasping connections. The one which concerns us here is the way which corresponds to the disjunctive proposition: the recognition of alternative exclusive possibilities. The use of the language of φαντασία in this connection is suggestive, in one respect, of Aristotle's use of the same term in *De Anima* where it designates an activity characteristic of common (as distinct from *particular*) sense. And other evidence has suggested to some commentators that the ideas of the logical connections were originally a Peripatetic innovation.[167] Finally, a full understanding of the Stoic preoccupation with what appears to the present day philosopher as a rather specialized and arcane notion of disjunction cannot neglect its connection with a theme which recurs as a *leitmotif* in one form or another throughout the history of Greek philosophy. The

[167] See [Barnes, 1985] for a discussion of the evidence suggesting that Theophrastus was one Peripatetic source.

διεζευγμένον of the Stoics is a late practical refinement of the notion of the conflict of opposites, which can be traced through Heracleitus' doctrines of the unity of opposites to Anaximander's doctrine of the generation of opposites from the undifferentiated ἄπειρον , and is to be found in the central images of the mythic cosmogony of Hesiod. For the Stoics, it was at the heart of their ethics, physics and logic, and its recognition was a necessary constituent of the rational unity to be made of the conduct of human affairs and the operations of nature. We can construct a simplified model that realizes some such conception as the one they seem to have had in mind. According to such a picture, each succeeding state of the world makes some atomic sentences true and the rest false. So each moment of time may be thought of as a function or rule which takes sentences to truth values. Coming to an understanding of the intelligent character of the world amounts to grasping the principles by which these functions are selected in their turn. And in a poetic or spiritual frame of mind, we might imagine such rules as competing for selection and thus, since they represent incompatible assignments of truth-values, we might imagine them as being in conflict. Moreover, the image comes equally to mind of nature selecting its way among these competing functions according to some rational principle. In the sphere of individual action, the notion will readily suggest itself to us that in minute part we each bear some responsibility through our choices for the successive states of the universe. The apprehension of the distinctness of these state-functions within the subdomain of alternatives presented to us would, in this admittedly fanciful reconstruction, correspond to the apprehension of διεζευγμένον . Like the rows of a truth table, or the items of a menu, they would be represented as mutually exclusive alternatives; were we to articulate them, it would most naturally take the form of a string of alternatives separated by 'or': this set of atoms true or that or the other ... .

Now this fancy is an anachronism, though the Stoics seemed to recognize something like the possibilities represented by the rows of a truth table. But if we cannot understand the Stoic use of *or* in other terms than those of twentieth century logical theory, it would be less misleading to bring the notion of διεζευγμένον into the light of such simple-hearted model-theoretic ideas than to associate it with the substitutional idea of a particular logical form.

As much as one might wish to complain to the Stoics that there are connections, such as non-exclusive alternatives, which are not provided for in their scheme, such objections are not to the present point, for what we have wanted is an explanation of the Stoic use of ἤτοι ... ἤ ... which accords with the evident fact that their technical *use* does not constitute a technical *meaning*. Understood as representing the most succinct way in which we reflect in utterance the connection between exclusive alternatives viewed as such, the use is surely unobjectionable. The fact that we, and for that matter, the Greeks, had other less succinct ways of reflecting such connections and as well used the same connective for non-exclusive alternatives is neither here nor there. In any case, when such alternatives confront us, a complete analysis of the possibilities will always yield exclusive alternands, namely, those corresponding to the three 1's of the truth table of $\vee$. If α and β

present themselves to us as non-exclusive alternatives, our choice, when fully and analytically apprehended, is seen to be among the three exclusive alternatives: pursuing both α and β, pursuing α but not β, and pursuing β but not α. Though the origins were different and the motives, the method need not be thought entirely unlike Boole's. For he too took exclusivity, even the same arity-free idea of exclusivity, to be centrally important to his representation, but the exclusivity was constructed out of a non-exclusive disjunctive use of *or*.

12 STOIC DISJUNCTION AS A HYPER-RELATION

It is important to bear in mind that before the nineteenth century, logical theorists, though they spoke of form (as distinct from content) thought of logical connection in relational rather than formal terms. The character and status of the items between which the relations were thought to obtain varied through the history of the subject, but the relational character can be said to have persisted without challenge at least until George Boole's temporal semantics for the connectives, and in some branches of logic, notably Idealist logic, to have persisted as explicit doctrine well into the twentieth century. The 1929 symposium on negation (Mabbott *et al.* 1929) might be said to mark its final departure from academic philosophical logic. We can, however, capture the character of Stoic disjunction in the recent language of coherence measures.

Let Σ be a set of sentences. Then the coherence level of $\Sigma, \lambda(\Sigma) = \min \xi : \exists \pi \in \Pi_\xi(\Sigma) : \forall c \in \pi, c \nvdash \perp$, if that limit exists; else $\lambda(\Sigma) = \infty$.

Thus, for example, $\lambda(\{p, \neg p\}) = 2; \lambda(\{p \wedge q, p \wedge \neg q, \neg p \wedge q, \neg p \wedge \neg q\}) = 4; \lambda(\{\perp\}) = \infty$ and so on.

Let Σ be a set of sentences. Then the coherence dilution of $\Sigma, \delta(\Sigma) = \min \xi : \exists \Delta \subseteq \Sigma \, |\Delta| = \xi$, and $\Delta \vdash \perp$, if such a subset exists; else $\delta(\Sigma) = \infty$.

Thus, for example, $\delta(\{p \wedge \neg p\} = \delta(\{\perp\} = 1; \delta(\{p \wedge q, p \wedge \neg q, \neg p \wedge q, \neg p \wedge \neg q\}) = 2; \delta(\{\alpha, \alpha \to \beta, \beta \to \gamma, \gamma \to \zeta, \neg\zeta\}) = 5; \delta(\{p\}) = \infty$ and so on.

Again, maximum dilution is illustrated by the modest believer, whose only mistaken belief is that at least one of his beliefs is false. No proper subset of his beliefs is inconsistent, yet the set as a whole is.

Then a set-representation of an n-term Stoic disjunction can be given as the set $\Sigma = \{\sigma_1, \ldots, \sigma_n\}$ of its disjunctions where Σ satisfies:

$$\lambda(\Sigma) = \delta(\neg[\Sigma]) = |\Sigma| = n.$$

The weaker Stoic notion of paradisjunction can be given a set-representation that weakens the dilution requirement to

$$\delta(\neg[\Sigma]) \leqslant |\Sigma|.$$

Now it would not have taken the Stoics beyond the resources available to them to have introduced a measure on paradisjunctions representing the maximum number of disjuncts that could be true. Such a measure would indirectly have yielded a measure of dilution of incoherence capable of independent study. As an example, consider the set

$$\Sigma = \{p \to q, q \to p, p \wedge q\}$$
$$\lambda(\Sigma) = 1;$$
$$\delta(\neg[\Sigma]) = 2.$$

Σ is a set representative of a paradisjunction: one of its elements must be true but all of them can be. On the other hand, the conflict among the elements of $\neg[\Sigma]$ is less diffuse than among the negated disjuncts of a Stoic disjunction. If the ideal is the absence of conflict, evidently more dilute conflict is better than less. Thus the notion that inference should preserve dilution is in the logical spirit of Stoicism. We conclude with the observation that a system of inference that (a) permitted only dilution-preserving inferences, and (b) took those inferences $\Sigma \therefore \alpha$ as correct for which the dilution of $\Sigma \cup \{\alpha\}$ was greater than the dilution of $\Sigma \cup \{\neg\alpha\}$ would satisfy connexivist constraints on inference corresponding to the theses of Aristotle and Boethius discussed earlier (page 482). Such a system was nearly within reach of the Stoics, and would constitute a natural extension of their logical theory.

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ARABIC LOGIC

Tony Street

INTRODUCTORY COMMENTS

This chapter limits itself to logical writings in the peripatetic tradition produced in Arabic between 750 and 1350. It is intended to provide a tentative framework for analysis of these logical works by describing aspects of the historical and intellectual context within which they were written. This is done by testing the model put forward in Rescher's *Development of Arabic Logic* against accounts of the syllogistic in a number of authors.

By about 900, the Organon had been translated into Arabic, and was subject to intensive study. We have texts from that time which come in particular from the Baghdad school of philosophy, a school which at its best proceeded by close textual analysis of the Aristotelian corpus. The school's most famous logician was Alfarabi (d. 950), who wrote a number of introductory treatises on logic as well as commentaries on the books of the Organon.

Within fifty years of Alfarabi's death, another logical tradition had crystallized, finding its most influential statement in the writings of Avicenna (d. 1037). Although Avicenna revered Alfarabi as a philosophical predecessor second only to Aristotle, his syllogistic system differed from Alfarabi's on two major structural points. It is in consequence relatively straightforward to assign subsequent logicians to one or other tradition. Avicenna differed from Alfarabi in his approach to the Aristotelian text, and assumed even less than Alfarabi had that it contained a straightforward exposition of a coherent system merely awaiting sympathetic interpretation to become clear. Due perhaps to the flexibility of the larger philosophical framework with which it was associated, a framework which proved adaptable to the needs of Islamic philosophical theology, Avicenna's logic came in time to be the dominant system against which later logicians set forward their own systems as alternatives or modifications.

The success and rapid spread of Avicenna's philosophy and logic elicited a strong reaction from establishment theology, whose very intellectual vitality was perceived to be threatened. The clearest and most influential response to Avicenna was given about half a century after his death by Abū-Ḥāmid al-Ġazālī (d. 1111). A case had been made at least as early as Alfarabi that logic could help Muslim scholars in juristic and theological reasoning. Ġazālī accepted these arguments and went so far as to preface his juridical summa, *The distillation of the principles of jurisprudence*, with a short treatise on logic. Logic continued to face pious opposition after Ġazālī, but even scholars who were opposed to Greek philosophy in its various manifestations were agreed that, taken as a formal system, logic was unobjectionable.

Handbook of the History of Logic. Volume 1
Dov M. Gabbay and John Woods (Editors)

Logic after Ġazālī was regularly studied by Muslim scholars for use in theology and jurisprudence. It also continued to be studied by Muslim scholars who were interested in the deeper formal and philosophical questions Avicenna had raised. Ġazālī extended the number of students of logic by inspiring people to study basic logic as a kind of exercise in critical thinking, but had no impact on the vitality or nature of the existing logical tradition. It would seem that the most interesting discussions in Arabic on logic were conducted from the late eleventh century on. Scholarly studies particularly in Spain carried on in the tradition of the Baghdad school, culminating in the work of Averroes (d. 1198). The Averroist project to make literal and globally consistent sense of the Aristotelian texts was an extension of the methods of the Baghdad school, even though Averroes saw many flaws in the work of his predecessors. Elsewhere, however, Aristotle had ceased to figure as a major coordinate to which logicians referred in constructing their systems. This role was rather filled by Avicenna's system, which was modified, extended, and in part rejected.

By 1350, it is clear that the Avicennan tradition predominated over the Farabian and Averroist traditions throughout the Islamic world, and the systematic problems in Avicenna's formal syllogistic were taken as being settled. By this time, texts were being produced which continued to figure in the syllabus of the *madrasa* down until recent times. For centuries after, advanced logical investigations continued in the Islamic world, but the *madrasa* texts were always the way that Muslim scholars had come to be able to conduct those investigations.

A note on conventions

All dates are given in common era, except occasionally in the bibliography. I have denied diacritical machinery to all names of dynasties and places, and also to the scholars I refer to by names that are either simplified, or derived from the medieval Latin tradition (among others, Alfarabi, Avicenna and Averroes). All other names are given on their first occurrence within a section in sufficient fullness to identify the scholar in question, and afterwards in a shortened form; so, for example, Faḫraddīn ar-Rāzī becomes after first reference to him merely Rāzī. Due to the vagaries of my grasp of BIBTEX, all names are given in the bibliography without their final definite article.

In the translations, I have tended to standardize the names of Muslim scholars. Book titles are given in translation in the text, and in Arabic in the bibliography. Many of the texts presented here in translation have been translated before. When I refer only to a translated version of the text, or to the translated version before the Arabic original, I have followed that translated version verbatim. When I refer to the Arabic original before the translated version of a text, I have relied on the translated version, but departed from it in some way, *even if only slightly*. At those times that I have emended an Arabic text as I have translated it, I mark the point that I have modified it with an asterisk.

Two frequent intrusions in the chapter may prove annoying. The first is constant cross-referencing within the text—I hope this makes what is essentially a narrative somewhat more chapter-like. The second is the phrase 'at least on my reading'. The phrase is intended to be disarming. A number of the texts used here are only in manuscript, or

are part of a larger opus demanding extended treatment, and my interpretation of them is tentative.

I have instituted one semi-technical convention. When I refer to a logician as 'Avicennan', I mean that he has put forward a system including the three elements I identify on page 553 below. By contrast, when I refer to a logician as 'post-Avicennan', I just mean that he lived after Avicenna had died. By this convention, all Avicennans are post-Avicennan, but the reverse is not the case.

Two last points regarding conventions. Although I use 'Spain' to refer to what used to be called al-Andalus, I refer to the logicians working there as Andalusian. The Index gives occurrences of ancient and medieval logicians named in the chapter, and is intended to serve as a point of reference for a set of names which for the Anglophone can be fairly forgettable. Definite articles, even medial ones, are ignored in ordering index entries.

1 LIMITS, METHODS AND SOURCES FOR THE CHAPTER

In this chapter, I present a historical sketch of logical writing in Arabic. A number of works have dealt with the broad topic of Arabic logic in the recent past [Arnaldez, 1960–; Black, 1998; Gutas, 1993; Inati, 1996; Madkour, 1969²; Rescher, 1963c; ?; Rescher, 1967a], and though all have given at least a rapid historical outline of the subject, it seems to me still to invite more extended treatment. This is so above all because understanding the particular logical tradition within and against which a given logician writes determines absolutely our ability to go on to appreciate and assess the nature and quality of the work presented. When the output of a logician writing in Arabic seems incommensurable with the work of a contemporary logician writing in Latin, it is nearly always as a result of the different configurations of their respective logical traditions. In light of this, I have tried to pull the existing secondary literature together to make clear the delineation of logical traditions in the Islamic world. This has led me in sections 2, 3.1, 3.2, 3.3 and 4 to reparade material which appears as such (in more or less extended form) in earlier writers, whereas in sections 3.4, 3.5, 3.6 and 5, I deploy material in ways less frequently encountered. The upshot of all this is that I end up offering, with somewhat more detail of technical aspects and traditional affiliation, the account first given by Ibn-Ḫaldūn (translated at page 580 below). In the remainder of this section, I try to justify the limits, methods and sources I have used in writing the chapter.

Limits 'Arabic logic' is in four respects imprecise as a title for this chapter. Firstly, because the logical works studied here consist of those written between 750 and 1350 (and I concentrate only on those written between 900 and 1300), many Arabic logical works are left to one side; these include a number of modern works contributing directly to the post-Fregean logical enterprise. Secondly, many of the scholars studied here were Muslim. They contributed to a tradition of writing which was made possible in the last analysis by the Islamic conquests, a tradition which was carried forward in both Arabic and Persian. In light of considerations like these, some would argue that a title like 'Islamic logic' would be more appropriate. Thirdly, although it is clear that Stoic logic filtered through

to scholars working in Islamic law and theology, there is no tradition of translating Stoic works and commenting on them comparable with that devoted to Peripatetic works, and, except for brief mention at page 556 below, I have left them out of consideration. Lastly, for reasons I explain below in *Sources*, I confine myself to aspects of the syllogistic. A more precise though less attractive title for this chapter, given its restrictions, would be 'Peripatetic logical writings in Arabic produced in the realms of Islam between 750 and 1350, with special reference to the syllogistic.'

The study of medieval Arabic logic is not yet in a state remotely comparable with the cognate study of medieval Western logic. The vast majority of the relevant texts are still in manuscript, and a fair number of those that are available in print have not been edited adequately. Important preliminary studies have been carried out, but the sum of these is still a long way short of desiderata set down as long ago as 1965 [Mahdi, 1965]. This state of affairs has two consequences for the writer of handbook entries. The first is that, at best, only a sketchy and often conjectural outline of the history of the subject can be given. The second consequence, more philosophically disappointing, is that although we can point to various aspects of Arabic logical writings that are of philosophical interest, we are not in a position to say that a given topic or set of topics as treated by logicians writing in Arabic is more interesting or original than others.

Having noted the limits I have imposed on myself, or had imposed upon me by the state of the field and the reach of my competence, I should go on immediately to dispel the impression that I stop at 1350 because it is the end of original logical writing in Arabic, as it is sometimes said to be. I am perfectly prepared to entertain the possibility that logical production went through a radical decline in quality at this time. But it cannot simply be assumed to be the case because the preferred genre of logical composition came to be the commentary, or because (as one writer on Arabic logic put it): "Toute évolution sociale monte et descend, progresse et tombe en décadence" [Madkour, 1969[2], page 240]. We simply have to read these texts. Sadly, I cannot claim to have done so—my knowledge of Arabic logical texts written after 1350 is even sketchier than my knowledge of the texts written before 1350.

Still, plausible reasons can be given for stopping at 1350. By that time, it is clear that even in Spain, as well as in North Africa and Egypt, a system of logic which had descended from the Avicennan tradition had come to predominate. By that time, texts had been composed which continued to be commonly taught in the *madrasa* down until recent years. Further, by this time Quṭbaddīn ar-Rāzī at-Taḥtānī (d. 1365) had written his book purporting to settle conflicts between two philosophical traditions in Persia and Transoxiana—his contribution has been claimed to mark the end of a significant period in the history of Arabic logic [Rescher, 1964, page 81]. Finally, 1350 is sufficiently recent to include Ibn-Taymiyya (d. 1328), a great and very quotable hater of logic.

Aside from these limitations, this chapter is confined and configured by my own prejudices. I think that disproportionate scholarly effort has gone to the study of the Baghdad school at the expense of post-Avicennan logic. In consequence, I have dwelt rather more on the logicians of Persia and Transoxiana in the thirteenth and fourteenth centuries than most historians of Arabic logic do. I also say rather less about Averroes than most historians do. Western medievalists have tended to be more interested in the logicians and

philosophers of the Islamic world who were translated into Latin, such as Averroes. While there is nothing wrong with studying Averroes and in recognising more fully the extent of his contribution to Western logic, there is a danger in presenting a distorted picture of the relative range, intensity and quality of logical studies throughout the Islamic world. There is simply no doubt that the time and place of major output was in the east, in Persia, Khurasan and Transoxiana, in the twelfth and thirteenth centuries—it may well also be the time and place that most of the interesting insights and doctrines were formulated.

Methods Although a number of attempts have been made in the past to sketch an outline of the history of Arabic logic, two in particular exemplify how widely approaches to the task have differed. The first approach assumes that logic is somehow separate from other philosophical doctrines, and is not fitted out to serve given metaphysical purposes. Madkour writes of the logicians writing in Arabic that

> Il serait fastidieux de suivre ces logiciens dans leurs divers exposés; d'ailleurs, il n'y aurait pas grand intérêt à mettre un tel projet à execution; car si les philosophes musulmans diffèrent entre eux en ce qui concerne certains problèmes physiques ou métaphysiques, ils sont tous d'accord sur les grandes questions logiques. [Madkour, 1969^2, page 9]

This means that it only remains to find a paradigm author to give the systematic outline of Arabic logical doctrine. Madkour would have preferred if that author could have been Alfarabi, but given the fragmentary nature of his surviving writings, it has to be Avicenna

> ...qui représente à juste titre l'école arabe et offre une doctrine complète sur laquelle on peut se prononcer aisément. Ses écrits, que nous avons en main, présentent les différentes manières dont les philosophes musulmans ont traité la logique aristotélicienne; ils en contiennent des abrégés très précis et des commentaires assez étendus. Ibn Sīnā est surtout le philosophe de langue arabe, et sa logique est encore aujourd'hui enseignée dans les écoles musulmanes... [Madkour, 1969^2, pages 9–10]

The alternative approach assumes that logical differences map precisely onto the differences among philosophical schools; to write a logical history of the realms of Islam, one need only write their more general philosophical history. Thus we find Rescher in [Rescher, 1964] tracing the filiations of the philosophical schools, and placing logicians and their writings onto the genealogy produced—the logicians inherit as proponents or opponents of their logic the same philosophers who promote or oppose their metaphysics.

As I hope will become clear in the course of this chapter, Rescher's approach accounts far better than Madkour's for the logical texts produced by the various authors; in fact, I have adopted Rescher's model as a heuristic device to work with in writing this chapter. Three changes seem to me to be in order. Firstly, the notion of 'school' as a way of collecting groups of logicians is fine, so long as it is recognized that these 'schools', particularly later on, may have had no fixed point of convention, no set curriculum, and no doctrinal unanimity. They tend to be united only by pedagogical lineage, itself often tenuous. The Baghdad school is closest to being a school in our usual sense of the word, but

it is unclear that its teaching practice was common later on and further east. Secondly, the periodization of Rescher's study (800 to 900, 900 to 1000, and so on) does not even have the doubtful merit of following the temporal boundaries of ruling dynasties. Abbasid imperial policy began the earliest sustained efforts to translate logical works [Gutas, 1998], but by the end of the second Abbasid century (950), it was debate about that translated logical problematic conducted among logicians writing in Baghdad and elsewhere that determined the further fortunes of logic in the Islamic world. I have tried to structure this chapter around moments of particularly intense logical activity in the realms of Islam.

Thirdly, and mainly due to his conviction that divisions among logicians follow divisions among philosophical schools more broadly conceived, Rescher tended to write his history from biobibliographical accounts of Islamic philosophical history. I have supplemented his account with different sources. I try mainly to use the references logicians made in their writings to other logicians. Post-Aristotelian logicians generally speaking are given to referring to other logicians, and the logicians writing in Arabic are no exception. These references serve to modify aspects of Rescher's account. It has to be stressed, however, that the references I have gathered, taken together, are not sufficient to do anything more constructive than modify an existing account. But I believe that there are sufficiently many such references ultimately to produce a far sounder history than we have at present.

Sources A few words are in order concerning the sources. Ideally, of course, one would give oneself up, like a corpse to the body-washers, to the vast body of logical treatises and their varying formulations of different logical doctrines, only regaining critical consciousness to note the past and contemporary logicians to whom they refer. As a paltry beginning to that ideal task, I have decided to examine aspects of the syllogistic. I have chosen the syllogistic due to the concentration of existing scholarship; I do not think that any other single area in Arabic logic has been studied through so wide a range of writers, or so successfully, from both a technical and a philological point of view. It may not be the most interesting of the achievements of the logicians writing in Arabic, but it is central to their systems, and each one of them treats it.

When I talk about the concentration of scholarship on the syllogistic, I mean especially the editions we have of the earliest Arabic version of the logic [Dānišpažūh, 1978], and the achieved translation of the *Prior Analytics* [Badawī, 1948/52]; the careful study of the *Prior-Analytics* complex, its technical terms, and the use made of the *Prior Analytics* by Alfarabi [Lameer, 1994]; the translations of texts by Avicenna which exposit the central features of his theory of the syllogistic [Goichon, 1951; Inati, 1981]; the analysis of Averroes' changing treatments of the modal syllogistic [Elamrani-Jamal, 1995], and the editions and studies of his major extant texts on the subject [Averroes, 1983a; Averroes, 1983b]; and the description of, and semantics for, the syllogistic system common to most writers after the end of the thirteenth century due to Rescher and vander Nat [Rescher and vander Nat, 1974].

In testing and modifying Rescher's account, I have looked especially at the logicians who are writing at historically significant moments, moments when new directions are claimed to be either beginning or ending. For this reason, I have looked especially at Abū-

l-Barakāt al-Bağdādī (d. 1165), Fahraddīn ar-Rāzī (d. 1210), Nasīraddīn aṭ-Ṭūsī (d. 1272), and Quṭbaddīn ar-Rāzī at-Taḥtānī (d. 1365). Throughout, I have written with an eye to explaining how the widely used treatise by Nağmaddīn al-Kātibī (d. 1276 or 1294), *Logic for Šamsaddīn* [Kātibī, 1854], came to acquire the form and content it has. This has helped to shape the narrative of this chapter and the way I read all the logicians considered. Inevitably, people will object to a number of the sources I have not used (especially works by Alfarabi and Averroes), the writers I have neglected, and the range of logical disciplines in the tradition which I have ignored. For those who want to know in advance what is not treated in this chapter, section 6 provides a summary of its main points, and indicates some of the lines of research not broached here. I hope the bibliographical notes in the third appendix help direct people disappointed in this way to further relevant material or, at least, to lists of such material.

2 THE TRANSLATION OF THE ORGANON

For more than two centuries throughout the period of the Graeco-Arabic translation movement the Organon was translated and revised numerous times by succeeding generations of scholars in accordance with their philosophical and philological needs. By the time a settled version had been achieved, a number of other commentaries by Greek writers of late antiquity had also been translated to help make sense of it. In this section of the chapter, the history of the translators and their cumulative efforts in rendering the Organon and related works into Arabic will be presented in four stages: the Syriac translations, the earliest Arabic translations, the translations produced by the translation circles headed by al-Kindī and by Ḥunayn, and the later revisions. It must be stressed, however, that the process was not in fact a linear progression to a final and complete version of the Organon, but was much more fluid, being carried out by a number of translators with varying levels of technical skill and differing philosophical priorities. Some of the early translations found a permanent place among the writings Arabic logicians read as a matter of course in centuries to come, while other translations were subject to revision after revision.

The Syriac translations A great many of the early translators of logical works were Christians belonging to one or other of the Syrian Churches. This is because these churches had for many centuries taught some logic from the Organon. That said, they taught even less than had been taught in the Alexandrian curricula of the sixth century, and limited themselves to the *Categories*, *On Interpretation*, and the assertoric syllogistic in the *Prior Analytics* (which is to say, to the end of the seventh book of the first part). The reason Alfarabi gave for their stopping at that point was this:

> The Christians stopped instruction in Rome, but it carried on in Alexandria, until the Christian king looked into it. The bishops gathered and took counsel on what part of philosophical instruction should be left, and what should be stopped; they came to the opinion that the logical texts up to the end of the assertoric figures should be taught, but not what comes after that. They came to this opinion because they thought that the later parts were injurious to

> Christianity, while the earlier parts that they permitted contained things that would help towards the promotion of their religion. So this amount remained in public instruction; the rest was looked into privately, until Islam came, a long time after... ([Ibn-Abī-Uṣaybi'a, 1882, page 135.8–14]; cf. [Lameer, 1994, page xviii], and especially [Gutas, 1999, pages 163 & 164])

It has been suggested that it was the same reason behind the narrow compass of the old logic in the West [Pines, 1996]. Whatever the historical merits of this argument, it must be borne in mind that the Syriac tradition always had the early parts of the *Prior Analytics*, and that provided textual authority against the kind of speculation about conversion that a scholar like Abelard was free to pursue. The old logic of the Syriac churches had potentialities different from the old logic of the West. It is even more important to note that the story of the Christian interdiction on the study of modal logic and beyond bears the marks of polemical tensions that pre-date Alfarabi. Once these are cleared aside, we can discover that Syriac logic was a constriction on the Alexandrian logical curriculum; when logical studies were widened in Baghdad at the turn of the tenth century to cover the whole of the Organon, it was a major structural change indeed ([Gutas, 1999, page 186]).

By the sixth century, a decline in the knowledge of Greek among Syriac Christians meant that any part of the Organon and related logical writings that were to be studied had to be translated into Syriac. One of the greatest translators in the resulting translation movement was Sergius of Reš'ayna ('the Boethius of the Syriac tradition'; d. 536); he was followed by important commentators such as Severus Sebokht (d. c. 666) and Athanasius of Balad (d. 686) [Brock, 1993]. These men were among the first to attempt the difficult task of translating the Organon into a Semitic language. Subsequent translations of the Organon into Arabic almost always went by way of a prior Syriac translation.

The earliest Arabic translations The early Caliphate and the Umayyads neither discouraged nor advanced logical studies in their newly conquered territories. But in 750, the Abbasid dynasty came to power, and by 756 had founded the empire's new capital, Baghdad. For various reasons, translation activity served to further Abbasid imperial propaganda and was therefore encouraged. The translation movement began by drawing on the living Syriac pedagogical tradition in philosophy and, to a lesser extent, the Sassanian tradition, but it soon came overwhelmingly to surpass both these traditions in terms of range and quality of the translations it produced and the energies it devoted to the teaching and study of those works. In fact, the translations created and met cultural needs in such a way that they came to be sustained alongside and ultimately without official Abbasid support [Gutas, 1998].

For all the importance of the Syriac Christians in the movement, it may well be that it was a Zoroastrian convert to Islam who produced the first work on logic written in Arabic. Ibn-al-Muqaffa', who was executed in 757 for political reasons, translated an epitome of the *Categories*, *On Interpretation* and the first part of the *Prior Analytics*, and prefaced the whole work with a short introduction on the value of philosophy and a brief treatment of the predicables [Dānišpažūh, 1978, pages 1–93]. Ibn-al-Muqaffa' was not the

only scholar to translate this short textbook ([Dānišpažūh, 1978, page 93]; cf. [Lameer, 1994, pages 11–12]), but we have no clear idea about who wrote the original. Whatever its origin, however, in covering the categories, ending with the assertoric syllogistic, but stretching to include some coverage of the *Posterior Analytics*, the work belongs to the pre-Syriac Alexandrian tradition ([Gutas, 1999, pages 184 & 185]). Soon after this textbook became available, a translation of the *Topics* was commissioned by the Caliph al-Mahdī (d. 785), and the commission was carried out by the Nestorian Patriarch Timothy I and Abū-Nūḥ, a Christian secretary, working with a Syriac intermediary text [Gutas, 1998, page 61]. The way the *Topics* was translated became typical. Among other scholars working at these early translations were Theophilus of Edessa (d. 785), who worked as the Caliph's court astrologer, and Theodore abū-Qurra (d. 826), the Melkite Bishop of Ḥarrān. The Christian tradition of which these scholars formed a part continued into the later period, and included Ḥunayn ibn-Isḥāq (d. 873), Isḥāq ibn-Ḥunayn (d. 910), and Abū-Bišr Mattā (d. 940). ¿From the ninth century on, many scholars in the Christian tradition had come to write in Arabic by preference, and not merely in fulfillment of a translation contract; such scholars include Yaḥyā ibn-'Adī (d. 974) and 'Abdallāh ibn-aṭ-Ṭayyib (d. 1043) [Brock, 1993, page 9].

The translation circles How the translators were employed is not entirely clear. A few, like Theophilus of Edessa, were employed directly by the Caliph. Most seem to have enjoyed the patronage of courtiers and wealthy patrons, or even just worked on projects by contract. They formed groups, perhaps only loosely affiliated. By around the 840s, the first famous circle of translators had formed around the celebrated 'philosopher of the Arabs', al-Kindī (d. 873) and his student, as-Saraḫsī (d. 899). Although Kindī and Saraḫsī were Muslims, most of their colleagues were Christians. The circle seems to have been interested in texts we now associate with neoplatonism [Zimmermann, 1986], and one member of the group, Ibn-Nā'ima (fl. c. 830), translated the *Sophistical Fallacies* as well as the Plotinian *Theology of Aristotle*. This did not exclude interest in Aristotle, of course, and Kindī himself wrote an outline of the Organon [Kindī, 1950; Rescher, 1963b].

Due to recent important work by Endress [END,], we have a rich understanding of the activities and goals of the Kindī-circle. It was probably fairly loosely constituted, and the fact that Kindī revised a translation by Ibn-al-Bihrīz (d. c. 860), the Bishop of Mosul (who also produced the earliest surviving compendium of logical terms [Dānišpažūh, 1978, pages 97–126]), may not indicate that Ibn-al-Bihrīz was actually part of the circle. In fact, Ibn-al-Bihrīz enjoyed the patronage of the Buḫtīšū' family, one of the wealthy families supporting translation work, and we know that the Buḫtīšū' family particularly supported the circle working around Ḥunayn ibn-Isḥāq (d. 873). Ḥunayn was a Nestorian Christian, and the most famous of the translators. The dates at which he and Kindī were working must have overlapped. He, his son Isḥāq ibn-Ḥunayn (d. 910), and his pupils translated nearly the whole of the Organon. Ḥunayn and his pupils drew on earlier Syriac translations when available, and on the Greek commentaries of late antiquity. Ḥunayn's primary interests were medical, and he held Galen in high regard; in consequence, he translated many of Galen's logical works along with the medical works, including the *Institutio Logica*, a treatise on the number of syllogisms, fragments dealing

with *On Interpretation*, and fragments of *On Demonstration* [Bergsträsser, 1925, pages 47–48]. Galen may well have dominated logical studies in Baghdad for one or two generations after Ḥunayn, but a reaction ultimately set in, and neither Alfarabi and Avicenna acknowledged any debt to Galen's logical works ([Zimmermann, 1981, page lxxxi]; this does not mean that they did not share some of his ideas; see [Shehaby, 1973b, pages 5 & 6] and [Lameer, 1994, pages 10, 47]).

The period of revision The activities of the group of scholars who had worked around Ḥunayn and his son carried on after Isḥāq's death, and merged seamlessly with the activities of a new group of scholars in Baghdad who worked with the texts more critically and philosophically. Claiming to represent the true pedagogical lineage of the Alexandrian school, a certain Abū-Yaḥyā al-Marwazī taught the scholars who taught the great luminaries in the Baghdad school of the early tenth century: Abū-Bišr Mattā ibn-Yūnus (d. 940), and Alfarabi (d. 950).

Alfarabi claimed himself to be an Aristotelian and, in making this claim, he meant to be taken as a true Peripatetic, doing something rather more rigorous than his predecessors. This is not to say that Alfarabi did not take a great deal from the Syriac tradition. Among other things, the sources for the logic chapter of his *Enumeration of the sciences* are mediated through that tradition [Gutas, 1983, page 255 ff.]. But Alfarabi came to respone to two urges in his work as a logician. Firstly, as a Muslim, he felt a desire to explain the whole enterprise of logic in terms that exponents of the other Islamic disciplines would understand. Secondly, he became increasingly aware that the introductory Syriac treatises obstructed understanding of the original Aristotelian texts, and he wanted to rectify this.

What matters for present purposes is the second of these pressures. Alfarabi began a critical examination of the Aristotelian texts, often setting aside the prevailing interpretation. This must have galled some of his colleagues, and they seem to have cited him less than one would expect given the quality of his work ([Zimmermann, 1981, page cxi & note 1]; but see also [Marmura, 1983, page 763b]). Later historians of philosophy referred to Alfarabi as a leading philosopher, a 'head of school', distinguished by his critical attitude towards and interaction with the translated texts. His colleague Abū-Bišr, and one of his students, Yaḥyā ibn-'Adī (d. 974), were similarly distinguished, as were Yaḥyā ibn-'Adī's students, Abū-Sulaymān as-Siğistānī (d. c. 990), Ibn-Zur'a (d. 1008), and Ibn-Suwār (d. 1017).

With Ibn-Suwār, we may say that the translation and refinement of the Organon was complete. It is his copy of the Organon, with extensive marginal and interlinear notes, which was copied as the manuscript now in the Bibliothèque Nationale (codex ar. 2346), and which serves as the basis of our contemporary edition ([Badawī, 1948/52]; cf. now [Jabre, 1999], which further uses MS Istanbul Ahmet III 3362). Ibn-Suwār is a product of the Baghdad approach to the Organon, and his version of it bears elegant testimony to the intimacy of the connexion between translation and interpretation, philology and philosophy. For a long time, the interlinear notes and marginalia of codex ar. 2346 have been recognised as showing the growing philological acuity of the Baghdad school [Walzer, 1962]; what they also show is a sophisticated philosophical reaction to the text as it was being received into Arabic ([Hugonnard-Roche, 1993]; but see also [Lameer, 1996]).

We are fortunate to have from this time the catalogue of a Baghdad bookseller, Ibn-an-Nadīm (d. 995), called simply *The Index*, completed in 987 ([Ibn-Nadīm, 1871/72]; the logic chapters are translated in [Peters, 1968]). Ibn-an-Nadīm knew Ibn-Suwār personally, as well as other members of the school of which he was a part. *The Index* provides most of the information we have about the translators of the various parts of the Organon and of the various Greek commentaries on it that were also translated. Aside from the works of Galen, these Greek commentaries included works by Theophrastus, Porphyry, John Philoponus, Stephanus the Alexandrian, Ammonius, Themistius, Simplicius, Iamblichus, a mysterious Ālīnūs [Elamrani-Jamal, 1989–] and, perhaps most importantly, works by Alexander of Aphrodisias. ¿From *The Index* one also gets a sense of the nature of the shared enterprise that produced Ibn-Suwār's version of the Organon: draft translations, commentary, discussion, revised translations. Through this process, the translators also established a technical vocabulary with which to render the Organon [Afnan, 1964], itself one of the great achievements of the translators. This technical vocabulary, and the translations in their various stages of refinement, enabled and were in turn enriched by vigorous philosophical debate.

A note on the Arabic *Prior Analytics* Lastly, since this chapter focuses on the syllogistic, especially as presented in the first twenty-two books of the *Prior Analytics*, it is worth making a few specific comments about the translation of the *Prior Analytics*. The translator was the mysterious Tadārī, plausibly identified by Lameer as a certain Theodore, a Syriac Christian working with Ḥunayn's circle [Lameer, 1994, pages 3 & 4], though quite possibly working long before that time. Tadārī was making his translation against a background in which the syllogistic was known at least superficially due to works like the text by Ibn-al-Muqaffa'. His translation was corrected by Ḥunayn, and, somewhat strangely for the translations of the Organon, used as a basis for the Syriac translation by Ḥunayn and Isḥāq [Ibn-Nadīm, 1871/72]. The edition which we have [Badawī, 1948/52, vol. I, pages 103–306] is based on the Paris manuscript, and the parts directly relevant to this chapter are *Prior Analytics* 24^a10–40^b16 & 50^b [Badawī, 1948/52, pages 103–176 & 217–218]. ([Jabre, 1999] came to my attention too late to be used for this chapter.) Some reflections on the notes that would have been available to Alfarabi, and his attitude to them, are given in Zimmermann [Zimmermann, 1981, pages lxxiv–lxxv]; the *Prior-Analytics* complex has been reconstructed as far as is possible by Lameer [Lameer, 1994, chapter one]. One general comment can be made: the *Prior Analytics* came with a mass of interpretative material, not of all of which was mutually compatible.

3 ALFARABI AND AVICENNA

Alfarabi (d. 950) and Avicenna (d. 1037) are the two most important writers to consider in constructing a history of medieval Arabic logic. They constitute, with Aristotle, the three main reference points for later writers as they react against or conform with the philosophical options before them. Both Avicenna and Alfarabi came to the Baghdad translation of the Aristotelian corpus by way of the translated commentatorial material.

As they worked, both sought to set themselves apart from the bulk of the Syriac Christians who, numerically at least, had dominated philosophising in the Islamic world up to that point. In this, Avicenna was explicitly following Alfarabi as his venerated forebear. This is true even though Avicenna differed from Alfarabi at many points, especially in the logic. With Avicenna's awarding Alfarabi a pre-eminent place in the history of Aristotelian philosophy, and with his reformulation of the Aristotelian system, we may say that a truly naturalized tradition of logic in the realms of Islam begins.

We know hardly anything about Alfarabi. He was born somewhere in the East, perhaps in Transoxiana, perhaps around 870. He moved at some time to Baghdad, and it was probably there that he studied logic up to the end of the *Posterior Analytics* with Yuḥannā ibn-Ḥaylān. This took place some time within the reign of al-Muqtadir, which is to say between 908 and 932. Near the end of 942 he left Baghdad for Syria, and worked in Damascus and Aleppo. He visited Egypt towards the end of his life, then returned to Syria, and died in Damascus in 950 or 951 [Gutas, 1982–b]. Most of his surviving works are on logic, or deal with logic as a central theme.

By contrast, we know quite a lot about Avicenna, and we can be confident that most of what we know is accurate. Born in Bukhara some time before 980 [Gutas, 1987–88], Avicenna spent the first twenty years of his life on philosophical studies, most of which he undertook without a teacher. Faced with political upheaval in his homeland, Avicenna travelled from one principality to another: from Bukhara to Khwarezm, after some years on to Jurjan, then to Rayy, to Hamadhan, and finally to Isfahan. In each of these places, he supported himself by his skills in medicine and administration. He died in 1037, leaving behind a huge corpus of works, many of which deal with logic.

3.1 Approaches to the Aristotelian tradition

The ways Alfarabi and Avicenna approach the logic are determined by their attitudes to the broader Aristotelian tradition. Each of them wrote a short text designed to clarify his attitude to Aristotelianism.

Alfarabi's approach Alfarabi is part of a movement in Baghdad which began around 900 with his teacher Yuḥannā ibn-Ḥaylān and his senior colleague, the Christian Abū-Bišr Mattā. Alfarabi described his philosophical pedigree in a short tract, *On the appearance of philosophy*, in which he claimed his teachers and himself to be Aristotelians, alumni of the Alexandrian school, a school whose move to Baghdad he also traced in the tract. There are many difficulties in reaching a good understanding of the considerations that went into the composition of this tract (see now [Gutas, 1999]), but it is possible to state concisely what being an Aristotelian meant for Alfarabi's conception of his forebears.

> If, as would appear, the pillars of the Baghdadian renaissance were Alexander and Themistius, neither of whom had been connected with Alexandria, it makes sense to talk of a continuity of Alexandrian tradition only in al-Farabi's scheme of the history of philosophy according to which *all* Greek Aristotelians, on the strength of their spiritual connexion with the legendary

school of Aristotle at Alexandria, would qualify as representatives of 'Alexandrian' tradition. [Zimmermann, 1981, civ–cv]

For present purposes, the historicity of Alfarabi's *On the appearance of philosophy* is beside the point; what matters is that Alfarabi was consciously trying to revive a true, textual Aristotelianism after a period of rupture [Hasnawi, 1985]. He was doing this, moreover, without making any mention of Kindī and his circle, or of the sui generis Muḥammad ibn-Zakariyā ar-Rāzī [Gutas, 1999, page 155]—Alfarabi obviously considered these philosophers part of the problem.

Another motivation behind many of Alfarabi's formulations was his consciousness of working in an Islamic community. At the time that Alfarabi was studying and teaching in Baghdad, the various Islamic disciplines were achieving their classical articulation. Alfarabi worked towards both making philosophy resemble the Islamic disciplines in its historical claims, and making its utility for and complementarity with those disciplines obvious [Gutas, 1982–a, page 219].

Avicenna's approach Avicenna was just as ardently Aristotelian as Alfarabi, but his Aristotelianism was constituted and implemented in different ways. It was constituted differently in that Avicenna's respect for Aristotle was not alloyed with a correspondingly high respect for Plato: "if the extent of Plato's achievements in Philosophy is what came down to us of him, then his wares were paltry indeed and philosophy in his time had not matured to the point of reaping" [Gutas, 1988, page 38]. Further, as has been mentioned, Avicenna had woven Alfarabi into his litany of great past philosophers, and reassigned lesser positions to some members of the Greek schools; on his work at one point in his later life, he wrote:

> ...[I] am occupied with men like Alexander [of Aphrodisias], Themistius, John Philoponus, and their likes. As for Abū-Naṣr al-Fārābī, he ought to be very highly thought of, and not to be weighed in the same scale as the rest: he is all but the most excellent of our predecessors. ([Gutas, 1988, page 64]; cf. [Badawī, 1948, page 122.2–4])

Avicenna's respect for Alfarabi was joined to an explicit contempt for the Syriac Christian philosophers. One of Avicenna's students remembered in his memoirs that Avicenna condemned Ibn-Suwār (whose version of the Organon is so important for modern scholars; see above page 532) and his colleagues, who, "because their field is so narrow, adhere more closely than others to the [traditional] transmissions of certain books."

> Upon my life, these people relax and are satisfied with whatever they imagine to be the case which is easily treated, dismissing logic absolutely. With regard to the matters of syllogisms, their dismissal is complete and they pay no attention whatever to them—and not only today, but they have been doing this for quite some time. As for the forms of syllogisms, specifically these people have disregarded them. Whenever they treated them, they strayed from the right path because they never acquired the habit of dealing with them and they never suffered the pains of analyzing the details of problems

> so that they may gain a syllogistic habit; their sole reliance, instead, is upon ideas not subject to rules. [Gutas, 1988, pages 68–69]

It is in the implementation of his Aristotelianism, however, that Avicenna differed more significantly from Alfarabi. Whereas Alfarabi constructed the story of his philosophical education to tie himself to a school and its teaching tradition, Avicenna constructed the story of his education to sever himself from any teaching tradition at all. He designed his autobiography to present himself as an autodidact successful by virtue of intuition. Gutas has summed up the major effect that the doctrine of intuition which undergirds the autobiography has for Avicenna's reading of Aristotle.

> The perspective of the Autobiography, therefore, is that of a philosopher belonging to no school tradition, who established truth on his own by means of his Intuition, equalling Aristotle in this regard, if not surpassing him, and whose independent Verification of the truth, which reproduces for the most part the philosophical sciences as classified originally by Aristotle, puts him in a position both to teach this more accurate version of the truth, and to judge the philosophical attainments of others. [Gutas, 1988, pages 197–198]

The content of the doctrine of intuition need not concern us here—what matters is that its effect was to position Avicenna relative to the Aristotelian corpus differently from Alfarabi. When Avicenna collided with a crux in the text, he did not have to resort to exegetical strategies to find his way out. In fact, throughout *The Cure* it is clear that he believed he had worked out the unified vision that motivates Aristotle's presentation, and this allowed him to elide, transform and augment the system of the *Prior Analytics*.

3.2 *Alfarabi and the logical treatise*

Alfarabi's attitude to Aristotle seems to have become clearer over time, and in consequence his position changes from one work to another. And Alfarabi had any number of opportunities to change his position: he wrote many works, nearly half of which seem to have been addressed principally to logic. In some places we find undigested stretches of logical doctrine which do not fit well with the rest of what he is doing [Zimmermann, 1972]. Further, we find that from one logical treatise to the next, some terms are being used more precisely, others are being discarded, and doctrines are being clarified as their relevance to each part of his project is established [Lameer, 1994, e.g pages 202, 259–289]. Alfarabi writes in the tradition of the Alexandrian summary, as a Muslim in an Islamic society, and as an Aristotelian concerned to recover the true sense of Aristotelian texts. Only the first two activities will be considered in this section—Alfarabi the Aristotelian commentator is considered in sections 3.4 and 3.5 below.

The fact that Alfarabi's final views on logic only came to be delineated over time makes it difficult to describe his logic generally. Further, we have lost a number of his works, especially the long commentaries. Most significantly for the line of investigation I am following in this chapter, we have lost the first section of the *Long Commentary on the Prior Analytics*, although we can reconstruct enough of this work for present purposes

from references in Avicenna and Averroes. We do have the long commentary to *On Interpretation*, and shorter commentaries on the other books of the Organon; and we have many works in which logic is a major or the major subject under discussion (for a short overview, see [Gutas, 1993, pages 47–50]; cf. [Lameer, 1996, page 97]). But these works were addressed to various audiences, and it is not always easy to say what the relative importance is of various doctrines, nor, because so many of the longer works are as yet unavailable, whether Alfarabi at the stage of his most mature reflections would have wanted to affirm any given doctrine. In short, we are not now, and probably never will be, able to describe Alfarabi's logic with confidence. Still, we can name a number of doctrines later logicians adopted from him.

Logic and language Alfarabi presented his definition of logic by contrasting it with grammar. To hold that this is the best way to go about such a definition is an important philosophical claim, which Avicenna was later to reject; but it is also a political claim, finding a position for Greek logic within Islamic society. Grammar is not uncommonly contrasted with logic—it is implicitly contrasted with logic and rhetoric throughout the medieval Western tradition of the trivium—but events in Alfarabi's Baghdad had made such a contrast especially urgent. Alfarabi's senior colleague, Abū-Bišr Mattā, had been ignominiously routed by a grammarian, Abū-Sa'īd as-Sīrāfī (d. 978), who doubted the scholarly viability of an independent subject like logic given that people had grammar (see below page 554). Subsequent treatments of logic tended to inherit from discussions like this an apologetic edge, trying to find a task for logic separate from but complementary with grammar. Alfarabi wrote in his *Introductory epistle on logic*:

> Our purpose is the investigation of the art of logic, the art which includes the things which lead the rational faculty towards right thinking, wherever there is the possibility of error, and which indicates all the safeguards against error, wherever a conclusion is to be drawn by the intellect. Its status in relation to the intellect is the status of the art of grammar in relation to language, and just as the science of grammar rectifies the language among the people for whose language the grammar has been made, so the science of logic rectifies the intellect, so that it intellects only what is right where there is a possibility of error. Thus the relation of the science of grammar to the language and the expressions is as the relation of the science of logic to the intellect and the intelligibles, and just as grammar is the touchstone of language where there is the possibility of an error of language in regard to the method of expression, so the science of logic is the touchstone of the intellect where there is the possibility of an error in regard to the intelligibles. [Dunlop, 1956, page 230 (Arabic, page 225)]

So grammar deals with the manipulation of expressions in a particular languages, whereas logic deals with the manipulation of meanings common to all peoples. Alfarabi's general point was accepted among his colleagues in Baghdad, though the extent to which the intelligibles can be compared to separate utterances was disputed, especially by Avicenna.

The context theory In the *Introductory epistle* we also find Alfarabi presenting his logic according to what is known as the context theory, which he inherited from the Alexandrian tradition. Thus he took the *Categories*, *On Interpretation*, and the *Prior Analytics* to have general application across all stretches of discourse, and each of the following five books of the Organon to have only specific utility for a particular mode of discourse. (Although he did accept the *Categories* as a logical work, Alfarabi recognised the force of arguments that sought to classify it as metaphysical.)

According to the context theory, syllogisms with premises of differing epistemic grades constitute distinct stretches of discourse, and may belong to demonstrative philosophy, dialectic, sophistry, rhetoric or poetry (cf. [Gutas, 1983, pages 256–257 and diagrams IV & V]). That is to say, syllogistic contributes to the analysis of every stretch of discourse.

> Syllogism is employed either in discoursing with another or in a man's bringing out something in his own mind... Philosophical discourse is called demonstrative. It seeks to teach and make clear the truth in the things which are such that they afford certain knowledge. Dialectical discourse seeks to overcome the interlocutor in the things which are known and notorious. Sophistic discourse seeks to overcome the interlocutor by a supposed victory in the things which are thought ostensibly to be known, without being so. The aim is to draw the interlocutors and hearers into error, likewise falsification and trickery, and that the speaker should produce the opinion of himself that he is one who possesses wisdom and knowledge, without being so... Rhetorical discourse seeks to satisfy the hearer by what will partially content his soul, without reaching certainty. Poetical discourse seeks to represent the object and suggest it in speech, as the art of sculpture represents different kinds of animals and other objects by bodily labours. The relation of the art of poetry to the other syllogistic arts is as the relation of sculpture to the other practical arts, and as the relation of chess-playing to the skilful conduct of armies. [Dunlop, 1956, page 231 (Arabic, page 226)]

The context theory depends on a division of discourse according to the 'matter' which makes it up—divisions of this matter came to be disputed, and even Alfarabi seems to change his mind from one text to the next. However the material aspects of discourse were divided, each of the resulting divisions was related to one or other of the faculties of the soul, and can be explained fully only in tandem with a treatment of psychological and epistemological doctrines ([Black, 1990, chapters 4 & 6]; cf. [Lameer, 1993]). The extent to which the syllogistic contributed to the analysis of various stretches of discourse was also disputed, and Avicenna doubted that rhetoric or poetics could really be treated formally as syllogistic. The context theory and its permutations continued to be a factor in logic manuals at least until the seventeenth century, though not always dictating the same structure in each treatise. It also had consequences for how logic related to the Islamic disciplines, such as those devoted to the analysis of dispute, and to rhetoric; as the Islamic disciplines came to exert a stronger claim to these fields, their treatment within logical treatises became sketchier and sketchier (see Ibn-Ḫaldūn's brief comments on this phenomenon, page 580 below).

Syllogistic analysis of arguments In Alfarabi's works we find another feature which recurs in a great many Arabic logical texts: the attempt to show that all valid argument-forms relate in some way to the syllogism, an attempt first made by Aristotle. Like the claim that logic and grammar had different but complementary interests in language, the claim that the structure of the syllogism was important in understanding the structure of other arguments was directed at Muslim jurists and theologians. In this effort, the earliest relevant text Alfarabi wrote was *The short treatise on reasoning in the way of the theologians*

> ... in which he interpreted the arguments of the theologians and the analogies (*qiyāsāt*) of the jurists as logical syllogisms in accordance with the doctrines of the ancients. ([Alfarabi, 1986b, page 68.11–12]; cf. [Sabra, 1965, page 242a])

In this text, we find analyses of the paradigm, of the argument used by Muslim theologians called 'reasoning from the seen to the unseen' (*al-istidlāl biš-šāhid 'alā l-ġā'ib*), and of 'the juristic argument' (*al-qiyās al-fiqhī*) itself. Alfarabi takes the second kind of argument to reduce to the first, and offers an elaborate analysis of the third as involving a range of rhetorical argument techniques [Lameer, 1994, respectively, chapters 6, 7 & 8]. This began a trend which did in fact issue in the acceptance of logic as useful by an important Muslim jurist, Abū-Ḥāmid al-Ġazālī (see below page 554 ff.). It also led subsequent writers of logic manuals to consecrate at least a part of their manuals to the reduction of argument-forms to the syllogism, a reflex carried over from this early time when Muslim scholars contested the place of logic in Islamic society.

3.3 Avicenna and the logical treatise

At the latest, Avicenna came by his middle age to a settled view of the proper conception and formulation of logic. Like Alfarabi, a large proportion of his work was given over to logic (for a brief overview of his works and their genres, see [Gutas, 1993, pages 50–53]). Though we lack any of his early commentaries directly on the texts of Aristotle, we have all of *The Cure*, Avicenna's great philosophical opus. The first book of *The Cure* treats the subject-matter of Porphyry's *Introduction*, and each one of the next eight books covers the subject-matter of each of the parts of the Alexandrian arrangement of the Organon. We also have a number of shorter expositions, three of which I refer to in this chapter. Two of these shorter expositions, *The Book of Salvation* and *Pointers and Reminders*, present the system with all the sophistication we find in *The Cure*, while the third, in *Philosophy for 'Alā'uddawla*, presents a greatly simplified system.

Avicenna's books became important as paradigms for subsequent writers. So, for example, we find Abū-l-Barakāt al-Baġdādī (d. 1165) consciously modeling his major philosophical work on *The Cure*. Most important of all Avicenna's works, however, is *Pointers*. Because of its difficult and allusive style, it became the subject of many commentaries—these evolved in time into free-standing treatises which none the less preserved the order and emphases of *Pointers*. Many of the changes in the treatment of logic which Ibn-Ḫaldūn notes (see below page 580) are apparent already in *Pointers*.

Rejection of Farabian doctrine In defining logic, Avicenna differed from Alfarabi. Avicenna agreed that logic was a normative instrument to protect man from going astray in thinking ([Avicenna, 1971[2], pages 117–127]; cf. [Gutas, 1988, page 281]). But he did not characterise logic in the way Alfarabi did.

> There is no merit in what some say, that the subject-matter of logic is speculation concerning the expressions insofar as they signify meanings... And since the subject-matter of logic is not in fact distinguished by these things, and there is no way in which they are its subject-matter, [such people] are only babbling and showing themselves to be stupid. ([Black, 1991, page 54]; cf. [Avicenna, 1952, pages 23.5–6, 24.3–4])

One reason for this is that in Avicenna's psychology, language as a set of discrete expressions is not essential for the intellect in its operations; it is only accidentally the path that humans have to follow.

> ... [I]f it were possible for logic to be learned through pure cogitation, so that meanings alone would be observed in it, then this would suffice. And it if were possible for the disputant to disclose what is in his soul through some other device, then he would dispense entirely with its expression. ([Black, 1991, pages 54–55]; cf. [Avicenna, 1952, page 22.14–17])

In consequence, intelligibles are not able to be likened to expressions in a language, which must by their essence be uttered and grammatically ordered through time.

Modifications to Alfarabi's doctrines Avicenna differed from Alfarabi in holding that logic does not deal with expressions in so far as they signify meanings. Rather, according to Avicenna, logic deals with meanings which classify meanings—logic does not deal with a proposition's subject in terms of the meaning it signifies, but as a subject-term. This is the famous doctrine that the subject-matter of logic is the second intentions (and here I quote the only doctrine of a medieval Arabic logician that is given in Kneale and Kneale [Kneale and Kneale, 1962, page 230]):

> As you have known, the object of logic is the second intentions (*al-maʿānī al-maʿqūla al-thāniya*)—those that depend upon (*tastanid ilā*) the first intentions—insofar as they may be of use in arriving at the unknown from the known, and not insofar as they are thoughts (*maʿqūla*) having an intellectual existence that is not attached to matter at all or attached to non-corporeal matter. ([Sabra, 1980, page 753]; translation modified slightly)

In this way, Avicenna was able to define logic not only as a normative instrument, as noted above, but also as an independent science with its own subject-matter, namely, the second intentions. For all the strong language used in clearing Farabian teaching away to make space for this doctrine, it would appear that Avicenna is in fact developing ideas found in Alfarabi [Sabra, 1980, pages 755–756].

It is fairly easy to compare Avicenna with Alfarabi on some other points. In terms of traditional allegiances, Avicenna was much more forthright in dismissing the logical

writings of the Syriac Christians than Alfarabi had been (see page 535 above). Avicenna echoed Alfarabi in questioning the propriety of placing the *Categories* within the Organon, and decided that it should only be treated within the other logical texts due to immemorial custom. But it is no help in understanding the syllogism:

> The student of logic, after learning what we have told him about regarding the simple terms, and learning the noun and the verb, can go on to learn propositions and their parts, and syllogisms, and definitions and their kinds, and the matters of syllogisms and the demonstrative and non-demonstrative terms and their genera and species, even if it does not occur to him that there are ten categories. ([Avicenna, 1959, page 5.1–4]; cf. [Gutas, 1988, page 265])

It is worth noting that in this decision, and in his excision of the assertoric syllogistic (see below page 548), Avicenna was cutting out of his logic the two things to which the Syriac Christians devoted most of their efforts.

On the other hand, like the Syriac Christians, and like Alfarabi, Avicenna at the beginning of his career accepted the context theory, though he dispensed with it later on ([Gutas, 1988, page 18, note 6]). In *Pointers*, however, he placed the major consideration of the material aspects of discourse, and its consequences for dividing kinds of discourse, at the end of his treatment of propositions [Avicenna, 1971^2, pages 341–364]. This became the standard place and way to treat the context theory in short treatises thereafter.

Avicenna's elimination of the categories from his logic texts, and the method by which he dealt with the context theory, were both influential. But perhaps most important for the structuring of logical treatises after him was a distinction he found in Alfarabi and used in his own writings: the distinction between *taṣawwur* and *taṣdīq*. This is a distinction dividing knowledge into 'conceptions' and 'judgements to which one assents'. Alfarabi mentioned the distinction in his treatment of demonstration, writing that knowledge "is of two kinds, conception and assent" [Alfarabi, 1986a, page 19.5], and later implicitly assigning the logical operations of definition and syllogism to the attainment of, respectively, conception and assent [Alfarabi, 1986a, page 45.1]. In all of Avicenna's writings, by contrast, the distinction is made at the very outset. Among conceptions are for example 'house' and 'man', and so forth. Among judgements to which one assents are included for example the judgement that a house is where people dwell, and that man is a rational animal. Indeed, all knowledge is either conception or assent. All investigations are directed

> either to a conception to be acquired, or to an assent to be acquired. It is customary to call the thing which leads to the desired conception an explanatory phrase, which includes definition and description and the like; and the thing which leads to the desired assent a proof, which includes syllogism and induction. [Avicenna, 1971^2, pages 136–137]

Logic then is concerned to prevent one going astray in thinking about conceptions and assent; that is, it provides a theory of definition, and a theory of proof [Sabra, 1980, page 761].

Finally, like Alfarabi, Avicenna agreed that the argument-forms used in law and theology were best analysed by reference to the syllogism ([Avicenna, 1971^2, pages 365–373]; see also [Avicenna, 1971, pages 38–40]). But unlike Alfarabi, apologetics for logic relative to the Islamic disciplines are not central to what he wrote.

Points for comparison There are problems in making further comparisons between Avicenna and Alfarabi. Alfarabi modified his logical doctrines throughout his life, Avicenna by and large did not; many texts of Alfarabi are missing, whereas we have the whole of the Avicennan system (even if we haven't yet worked through it); Alfarabi wrote both commentaries on Aristotle and apologetics for logic to propitiate the lawyers, Avicenna wrote neither (at least in later life).

A comparison, then, is difficult. Luckily for the narrow confines of this chapter, however, Avicenna directed comments to the Farabian system, presumably as developed in the lost *Long commentary on the Prior Analytics*, while he was dealing with important points in his own syllogistic. Modern scholars have tended to overlook these comments because Avicenna referred to Alfarabi as 'the eminent later scholar' (*al-fāḍil min al-muta'aḫḫirīn*), and many have thought that by this he meant Alexander of Aphrodisias (see for example [Maróth, 1989, page 7]). The eminent later scholar is, however, Alfarabi (see [Dānišpažūh, 1989, vol. 3, Dībāğa 14]; cf. [Averroes, 1983b, page 101.3–5], and most recently [Street, 2001]). Avicenna referred to Alfarabi while developing his modal logic, and at one point in developing his hypothetical syllogistic. Because Avicenna dictated by his changes what were to be the fundamental questions for later logicians, these are the major points of discussion by Averroes, and so we find the later tradition effectively evaluating the earlier traditions with reference to this material.

Four of Avicenna's references to Alfarabi are particularly helpful for comparing the systems the two men built. The first point of comparison is made somewhat complex by the fact that the two men meant different things by 'absolute proposition' (*qaḍiyya muṭlaqa*)—I return to this below (see page 547)—and Alfarabi argued that an absolute e-proposition converts as an absolute e-proposition, whereas Avicenna argued that e-conversion fails for the absolute. This is symptomatic of more fundamental and far-reaching differences in how the two men went about laying the foundations for their modal systems. The second reference is to the fact that Alfarabi accepted Barbara LXL (as did Avicenna). This raises a problem of consistency for Alfarabi relative to the stratagem he adopted to save the conversion of the two-sided possible proposition—this is the third important reference to Alfarabi. Lastly, Avicenna rejected an ascription to Alfarabi of a long text on the hypothetical syllogistic; this reference allows us to put to rest claims of a missing long treatment by Alfarabi of the hypothetical syllogistic.

Broadly speaking, then, this provides material for a comparison of how the two men take the modal syllogistic, and how they take the hypothetical syllogistic. I deal with the hypothetical syllogistic first.

3.4 *The hypothetical syllogistic*

I stress at the outset that I do not intend to analyse the hypothetical syllogistic. Alfarabi probably never treated it in enough detail to ground such an analysis, and the differences between Avicenna and Averroes touch on deep issues which I do not properly understand. I will only be using debate about the hypothetical syllogistic as one index for the traditions to which Avicenna and Alfarabi belonged, and for the way later logicians worked within their respective traditions.

I use 'hypothetical syllogistic' loosely here, as a term sufficiently broad to cover the two quite different approaches of Avicenna and Alfarabi. In translating Alfarabi's *qiyās šarṭī* as 'hypothetical syllogistic', I am prescinding from the debate about better possible translations (raised because it does not extend to cover as many inferences in Alfarabi's usage as 'hypothetical syllogistic' does in Alexander's usage, but rather seems limited like Galen's; see [Lameer, 1994, pages 45–46]). Again, in the case of Avicenna, there is no technical phrase that corresponds directly with 'hypothetical syllogistic'—some of his *iqtirāniyyāt* and all of his *istiṯnā'iyyāt* together would constitute what I mean here by hypothetical syllogistic (for Avicenna's technical terms, see below page 546; for aspects of the usage of *istiṯnā'*, see [Gyekye, 1972]). I use hypothetical syllogistic in the same way Barnes does:

> A hypothetical syllogism is an argument at least one of whose premisses is a hypothetical proposition. A proposition is hypothetical if it is a compound of at least two propositions...
>
> Hypothetical syllogistic contrasts with categorical syllogistic, for a syllogism is categorical if all its component propositions are "simple," i.e., if none is compounded of two or more propositions. [Barnes, 1985, page 129]

Alfarabi on the hypothetical syllogistic Alfarabi made the following remark regarding the hypothetical syllogistic in his *Long Commentary on On Interpretation*:

> He (Aristotle) examines the composition of hypothetical (statements) not at all in this book, and only slightly in the *Prior Analytics*. The Stoics, on the other hand, Chrysippus and others, examined it thoroughly to the point of excess, made a thorough study of hypothetical syllogisms—as Theophrastus and Eudemus had done after Aristotle's time—and claimed that Aristotle wrote books on hypothetical syllogisms. But we have no knowledge of any separate treatment by him (Aristotle) of hypothetical syllogisms in his books on logic; this (claim) is found rather in the commentaries of the commentators who give an account of them (hypothetical syllogisms) on the authority of Theophrastus only. [Fortenbaugh and others, 1992, page 239]

Similarly, Alexander of Aphrodisias had said in his own short comments on the hypothetical syllogistic that "no book of his (Aristotle) on the subject is in circulation. Theophrastus, however, refers to them in his own *Analytics*—and so do Eudemus and some others of Aristotle's associates" ([Barnes, 1985, page 125]; cf. [Shehaby, 1973b, page 24, note

11]). In short, Alfarabi belonged to a tradition which was unacquainted with the existence of a separate, genuinely Aristotelian treatise on the hypothetical syllogistic, and which seemed reluctant positively to postulate the existence of such a treatise.

We can also say something concrete about Alfarabi's own hypothetical syllogistic. Alfarabi presented in his treatises (and here I take *The short treatise on reasoning in the way of the theologians* as an example) a definition of the syllogism:

> A syllogism is a phrase, composed of propositions laid down from which, if so composed, some other thing follows of necessity by virtue of these very things themselves, and not accidentally. And whatever comes to be known through a syllogism is called a 'conclusion' or 'what follows'...
>
> The least from which a syllogism may be composed is two propositions sharing in a single part; and syllogisms may be composed from hypothetical or categorical propositions. ([Alfarabi, 1958, page 250.12–apu]; cf. [Lameer, 1994, pages 16–17])

Alfarabi delivered as his hypothetical syllogistic the five Stoic indemonstrable inference schemata ([Alfarabi, 1958, pages 257.6–260.10]; cf. [Lameer, 1994, page 45]), and did not take it to contribute to the analysis of the deduction involving a contradiction ([Alfarabi, 1958, pages 260.11–261.7]; cf. [Lameer, 1994, pages 50–54]).

It has been speculated that Alfarabi's lost first part of the *Long Commentary on the Prior Analytics* covered the hypothetical syllogistic in considerably more detail [Maróth, 1989], but it is unlikely that it did. Avicenna almost certainly had read that commentary, yet we find in *The Cure* that

> ...we came across a book on conditional (propositions and syllogisms) attributed to the most excellent among later (scholars). It seems to be wrongly imputed to him. It is neither clear nor reliable. It neither gives an extensive survey of the subject nor does it achieve its purpose. It gives a mistaken exposition of conditional propositions, of a large number of syllogisms which accompany them, of the reasons for productivity and sterility, and of the number of moods in the figures. The student should not pay any attention to it—it is distracting and misleading. ([Shehaby, 1973b, page 159]; cf. [Avicenna, 1964, page 356.10–15])

On coming across an alternative treatment of the hypothetical syllogistic, Avicenna thought it was not Alfarabi's, but he did not know. If Alfarabi had treated the hypothetical syllogistic at any length in his *Long Commentary*, Avicenna would have known for sure whether or not the attribution of the book to Alfarabi was correct. Later, Averroes would have referred to Alfarabi's longer treatment of the hypothetical syllogistic when treating the problem in one of his essays (see below page 565). Avicenna did not know for sure, Averroes did not refer to the longer treatment. We have already the main burden of what Alfarabi wanted to present about the hypothetical syllogistic in developing his syllogistic.

Avicenna on the hypothetical syllogistic Alfarabi did not think Aristotle had written a separate treatise on the hypothetical syllogistic, and he did not think that the hypothetical

syllogistic merited extensive treatment. But he was aware of another, divergent tradition on this point. Against Alfarabi's comments in *On Interpretation* we must compare Avicenna's comments in *The Book of the Syllogism* from *The Cure*, on the proof by reduction and its proper analysis.

> The only thing invoking this pointless exertion from people is the fact that they have lost the work that Aristotle wrote detailing the hypothetical syllogistic. ([Avicenna, 1964, page 397.4–5]; cf. [Shehaby, 1973b, page 190])

Avicenna, that is to say, was part of the tradition which claimed extensive treatment of the hypothetical syllogistic by Aristotle, a tradition known to Alfarabi when he was writing his long commentary on *On Interpretation*.

This point makes a radical difference to how the two men write their treatises on logic. Avicenna devoted substantial portions of *The Cure* to the hypothetical syllogistic (translated in [Shehaby, 1973b]). He was clear that he got his treatment from elsewhere.

> In our native country we came to know a long annotated book on this subject which we have not seen since we left our country and travelled around to look for a means of living. However, it might still be there. ([Avicenna, 1964, page 356.7–]; cf. [Shehaby, 1973b, page 159])

Two observations should be made at this point. Firstly, Avicenna did not create differences from the Farabian system, but followed existing ones. Secondly, he was concerned to modify the syllogistic so he could accommodate the doctrine of this non-Aristotelian text.

Avicenna on proof by reduction To repeat: there are many important aspects of Avicenna's doctrines on hypothetical propositions and hypothetical syllogistic which cannot be considered in this chapter. Here are three, which I mention because Averroes commented on them (see below page 566 f.). Firstly, Avicenna wrote at one point

> All conditional and disjunctive propositions, and in particular the conditional in which the antecedent and the consequent share one part, can be reduced to categorical propositions—as when you say, for example, "If a straight line falling on two straight lines makes the angles on the same side such and such, the two straight lines are parallel." This is equivalent in force to the categorical proposition: "Every two straight lines on which another straight line falls in a certain way are parallel." ([Avicenna, 1964, page 256.11–15]; cf. [Shehaby, 1973b, page 55])

though his full doctrine on this matter is nuanced and complicated (see e.g. [Avicenna, 1964, page 264 f.]; cf. [Shehaby, 1973b, page 62]). Secondly, Avicenna held that a syllogism which conveyed new knowledge had to depend in the final analysis on a categorical syllogism, which is therefore in this sense primary [Avicenna, 1964, page 415 f.]. Thirdly, Avicenna's analysis of hypothetical syllogisms and categorical syllogisms includes claims about the epistemic immediacy of the inferences [Avicenna, 1964, page 416.12 ff.]. All these points deserve careful study, which they do not receive in this chapter.

What I do examine (and it has been examined before in [Shehaby, 1973b, page 277 f.]) is the way Avicenna accommodated the hypothetical syllogistic so that it goes to his analysis of proof by reduction. I present very briefly Avicenna's placement of the hypothetical syllogistic and its use in explaining the way Baroco is proved. I do so because it exemplifies how seamlessly Avicenna fitted extra-Aristotelian doctrine into his syllogistic. To do this, I turn from Avicenna's long exposition of the hypothetical syllogistic in *The Cure* to the more managable exposition in *Pointers* [Avicenna, 1971^2; Goichon, 1951; Inati, 1981], which is limited in its presentation to just those three parts of the hypothetical syllogistic actually used in the explanation of Baroco [Ṭūsī, 1971, page 441].

As first studied in [Rescher, 1963d], Avicenna's hypothetical propositions are quantified. *Always: when A is B, then D is H* is an a-conditional; *sometimes: when A is B, then D is H* is an i-conditional; *never: when A is B, then D is H* is an e-conditional; and *sometimes not: when A is B, then D is H* is an o-conditional. They and categorical propositions contribute to inferences, which Avicenna divided into conjunctive (*iqtirānī*) and exceptive (*istiṯnā'ī*). This division is one of the points in his logic where he claimed for himself the doctrine put forward:

> According to what we ourselves have verified, syllogistic divides into two, conjunctive (*iqtirānī*) and exceptive (*istiṯnā'ī*). The conjunctive is that in which there occurs no explicit statement [in the premises] of the contradictory or affirmation of the proposition in which we have the conclusion; rather, the conclusion is only there in potentiality, as in the example we have given. As for the exceptive, it is that in which [the conclusion or its contradictory] occurs explicitly [in the premises]. [Avicenna, 1971^2, page 374]

Avicenna built his wholly hypothetical conjunctive syllogistic from quantified conditionals:

> From the conditionals may be composed the three figures, just like the figures of the categorical—they share in a consequent or an antecedent, and differ in a consequent or an antecedent, just as the categoricals share in a subject or a predicate, and differ in a subject or a predicate. The status [of one] is the status [of the other]. [Avicenna, 1971^2, pages 435–436]

An example of such a syllogism would be (Barbara):

> Always: when A is B then J is D, and
> Always: when J is D then H is Z, which produces
> Always: when A is B then H is Z

There are also a number of rules, including ecthesis, which deliver fourteen moods in the wholly hypothetical syllogistic ([Avicenna, 1964, pages 295–304]; cf. [Shehaby, 1973b, pages 91–99]).

Avicenna then considered the conditional premise conjoined with a categorical:

> The conditional may be joined with a categorical. The most natural of these [conjunctions] is when the categorical shares the consequent of the affirmative conditional, in one of the ways categoricals share [a term with each

> other]. Then the conclusion will be a conditional whose antecedent will be the very antecedent [of the first conditional], and whose consequent will be the conclusion of the composition of the consequent conjoined with the categorical. An example is: *If A is B then every J is D*, and *Every D is H** it follows that *If A is B, then every J is H*. It is up to you to enumerate the rest of the divisions from what you have learned. [Avicenna, 1971[2], pages 440–441]

Lastly, in a separate section, Avicenna listed the exceptive hypothetical syllogisms, which include modus ponendo ponens, modus tollendo ponens, modus tollendo tollens, and modus ponendo tollens [Avicenna, 1971[2], pages 448–452].

With this material, we may follow how Avicenna analyses Baroco [Avicenna, 1971[2], page 453 f.]. In schematized form:

> To prove: Given *all Bs are Ds* and *some Js are not Ds*, then *some Js are not Bs*.
>
> 1. Conjunctive:
> When it is not the case that *some Js are not Bs*, then *all Js are Bs*. And *all Bs are Ds*.
> Therefore: when it is not the case that *some Js are not Bs*, then *all Js are Ds*.
>
> 2. Exceptive:
> When it is not the case that *some Js are not Bs*, then *all Js are Ds*.
> But it is not the case that *all Js are Ds*.
> Therefore: It is not not the case that *some Js are not Bs*.

I would think that this could be extended programmatically to cover all proofs by reduction, although Avicenna did not himself do so. In any event, Avicenna's analysis failed to impress the majority of logicians who followed him. The analysis allows us to observe attitudes to Avicennan logic in the later tradition, however, as will become apparent (see page 576 below). It is also significant, as noted above, because it shows just how Avicenna fitted a non-Aristotelian tradition into his treatment of the categorical syllogistic.

3.5 *Avicenna on Alfarabi on the modal logic*

Avicenna and the tradition to which he belonged had a very different approach to the hypothetical syllogistic from the one that Alfarabi and the Baghdad school had. There is an even larger difference in the treatment of the categorical syllogistic, both as it is made up of unmodalized and modalized propositions.

Differences concerning the absolute Basic to the many differences between Avicenna and Alfarabi in treating the modal logic is their difference regarding the absolute proposition (*al-qaḍiyya al-muṭlaqa*), the 'existential' proposition (*al-qaḍiyya al-wuǧūdiyya*), and their truth-conditions. Why these terms came to be used the way they were need not concern us here (but see [Lameer, 1994, page 55 ff.], whence I draw the information in this paragraph). In any event, *muṭlaqa* was used in the mid-800s to render Aristotle's *tou hyparchein protasis* as well as in connexion with the synonymous *hyparchousa protasis*,

whereas *wuǧūdiyya* seems to have been preferred later (about 900) to render *hyparchousa*. Lameer's results not only make clear the importance that the lexical preferences of the translators have for coming to grips with the technical terms of the Arabic philosophical tradition; they also make clear the fact that Alfarabi's usage of both *al-qaḍiyya al-muṭlaqa* and *al-qaḍiyya al-wuǧūdiyya* may be translated as 'assertoric proposition'; and, finally, that Alfarabi begins his logical work by constructing an assertoric syllogistic [Lameer, 1994, chapter four].

By contrast, Avicenna did not put forward an assertoric syllogistic and then modalize it. Propositions 10 and 24 in appendix two serve roughly to show how Avicenna took the absolute (*al-qaḍiyya al-muṭlaqa*) and the 'existential' proposition (*al-qaḍiyya al-wuǧūdiyya*; henceforth referred to as the special absolute), in that their contradictories are perpetuals or disjunctions of perpetuals (proposition 5 in appendix two). It is easy to understand the way Avicenna understood the absolute proposition—his favourite example of it is *all men sleep*. That is to say, an absolute a-proposition is taken as concealing an 'at least once'; *all Frenchmen drink wine* is not naturally taken to mean that they drink wine constantly, but at least once in their lives. The unmodalised e-proposition by contrast can be taken to convey perpetuity: *no teetotaller drinks wine* means that no teetotaller ever drinks wine. Avicenna took the customary understanding of an e-proposition to be perpetual, but stipulated that for logical purposes, it was to be taken as concealing the same temporality as the a-proposition. The squares generated by the absolute and the perpetual are isomorphic with the squares generated by one-sided and two-sided possibility taken with the necessary proposition (respectively, propositions 13, 26 and 1 in appendix two), which are sometimes referred to as the classical squares of modal opposition [Thom, 1996, pages 13 & 15].

Further, Avicenna rejected the conversion of the absolute e-proposition. Whereas contradiction for his simple propositions (*ḏātī*; see below 550) depends on the modality or temporality of the predicate, conversion depends further on the modality of the subject-term. This is where we have the opportunity directly to compare what he was doing with what Alfarabi was doing. Avicenna rejected the conversion by citing the counterexample, *no man is laughing*. The only way to have the conversion go through according to him is to take all men at the time or times that they are not laughing. One way then is to have *at time t: no men are laughing*, which would convert as *at time t: no laughing thing is a man*. Avicenna rejected this as a solution, because the men of a given time are not all men, as should be the case in a proposition ready for logical treatment. (See further on the conditions under which a proposition may be read at page 550 below.)

Proofs for e-conversion did not impress Avicenna. He had seen Alexander's proof for e-conversion as given in *On the conversion of propositions*, which runs as follows:

> It may also be possible to prove conversion of the e-proposition by reduction to the absurd. So if A belongs to no B, and B belongs to some A, these combine by Ferio to mean that some A does not belong to A.* Since this is impossible, its contradictory is necessary, which is that B belongs to no A. [Alexander, 1971, page 65.6–9]

Whether Avicenna thought Alexander was entitled to the proof is unclear, but Alfarabi had adopted the proof, and Avicenna held it to be inconsistent with other doctrines he held.

> He who is eminent among the later scholars made this claim [for e-conversion] with a good argument: [no Js are Bs converts to no Bs are Js,] if not, then some Bs are Js; but no Js are Bs. This is a perfect syllogism [Ferio], self-evidently productive. It is only made known afterwards by way of reminding us, not to convey knowledge of which we are ignorant. From the above it follows that some Bs are not Bs—this is absurd. [Avicenna, 1964, page 81.1–4]

The reason Alfarabi could not use, for example, the stratagem of the as-of-now proposition to escape the counterexample was because he, like Avicenna, held that the subject-term in a proposition ready for logical treatment could not be limited to those things that fall under it at a given time (see next section, and also page 562 below).

> On the other hand, if the subject is taken in the way chosen by the eminent later scholar, such that J is whatever can be J, so that everything that can be J, even if it exists or not or it is not the case that it is J, enters under it. Let what follows from this be investigated... [Avicenna, 1964, page 85.5–7]

Differences concerning the modal logic Alfarabi's ampliation of the subject-term got him all the inferences in Aristotle, though often, as was pointed out later by Averroes, with such obviousness that the proofs put forward by Aristotle become pointless (see page 562 below). One inference it gave Alfarabi was Barbara LXL, against the well-known Theophrastean objection.

> Know that the eminent scholar with whom I am most concerned to conduct my discussion agrees with what I say; indeed, the First Teacher believes that if the necessary major in the first figure is joined to a non-necessary minor, the conclusion is necessary. Let us assume all Js are Bs non-necessarily, and all Bs are As necessarily, and it yields what the eminent scholar and the First Teacher both agree on, and what you have learned. [Avicenna, 1964, page 148.9–12]

Once again, however, Alfarabi is accused by Avicenna of holding incompatible doctrines. Straight after noting that Alfarabi accepts Barbara LXL, Avicenna goes on:

> But why doesn't one of them go on to say that this is not a necessary proposition, but rather must be: all Bs are As necessarily in so far as they are Bs... If this is taken into account, then what the detractors say against those who produce a necessary from these premises turns out to be true. That is because what the detractor is saying in this matter is like what the eminent scholar had to say about the conversion of the possible proposition... [Avicenna, 1964, page 148.13–pu]

The 'detractor', a Theophrastean, would have as the conclusion *all Js are As necessarily in so far as they are Bs.* Alfarabi is open to this objection because of the way he used a reduplicative proposition to have the conversion of the two-sided possible go through. His arguments on this point are reported by Avicenna (who by contrast held the two-sided possible to convert as a one-sided possible [Avicenna, 1971[2], page 340]) as follows:

> That which a certain eminent scholar said is this: every animal is possibly sleeping in so far as it is sleeping, so some of that which is sleeping is in so far as it is sleeping possibly an animal, because its being an animal does not belong to it in so far as it is sleeping—this is sheer sophistry. As for that which rightly should be known about this matter, it is something the proof for which has been given above. That which we ought to repeat and set down here is that the utterance 'in so far as it is sleeping' is said either as part of the predicate, or as part of the subject. If it is part of the predicate, then it must first off in conversion be made part of the subject, thus: Some of that which is sleeping in so far as it is sleeping is possibly an animal; this is as you hear it [that is, it is gibberish]. Given that it is true, it is not what we are talking about...
>
> But you know, O eminent one, that the sleeping taken without condition is other than the sleeping taken with the condition of its being sleeping, and in so far as it is sleeping... [Avicenna, 1964, pages 209.7–210.5]

Avicenna, that is to say, took Alfarabi to task for using one solution to get out of the problem of the conversion of the two-sided possible, but not continuing to bind himself by that solution in solving the problem of the two Barbaras. Avicenna placed both stricter and looser demands on logical exegesis: it need not follow Aristotle everywhere, but it must be internally consistent. More generally, Alfarabi was trying to find a way to make sense of the Aristotelian text, proposing solutions to local problems, and hoping that the ideas behind the text would ultimately shine through, whereas Avicenna did not think that there was any point in trying to give a literal exegesis of Aristotle's syllogistic.

> You should realize that most of what Aristotle's writings have to say about the modal mixes are tests, and are not genuine opinions—this will become clear to you in a number of places... [Avicenna, 1964, page 204.10–12]

Avicenna's modal syllogistic So how did Avicenna build his modal syllogistic? The first important feature to note is that having taken the absolute as a temporal, he placed it within his syllogistic alongside the modals.

The second important feature in Avicenna's syllogistic is the conditions under which a proposition can be read. There is no distinction made in Arabic logic corresponding to the Western distinction between divided and composite readings. The distinctions Avicenna proposed, however, became just as important and pervasive for logicians writing in Arabic. There are four intrinsic and two extrinsic conditions under which propositions can be read. (Although this passage is given for a necessary proposition, these conditions are applied to propositions with other modal operators.)

> Necessity may be (1) absolute, as in *God exists*; or it may be connected to a condition. The condition may be (2) perpetual for the existence of the substance (*ḏāt*), as in *man is necessarily a rational body*. By this we do not mean to say that man has been and always will be a rational body, because that would be false for each given man. Rather, we mean to assert that while he exists as a substance (*mā dāma mawǧūda ḏ-ḏāt*), as a human, he is a rational bodyŁ Or the condition may be (3) perpetual for the subject's being described in the way it is (*dawāma kawni l-mawḍū'i mawṣūfan bi-mā wuḍi'a ma'ahu*), as in *all mobile things are changing*; this is not to be taken to assert that this is the case absolutely, nor for the time [the subject] exists as a substance, but rather while the substance of the moving thing is moving. Distinguish between this condition and the first condition, because the first has set down as the condition the principle of the substance, 'man', whereas here the substance is set down with a description (*ṣifa*) that attaches to the substance, 'moving thing'. 'Moving thing' involves a substance (*ḏāt wa-ǧawhar*) to which movement and non-movement attach; but 'man' and 'black' are not like that.
>
> Or it may be a condition (4) of the predicate; or (5) of a definite time, as in an eclipse; or (6) of an indefinite time, as in breathing. [Avicenna, 1971[2], 264–266]

This passage draws on earlier Peripatetic writings (for an analysis of its probable sources, see [Bäck, 1992]), and it is best understood as the way Avicenna laid out various modal notions. Avicenna's interests were, with one exception, exclusively in propositions read under the second condition (the *ḏātī*, or substantial reading) and the third (the *waṣfī*, or descriptional reading), but later Avicennan logicians also investigated the fifth (the *waqtī*, or temporal) and the sixth (the *muntashir*, or spread). (See the renditions in appendix two at page 592 below; $\mathcal{E}$ renders the *ḏātī*, C the *waṣfī*, and $\mathcal{T}$ and $\mathcal{S}$ the fifth and sixth *waqtī* readings.)

I think that the *ḏātī* reading, although it turns on a distinction different from the one which delivers Abelard's divided reading, is functionally the same as the divided. Two examples may help clarify the distinction between it and the *waṣfī*. *All bachelors are necessarily unmarried* is true as a *waṣfī*, because 'bachelors' picks out men just while they are unmarried: *all men while bachelors are necessarily unmarried*. As a *ḏātī*, however, it is false: all bachelors are men, and it is untrue that all men are necessarily unmarried. By contrast, (and this is the most common Avicennan example) *all who sleep wake* is true as a *ḏātī* (because every animal that sleeps also wakes up from time to time), but false as a *waṣfī* (because nothing can be awake while sleeping). *No As are Bs while As*, Avicenna claimed in *Pointers*, would convert as *no Bs are As while Bs*, and would contradict *some As are Bs while As*; and would save the second figure for the account in the *Prior Analytics*. Avicenna took great pride in the fact that his two readings of the propositions allowed him to square a set of examples where other logicians had failed (see also page 578 below).

> The people who went before us were not able to reconcile us to their view by their examples and usage. The explanation of this is lengthy. [Avicenna, 1971[2], page 314]

Avicenna later in his work also investigated the way a *waṣfī* major and a *ḏātī* minor function in a syllogism, but those investigations deserve extended study, and are beyond the scope of this chapter (though see now [Thom,]). Later logicians challenged Avicenna's claims for the way the *waṣfī* contributes to an inference (see below page 575 f.), and developed his insights extensively; even later, they included his extrinsic temporal conditions in their investigations.

And the syllogistic with purely *ḏātī* premises? Avicenna developed his *ḏātī* modal syllogistic as two isomorphic systems, one using temporal propositions (functioning in contradiction and conversion like propositions 5, 10 and 24 in the appendix) and the other using modal propositions, which function in contradiction and conversion like these (the numbers indicate the propositions in appendix two which replace Avicenna's modals in later logical writings):

1.* $(\forall x)[\Diamond A_x \supset \Box B_x]$

13.* $(\forall x)[\Diamond A_x \supset \Diamond B_x]$

26.* $(\forall x)\{\Diamond A_x \supset [\Diamond B_x \,\&\, \Diamond \sim B_x]\}$

But Avicenna wanted the two sub-systems to interract in ways that show that these renditions are not right—for example, Avicenna wanted syllogisms with possible minors, in particular Barbara XMM, yet argued that absolute a- and i-propositions convert as absolute propositions. The system deserves serious study (for a description of the whole system with *ḏātī* premises, see [Street,]).

It is important to note that Avicenna, though using the perpetual to provide a contradictory for the absolute, did not investigate how the perpetual contributes to other inferences. I think that Avicenna wanted to provide a syllogistic that looks like it is treating only the propositions that Aristotle examined. Later logicians were far less concerned with preserving that sort of contact with the Aristotelian tradition, and investigated the perpetual as a fully-fledged member of their set of propositions. Working out Avicenna's system is the first and major problem for the study of medieval Arabic logic. It may be complicated by non-logical factors: Avicenna saw himself as a second Aristotle, and it may well be that certain features of his own logic are tests set to puzzle his readers.

3.6 *Baghdad and the East*

The history of post-Avicennan logic is the history of the eventual conquest of a system derived from Avicenna's system over the logic taught in the Baghdad school. In this section I offer a few reflections on the respective provenance and strengths of each tradition.

Firstly, I think that there are some grounds to believe that Avicenna was not offering an entirely new system to which people had to be converted, but was merely setting in sharper format a system which was already broadly accepted in Khurasan. At least on

my reading of *Pointers*, Avicenna only claimed as his personal contributions to the formal logic he presented the following: the coinage of the term *wuǧūdiyya*, the use of the *waṣfī* as a plausible way to save Aristotle's position on the absolute, and the division of the syllogistic into conjunctive and exceptive [Avicenna, 1971[2], respectively pages 309, 314, 374]. The last two contributions ramify through the rest of the system. Even without them, however, the fact remains that the absolute would be treated quite differently from the way Alfarabi had treated it, and attention would be paid to a body of hypothetical syllogistic ignored by the Baghdad philosophers. This is at least compatible with the conclusion that the logic studied in Khurasan was quite different from Baghdad logic before Avicenna arrived. Avicenna came to an Aristotle mired in nearly one and a half millenia of interpretations, and the specificities of the tradition in Khurasan may have been paramount in determining what he did with Aristotle, and perhaps also in determining what those who came after him did. This speculation is not meant to deny that Avicenna's formulation of that logic was the strictest, and the one to which subsequent logicians, both friendly and unfriendly to the project, had first recourse.

Other reasons for the wide acceptance of the Avicennan tradition of logic have to do with the general fortunes enjoyed by the larger philosophical system Avicenna put forward. That system, having been presented at many points in Avicenna's writings as congenial with Islam, proved to be so adaptable to the needs of Islamic philosophical theology that by the beginning of the twelfth century, people understood by 'philosophy' Avicenna's synthesis. Even more generally, although Avicenna was born into the dying days of one dynasty (the Samanids), he moved and worked through the halcyon realms of the dominant force of the era, the Buyids. Baghdad had lost much of the political and cultural prestige it had enjoyed in the tenth century, and the dynamics of political hegemony in the Islamic world were driving from the East through to the West.

Finally, it must be borne in mind that although places like Khurasan and other eastern realms quickly became almost wholly Avicennan, what that means is much more complex than appears on the face of it. None of these logicians, so far as I am aware, adopted the Avicennan syllogistic in its entirety, though most adopted Avicenna's three most characteristic doctrines. So these logicians nearly all agreed that the syllogistic divides broadly into conjunctive and exceptive; they further agreed that the hypothetical syllogistic is important, and nearly all devoted analyses to aspects of the hypothetical syllogistic. That said, they did not necessarily agree on precisely what it is that matters most about the hypothetical syllogistic, or how exactly it relates to the categorical syllogistic. Secondly, the logicians in these regions nearly all delivered a syllogistic system that uses an Avicennan absolute proposition. Thirdly, they all investigated propositions read under some or all of Avicenna's conditions. This is the sense in which these logicians are Avicennan.

Baghdad logic, by contrast, did not prosper. One thing that did not limit its influence was the fact that it was a Christian school in an Islamic society, constituted mainly by Christians, with only a few Muslims such as Alfarabi and Abū-Sulaymān as-Siǧistānī among its members. Avicenna's disciples included a Zoroastrian and a Christian; philosophy throughout the Islamic world tended to be accepted, by those people who accepted it at all, as a discipline which would attract people from various faith-communities. Again, it was not internal dissension that weakened Baghdad; in spite of the sectarian differences

between the various Syriac confessions, it appears that in Baghdad a collegial and cordial spirit prevailed [Zimmermann, 1981, page cxii]. In fact, Christian and Muslim logicians faced parallel opposition within their respective faith-communities, and the *apologia* for logic was a genre they were both forced to write (for a Christian example, see [Rescher, 1963f]).

Baghdad was unlucky in the successors it had to Alfarabi. Yaḥyā ibn-ʻAdī wrote extensively and competently on logic, although he apparently argued that modal logic was ill-conceived [Endress, 1977, chapter 3, and page 51]. The only Baghdad philosopher Avicenna admired was Alfarabi; the rest of the Syriac Christians he dismissed as wooden-minded in logic (see above page 535). He famously and witheringly said of Ibn-aṭ-Ṭayyib, his contemporary and head of school in Baghdad, that his work was best sent back to the bookseller, whether or not a refund was offered [Gutas, 1988, page 68]. Alfarabi had been a continuation of the Syriac tradition, but Ibn-aṭ-Ṭayyib was a mere replication of it. (Of course, this is merely to repeat Avicenna's judgement and, as Lameer reminds us, we have yet properly to check Ibn-aṭ-Ṭayyib's writings; see [Lameer, 1996, page 96].) Still, for all the undoubted decline in its philosophical fortunes, Baghdad still produced a considerable logical posterity. The Andalusian Muḥammad ibn-ʻAbdūn (d. c. 995) came to Baghdad to study with Abū-Sulaymān, whereupon he went back to Spain and inaugurated a tradition of logicians who can only be understood against the tradition of Alfarabi. In fact, what modal logic we can guess was being taught in Baghdad was probably the modal logic we find in early Spain (see below pages 561 & 567).

For the most part, the Baghdad school continued to concentrate on a range of tasks among which exegesis, or really, summary, figured prominently. If Galen's logic was still read at all, it was here in Baghdad—Avicenna had followed Alexander, and dismissed him as "the man who was strong in medicine but weak in logic" ([Shehaby, 1973b, page 5]). But even in Baghdad teaching changed after Avicenna. Avicenna's philosophy was the most significant intellectual challenge the Baghdad philosophers had to face, and even during Avicenna's lifetime, they tried to meet that challenge. After Avicenna a new strain is apparent in Baghdad logic. References and reflexes can be found in writings from Baghdad in which the Avicennan position is set down and then dismissed. This apologetic becomes a new theme in the Baghdad school, and subsequently in Spain. In the end, however, even among the Syriac Christians, Avicenna's system prevailed (see e.g. [Jannssens, 1937]).

4 LOGIC AND THE ISLAMIC DISCIPLINES

The Islamic disciplines include law and jurisprudence, Koranic exegesis, analysis of traditions relating to the Prophet, theology and grammar. Together these disciplines function to determine the Islamicity of the spiritual and public life of a society. The central doctrines and techniques of these disciplines reached what later came to be considered their classical formulation by the end of the second Abbasid century, which was about the same time that a truly naturalized Arabic logic was being achieved. People began to ask whether logic was a discipline constituted in ways similar to the Islamic disciplines, whether it was useful for the Islamic disciplines, whether, indeed, it was even compatible

with them. Some of Alfarabi's logical writings are attempts to answer these questions: he linked logic into an ancient tradition, and showed great concern to make its technical terms perspicuous to speakers of pure Arabic, both important matters in the constitution of an Islamic discipline; he was concerned to make the forensic utility of logic obvious; and he stressed its parallels to grammar (see above page 536 ff.).

The single most important voice in the arguments over the centuries about logic and its relation to the Islamic disciplines is that of the famous jurist and theologian, Abū-Ḥāmid al-Ġazālī (d. 1111). Some fifty or so years after Avicenna's death, Ġazālī argued that logic was both licit and useful for theology and jurisprudence. In this he was following the example of Alfarabi. The clarity of Ġazālī's prose style and the depth of his spiritual insights have won him enormous prestige in the Islamic community. His arguments that there is nothing inimical to religious belief in logical studies derived much of their force from that prestige. He did not end the attacks on logic, but his arguments in support of logic are probably the most decisive factor in its inclusion as a subject for study in the *madrasa*.

Logic and grammar What sort of resistance did logic face? The most frequently cited example of the clash between the Islamic disciplines and logic is the debate in Baghdad conducted between Abū-Sa'īd as-Sīrāfī (d. 978) and Alfarabi's senior colleague, Abū-Bišr Mattā (d. 940). The study of this debate is now a something of a minor industry within the field of Islamic studies, and it is undoubtedly important for revealing how logic was received among some of the educated classes of Baghdad at the end of the translation movement described in section 2 above. The debate was convened by the vizier, who invited Abū-Bišr to defend the value of Aristotelian logic relative to that of the Arabic grammatical tradition. Abū-Sa'īd, a young but promising grammarian, stepped forward to put the case for grammar, and won the debate by acclaim, humiliating Abū-Bišr in the process. It should be said, however, that important as the debate may have been, and much studied as it is, there continues to be scholarly disagreement as to what point it is that Abū-Sa'īd was trying to make. One account of Abū-Sa'īd's attack has him defending the ambient Platonism of Baghdad against the rising peripateticism of Abū-Bišr and his colleagues [Mahdi, 1970]; more commonly, he is taken to represent the practitioners of the Islamic disciplines and their worries about the far-reaching claims made for logic ([Elamrani-Jamal, 1983, pages 61–71]; briefly in [Arnaldez, 1960–], at length in [Endress, 1986]). It has also been pointed out that we may be trying to extract more from the debate than the occasion of its convention (an amusement for the vizier) allows [Frank, 1991]; whatever the specific points in Abū-Sa'īd's arguments, one is left with the impression that logic is not so much dangerous as merely laughable.

In the debate, we find Abū-Sa'īd again and again chiding Abū-Bišr for his bad Arabic, and for his naive confidence that knowledge of logic can somehow protect him from error in thinking and end dispute in philosophy.

> The world remains after Aristotle's logic as it was before his logic. Resign yourself, therefore, to dispense with the unattainable, since such a thing is wanting in the creation and nature of things. If, therefore, you were to empty

> your minds of other things, and devote your attention to the study of the language in which you are conversing and disputing with us, and instruct your friends in words which the speakers of that language can understand, and interpret the books of the Greeks in the style of those who know the language, you would learn that you can dispense with the ideas of the Greeks as well as you can dispense with the language of the Greeks. [Margoliouth, 1905, pages 115–116]

Abū-Sa'īd's most stinging taunts were directed against the technical jargon that Abū-Bišr and his colleagues were using, which Abū-Sa'īd showed to be more a hindrance than a help to clear thinking. He also mocked the claims the philosophers made for the importance and utility of logic. Among other things, he cast doubt on the coherence of their claims that there is a higher grammar, common to all languages.

Logic and theology The theologians of tenth-century Baghdad also studied argument-forms, and had an elaborate set of terms to classify arguments as good or bad. These terms, however, map so precisely onto Stoic terms and function in such a similar way that it has been concluded that theological logic almost certainly derives from Stoic logic [van Ess, 1970]. That said, the process by which Stoic logic came to Baghdad is far from clear, and it is only on the grounds of structural and terminological similarity that the claim can be made. (To get an idea of how speculative these assessments of Stoic origin are, see [Shehaby, 1973a] and especially the discussion following it; see now [Gutas, 1994].) Whatever their origin, theologians had methods to evaluate arguments which were not Aristotelian.

Still, it is one thing to have a system for evaluating arguments, but quite another to say that no other system should be studied; yet that is what some theologians did argue. We can get some idea of why they did so from the great fourteenth-century intellectual historian, Ibn-Ḫaldūn (d. 1406). Ibn-Ḫaldūn had himself written a short treatise on logic in his younger days, and there are many references to logic running through his *Prolegomena* [Ibn-Ḫaldūn, 1958]. He was, in short, an interested and quite probably competent witness to the state and history of logic in his time. The passage he wrote devoted solely to logic divides into two sub-histories, one on the Organon in the Islamic world, and the other on the tensions between the logicians and the theologians.

> It should be known that the early Muslims and the early speculative theologians greatly disapproved of the study of this discipline. They vehemently attacked it and warned against it. They forbade the study and teaching of it. Later on, ever since Ġazālī [d. 1111] and Faḫra ddīn ar-Rāzī [d. 1210], scholars have been somewhat more lenient in this respect. Since that time, they have gone on studying logic, except for a few who have recourse to the opinion of the ancients concerning it and shun it and vehemently disapprove of it. Let us explain on what the acceptance or rejection of logic depends, so that it will be known what scholars have in mind with their different opinions... ([Ibn-Ḫaldūn, 1858, page 113.13–u]; cf. [Ibn-Ḫaldūn, 1958, pages 143–144])

Ibn-Ḫaldūn went on to give a short history of Islamic theology, the arguments that it developed in defence of the articles of faith, and the atomistic and nominalist metaphysics which was simultaneously refined to support those arguments, that is to say, classical kalām atomism.

> It then came to be the opinion of Aš'arī, Bāqillānī, and Isfarā'īnī [famous exponents of the classical kalām], that the evidence for the articles of faith is reversible in the sense that if the arguments are wrong, the things proven by them are wrong. Therefore, Bāqillānī thought that the arguments for the articles of faith hold the same position as the articles of faith themselves and that an attack against them is an attack against the articles of faith, because they rest on those arguments. ([Ibn-Ḫaldūn, 1858, page 114.13–16]; cf. [Ibn-Ḫaldūn, 1958, pages 144–145])

But Aristotelian logic, Ibn-Ḫaldūn went on, assumes the five universals and the commonplaces for topical reasoning, and assumes further that they have an extramental existence. This assumption is incompatible with the theologians' denial that universals have a real existence. If the theologians are right, then

> ...all the pillars of logic are destroyed. On the other hand, if we affirm their existence, as is done in logic, we thereby declare wrong many of the premises of the theologians. This, then, leads to considering wrong their arguments for the articles of the faith, as has been mentioned before. This is why the early theologians vehemently disapproved of the study of logic and considered it innovation or unbelief, depending on the particular argument declared wrong by the use of logic.
>
> However, recent theologians since Ġazālī have disapproved of the idea of the reversibility of arguments and have not assumed that the fact that the arguments are wrong requires as its necessary consequence that the thing proven by them be wrong. They considered correct the opinion of logicians concerning intellectual combination and the outside existence of natural quiddities and their universals. Therefore, they decided that logic is not in contradiction with the articles of faith, even though it is in contradiction to some of the arguments for them. In fact, they concluded that many of the premises of the speculative theologians [who followed classical kalām] were wrong. For instance, they deny the existence of atomic matter and the vacuum and affirm the persistence of accidents and so on. For the arguments of the theologians for the articles of the faith, they substituted other arguments which they proved to be correct by means of speculation and syllogistic reasoning. They hold that this goes in no way against the orthodox articles of faith. This is the opinion of Rāzī, Ġazālī, and their contemporary followers. ([Ibn-Ḫaldūn, 1858, page 115.13–116.8]; cf. [Ibn-Ḫaldūn, 1958, pages 146–147])

In other words, Ġazālī and Rāzī were worried at equating the credibility of their faith with the credibility of kalām atomist theory. It is the theological decision to overturn

the theory of the reversibility of arguments which found space for logical studies in a theological education.

That at least is Ibn-Ḫaldūn's account. It suffers from some problems which cannot be considered here (though compare the account in [Marmura, 1975]). In any event, Ġazālī not only argued in defence of logic, and used it in his theological works; he went further and argued in *The Just Balance* [Ġazālī, 1959] (translated [Brewster, 1978; McCarthy, 1980], and studied [Kleinknecht, 1972]) that the Koran sanctioned its use, if in slightly coded language. But the fact that theologians condoned it may not have been the deciding factor in the acceptance of logic.

Logic and law The law is sometimes said to be the most important of the Islamic disciplines. Lawyers had been reflective about their system of reasoning, and analogy (which they called *qiyās*, the same word used by the logicians for 'syllogism') as well as other techniques were the centre of a series of sophisticated discussions. Ġazālī was first and foremost a lawyer, and held his chair in Šāfi'ī jurisprudence at the Niẓāmiyya in Baghdad from 1091. By showing legal arguments ultimately to depend on the syllogism, and by prefacing his last juridical summa, *The Distillation of the Principles of Jurisprudence* [Ġazālī, 1938], with a logical treatise, Ġazālī did more than any other earlier scholar to have logic made part of *madrasa* studies.

In *Distillation*, Ġazālī referred to two of his earlier works on logic, *The Touchstone for Speculation* [Ġazālī, 1966] and *The Yardstick of Knowledge* [Ġazālī, 1961b], both of which he wrote after writing his famous *Intentions of the Philosophers* [Ġazālī, 1961a]. *Intentions* is in fact a pretty close Arabic paraphrase of Avicenna's Persian *Philosophy for 'Alā'addawla*, which contains a very elementary treatment of logic (English translation, [Avicenna, 1971]). In assessing Ġazālī as a logician it must be said that, from a formal point of view, he never rises above *Intentions*.

The first section of *Touchstone* follows the structure of *Intentions*, but in the second and third sections, Ġazālī's interest in cognitive aspects of premises, in the pragmatics of argument, and in legal problems, comes to the fore. Throughout the book, there is a concern to find new ways of putting the philosophers' terms of art, ways that correspond to terms used in the Islamic disciplines (though Ġazālī was careful to point out when there are differences between logical and grammatical usage). In *Touchstone*, Ġazālī had more fully than any other logician up to that time gone into the problems of naturalising logical terminology; and he had more comprehensively shown how it can contribute to the pragmatic needs of juristic reasoning. At the end of *Touchstone*, Ġazālī advised his readers to go to his *Yardstick of Knowledge* for fuller treatment of the material covered. *Yardstick* does indeed give a much fuller exposition of the subject, with all the technical terms normally used by the philosophers. None the less, the goals of *Yardstick* are identical with those of *Touchstone*:

> We shall make known to you that speculation in juristic matters (*al-fiqhiyyāt*) is not distinct from speculation in philosophical matters (*al-'aqliyyāt*) in terms of its composition, conditions, or measures, but only in terms of where it takes its premises from. [Ġazālī, 1961b, page 28.2–4]

The questions that were important for Avicenna in his reading of Alfarabi, and which came to be important generally for the major logicians of the post-Avicennan tradition (as examined in section 5 below), did not matter at all for Ġazālī. Even the more advanced *Yardstick*, though mentioning the distinction between *ḏātī*, *waṣfī* and temporal (*waqtī*) readings in propositions, and the modalities [Ġazālī, 1961b, pages 88–90], never considers how they contribute to an inference. There is never a doubt raised about whether the unmodalised proposition (the only kind Ġazālī considered) will function like the assertoric in the early books of the *Prior Analytics*. The relation between the categorical and hypothetical syllogistic is treated insouciantly (the categorical "is sometimes called an *iqtirānī* syllogism, sometimes a *ǧazmī*" [Ġazālī, 1961b, page 98.14–apu]), the hypothetical syllogistic is exemplified only by unanalysed propositions [Ġazālī, 1961b, pages 111–114], and the deduction involving a contradiction is treated without any reference to the hypothetical syllogistic [Ġazālī, 1961b, page 114].

I think a few conclusions may be drawn from these considerations. For all Ibn-Ḫaldūn says that theological reasons made logic acceptable to Ġazālī, juridical considerations seem more significant. Ġazālī's contribution to logic was mainly on the level of defusing objections to its study by domesticating its jargon, and showing by legal examples its utility [Hallaq, 1990, page 315]. Although Ġazālī is sometimes said to be Avicennan [Rescher, 1964, page 49], this is true only in an attenuated sense. Even though his treatises derive their logical content from *Philosophy for 'Ala'addawla*, his significant work was done in the spirit of Alfarabi's apologetics. He is in this sense more Farabian than Avicennan. Further, his work was not paradigmatic for later theologians. Other theologians began to study logic from the twelfth century onwards, but they did not all study it in the same way, or for the same purposes. This is particularly clear from a comparison of Ġazālī's interests with those of Rāzī (see below page 572 ff.). Ġazālī was a promoter of logic, but not a practitioner. Other, later theologians were often both.

Continued oposition to logic Ġazālī's achievement was not the end of all opposition to logic among Muslim scholars, though future attacks on logic never seriously affected the study of the discipline. We find pious opposition taking a number of forms. One famous example is a legal opinion issued by Ibn-aṣ-Ṣalāḥ (d. 1245) on the reprehensibility of logical studies.

> As far as logic is concerned, it is a means of access to philosophy. Now the access to something bad is also bad. Preoccupation with the study and teaching of logic has not been permitted by the law-giver, nor has it been suggested by his Companions or the generation that followed him, nor by the learned imams, the pious ancestors, nor by the leaders or pillars of the Islamic community whose example is followed. God has protected them from its danger and its filth, and cleansed them of its uncleanness. The use of the terminology of logic in the investigation of religious law is despicable and one of the recently introduced follies. Thank God, the laws of religion are not in need of logic. Everything a logician says about definition and apodictic proof is complete nonsense. God has made it dispensable for those

> who have common sense, and it is even more dispensible for the specialists in the speculative branches of jurisprudence. [Goldziher, 1981, pages 205–206]

Perhaps the most famous opponent of logic is the fideist jurist, Ibn-Taymiyya (d. 1328), who was writing near the end of the period covered in this chapter. Ibn-Taymiyya wrote a lengthy condemnation of the use of logic, *Refutation of the logicians*, which in a later abridged form has been translated into English [Hallaq, 1993]. Logic is merely distracting where sound intuition can hit the mark; so much for the claims made about the universal utility of logic. On the more specific claim that syllogistic reasoning is valuable for jurisprudence, Ibn-Taymiyya has nothing but derision. In fact,

> their distinction between a categorical syllogism and analogy—that the former is capable of leading to certainty while the latter to nothing but probability—is invalid. In fact, whenever one of them leads to certainty so does the other, and whenever one of them leads to nothing but probability the other does likewise. When the evidence results in certainty or in probability, this is so not because its form is syllogistic and not analogical, but rather because the syllogism contains conclusive evidence. If either an analogy or a syllogism encompasses a matter that entails a certain judgement, then certainty is attained. [Hallaq, 1993, page 125]

For all his contempt for logic, Ibn-Taymiyya never impugned the formal aspects of the syllogistic, which had come by his day to be the major focus of the logical treatise. But what is the value of this formal study?

> The validity of the form of the syllogism is irrefutable ... But it must be maintained that the numerous figures they have elaborated and the conditions they have stipulated for their validity are useless, tedious, and prolix. These resemble the flesh of a camel found on the summit of a mountain; the mountain is not easy to climb, nor the flesh plump enough to make it worth the hauling. [Hallaq, 1993, page 141]

5 LOGIC AFTER AVICENNA

As has been noted (see page 552 f. above), Avicenna came to exercise an extraordinary influence over subsequent generations of philosophers. For many logicians, their work has to be understood as an attempt to extend or modify the Avicennan system. These logicians no longer referred to the *Prior Analytics* as they went about their tasks, but to *Pointers and Reminders*. Again, as has been noted, I refer to a logician as 'Avicennan' if he adopted the three central modifications Avicenna introduced into the formal syllogistic: the Avicennan truth-conditions for the absolute proposition; the readings under which Avicenna read modal and temporal propositions; the division of the syllogistic into conjunctive and exceptive syllogisms—though this way of deciding whether or not a logician is Avicennan has the consequence of making quite a few logicians Avicennan who none the less direct trenchant criticism against Avicenna. The Avicennan logicians contrast most starkly with those logicians for whom the primary task was the recovery of a

true Aristotelianism; Averroes was the major, though not unique, representative of this tradition. But by the time Averroes came to grapple with the Aristotelian logical texts, the Avicennan system was so dominant that the points to which Averroes had to direct most of his exegetical energies had been determined for him by that system. Averroes was something of an Avicennan in spite of himself.

There are other approaches to the writing of logical treatises in this period which are less easily categorised. Some of the Syriac Christians continued to write logical commentaries much as they always had, seemingly with little or no reference to Alfarabi or Avicenna. Other logicians in Baghdad in the twelfth century certainly had access to Avicenna's writings, and Alfarabi's manuscripts may well still have been available in Baghdad in the twelfth and thirteenth centuries. Given the few and stereotypical references to Alfarabi among the later logicians working in Iran and further east, however, we must wonder whether his manuscripts were available there. A tradition of logical studies in Spain directed much of its attention to Alfarabi's writings, and perhaps had less access to the Avicennan texts. By the end of the twelfth century, the only traditions that really mattered were the Averroist and the Avicennan. But by the end of the thirteenth century, only the Avicennan mattered.

5.1 Spain and the Averroist project

Early logical studies in Spain Logic was first taught in Muslim Spain, so the biobibliographers would have it, when a Andalusian, Muḥammad ibn-'Abdūn (d. c. 990), studied in Baghdad under Abū-Sulaymān as-Siǧistānī, and then returned to his homeland to start teaching the subject there (see generally [Dunlop, 1955]). Ibn-'Abdūn was among the teachers of Abū-'Abdallāh al-Kattānī (d. 1029) who in turn was one of the teachers in logic of Ibn-Ḥazm (d. 1064), a man more famous for his work in poetry, jurisprudence and theology, than for his logic. Still, his book, *An approach and introduction to logic* (*at-Taqrīb li-ḥudūd al-manṭiq wa-madḫaluhu*), is interesting because it is another effort to make Aristotelian logic acceptable to Muslim jurists (for a summary, see [Chejne, 1984]). This is very like the project of Ġazālī, and of Alfarabi. But Ibn-Ḥazm was, like Ġazālī, more a religious scholar than a logician, and his contemporaries and immediate successors tended to belittle his logical achievements. Ibn-'Abdūn was also among the teachers of the teachers of Ibn-Bāǧǧa (d. 1138) and Averroes (d. 1198), an altogether more glorious line of logicians.

Before turning to Ibn-Bāǧǧa and more particularly Averroes, it is interesting to note a treatise we have by one of Ibn-Bāǧǧa's contemporaries, Abū-ṣ-Ṣalt of Denia (d. 1134). Abū-ṣ-Ṣalt's presentation of the Aristotelian modal system in his *Setting minds straight* has been edited by González Palencia and its results noted by Rescher [González Palencia, 1915; Rescher, 1963a]. Abū-ṣ-Ṣalt stated the assertoric syllogistic [González Palencia, 1915, pages 20–29], then developed his modal syllogistic by setting down exactly the same moods and mixes we find in Aristotle, in exactly the ordering of the *Prior Analytics* [González Palencia, 1915, pages 29–46] (with two exceptions, being uniform necessity moods and mixed necessity and problematic moods; but Abū-ṣ-Ṣalt explained how to get an Aristotelian conclusion for each). Abū-ṣ-Ṣalt's treatment of conversion

[González Palencia, 1915, page 21] betrays no concern for the Avicennan counterexamples. He devoted two pages to the hypothetical syllogistic, giving it only with unanalysed propositions [González Palencia, 1915, page 46–47]; and he did not call on it to explain the workings of a deduction involving a contradiction [González Palencia, 1915, page 48]. I think his treatise gives us an opportunity to see the standard treatment of the later Baghdad school, and of the pre-Averroist school in Spain. It is nothing more than a set of notes summarizing early parts of the *Prior Analytics*.

Ibn-Bāǧǧa was regarded by no less than Ibn-Ḫaldūn as a philosopher of the calibre of Avicenna, Alfarabi and Averroes. He was seen by later scholars as inaugurating a new and more rigorous era of logical studies in Spain. I do not think that the present state of the field is such that we are able to judge Ibn-Bāǧǧa's logical writings. None the less, he obviously consecrated a great deal of effort to writing commentaries on Alfarabi's logical works (see for example [Alfarabi, 1986a]; a brief summary is given [Gutas, 1993, pages 54–55]). In this he prepared the ground for Averroes' early logical training.

Averroes and the logical tradition It is beyond dispute that the major logician writing in Muslim Spain was Averroes. Averroes has a vast output. Some of his work was directed to the familiar task of showing the study of logic to be not only licit but actually incumbent on Muslims [Hourani, 1961, pages 44–47]. Specifically on the *Prior Analytics*, we have a middle length commentary and a collection, *The Essays* [Averroes, 1983b], which address specific problems in the Aristotelian tradition. (For his logical writings, see [Gutas, 1993, pages 55–56].) *The Essays* are very focused, and what follows derives from them.

The main point I hope to emerge here is what it means to say that Averroes wrote in the Farabian tradition. It is much harder to consider Averroist logic (and Averroist philosophy generally) than Avicennan logic, because Averroes was constantly revising his system. Throughout his career, and even more ardently at the end, Averroes was trying to preserve the insights of Aristotle. In the *Essay on the modalities of conclusions following from the modalities of premises* [Averroes, 1983b, pages 176–186], Averroes wrote:

> These are all the doubts in this matter. They kept occurring to us even when we used to go along in this matter with our colleagues, in interpretations by virtue of which no solution to these doubts is clear. This has led me now (given my high opinion of Aristotle, and my belief that his theorization is better than that of all other people) to scrutinize this question seriously and with great effort. [Averroes, 1983b, page 181.6–10]

As a corollary of this constant revision, Averroes was changing his opinion and assessment of Alfarabi. This can obscure the extent to which his work derives from that of Alfarabi. Even though Averroes came to his logic by way of the Farabian treatises, he went on to distance himself from Alfarabi as his sense of the Aristotelian tradition began to emerge more clearly. In his early days, Averroes wrote in one of his short works on Physics that people wanting to understand the discipline should first learn some logic, preferably from one of Alfarabi's books [Elamrani-Jamal, 1995, page 51]. But at the end of his scholarly life, Averroes had reached a different assessment of Alfarabi, an assessment which we find in his *Essays*:

> One of the worst things a later scholar can do is to deviate from Aristotle's teaching and follow a path other than Aristotle's—this is what happened to Alfarabi in his logical texts, and to Avicenna in the physical and metaphysical sciences. ([Averroes, 1983b, page 175.6–8]; cf. [Elamrani-Jamal, 1995, page 52])

Averroes, that is to say, decided that Alfarabi was not Aristotelian enough (at this specific point, due to Alfarabi's ampliation of the subject-term and the resulting misformulation of the *dictum de omni*; see above page 549 f.).

As I say, Averroes' statements about Alfarabi can be misleading. As will emerge in the next two sub-sections, Averroes followed Farabian inspiration for important elements that feature in all of his syllogistic systems, and that become characteristic of systems which may be termed Averroist. We are faced with the irony that Avicenna claimed Alfarabi as his only worthy predecessor writing in Arabic, and then differed from him in every major point in the syllogistic, while Averroes upbraided Alfarabi's logical mistakes, but developed ideas he found in Alfarabi's writings into some of his most influential contributions to logic.

Averroes on absolute and modal propositions In two essays in particular, Averroes may be seen to be working under Farabian inspiration, and against the Avicennan system. One of these essays is given by its editor the title, *A criticism of Avicenna's doctrine on the conversion of premises*, the other, *On the absolute proposition*.

In his essay on the conversion of propositions, Averroes developed his distinction between reading a proposition, and more specifically a term, as either *per se* (*biḏ-ḏāt*) or *per accidens* (*bil-'araḍ*) (a distinction noted and studied in [Lagerlund, 2000, pages 32–35] and [Knuuttila, 1982, pages 352–353]). This strategy is motivated in the first place by the conversion of the contingent proposition, which, to preserve the Aristotelian claim, has to convert as a contingent proposition. The discussion opens by considering the counterexample *all men are contingently writing*, which on the face of it should convert to *some who write are necessarily men*. Averroes developed his solution, and then went on to consider the parallel discussion between Avicenna and Alfarabi, which was directed to the counterexample *all animals are contingently sleeping* (see above page 550). Alfarabi tried to save the conversion as a contingent proposition by reading it with a reduplicative phrase: *all animals are contingently sleeping in so far as they are sleeping*. Avicenna rejected this move, arguing that the proposition with the reduplicative phrase is not the same as the original proposition which was to be converted. Averroes argued that, on the contrary, the original proposition implicitly contains the reduplicative phrase,

> because the animal can only be sleeping in so far as it is sleeping and not in so far as it is a horse or a donkey or the various other species which sleep. Since this is the case, the condition is implicit whether it is expressed or not. The two propositions are one and the same, I mean, that in which the condition is expressed and that in which it is not. The fact that the condition is part of the predicate is self-evident, because the animal is not contingently sleeping

in so far as it is actually sleeping; but rather, if it is [sleeping], then [it is so due to an aspect had] in potentiality.

If this is the case, then the one actually sleeping in so far as it is actually sleeping is contingently an animal. But it is accidental for it that if it is an animal necessarily in so far as it is sleeping potentially, then it is a necessary* animal. So this premise is necessary *per accidens* and contingent *per se*. So if we say *every [creature] sleeping is an animal*, and we understand from it every [creature] potentially sleeping, it is necessary *per se*; but if we understand from it every [creature] actually sleeping, then it is necessary *per accidens*, contingent *per se*. Since *every animal is contingently sleeping* has the sense that it is contingent that it is sleeping actually, not potentially, then were we to understand from it the one sleeping potentially, the premise is necessary not contingent. Thus one must understand in the conversion of the contingent the [creature] actually sleeping, and in the conversion of the necessary the [creature] potentially sleeping. This view is correct, and it contains the solution to the doubt raised regarding the conversion of the necessary as a necessary. [...]

As for the doubt raised relative to the necessary, the solution is known from what Alfarabi said regarding the possible.

This doubt had to be singled out for treatment due to the prominence of Avicenna's questioning of it. [Averroes, 1983b, pages 104.4–105.apu]

I think Alfarabi equivocated in his modal usage between the convertend and the converse, and I think that Averroes did too in his modified version of the solution (which I have omitted from the quotation above). Still, the distinction between *per se* and *per accidens* is important in logical systems inspired by Averroes, and this passage serves to show that Averroes followed Alfarabi in adopting the distinction, the operation of which he then extended.

Averroes was, in this important respect, Farabian. He was, more importantly, not Avicennan in his modal logic, which he built on top of the assertoric syllogistic. In his *Essay on the absolute proposition*, he set down Avicenna's conditions for reading a necessary proposition [Averroes, 1983b, pages 120.11–121.5] and how they relate to different definitions of the absolute; "this is all just confusion and disorder" [Averroes, 1983b, page 122.1]. Averroes' own positions on the assertoric as he conceived it over his career are too complex to be stated compendiously. Two aspects of his position may however be noted. The first is that Averroes wanted to keep the conversions set down for the assertoric in the early books of the *Prior Analytics*, and thought that the distinction between the *per se* and the *per accidens* reading would help him. The second is that he was forced ultimately to admit that Aristotle's examples of assertoric propositions could not all be fitted on to the same set of truth-conditions, and he came at one point to speak of three assertorics, the mostly-assertoric, the leastly-assertoric and the equally-assertoric [Averroes, 1983b, pages 117.apu–118.2]. Both of these aspects of the Averroist position were considered subsequently by at least one logician working within his tradition.

Averroes on the hypothetical syllogistic Like Avicenna, Averroes was concerned to investigate the interrelation between the categorical and hypothetical syllogistics. His essay on this, *Discourse on the categorical and hypothetical syllogistic, with a criticism of the conjunctive syllogistic of Avicenna*, is deep and complex, and awaits serious study. I want merely to note some superficial points regarding the lines of argument he developed to underline another broad aspect of his logic which is Farabian.

Firstly, and unsurprisingly, Averroes' major goal in the essay is to show that Aristotle was correct in his views on the hypothetical syllogistic, and that "these syllogisms are not to be analysed into the figures" ([Badawī, 1948/52, vol. 1, pages 217.u–218.2]; cf. *Prior Analytics* 50^b2–3). Averroes took himself to have proven that the hypothetical is indeed ineliminable and irreducible to the categorical, and his essay concludes:

> So it has become clear that every syllogism and every syllogistic discourse is either hypothetical or categorical or a compound of the two (and that is called reduction (*ḥalf*)) according to what Aristotle said in the *Prior Analytics*. And that is what we intended to explain. [Averroes, 1983b, page 207.apu–u]

But Averroes was not merely interested in proving Aristotle right. He also wanted to defend Aristotle against any charges of carelessness in not treating the hypothetical syllogistic more fully. He did not, according to Averroes, because the hypothetical can prove no primary Question, and is consequently redundant in scientific writing:

> For this reason, Aristotle discarded it and did not set it down in the *Prior Analytics*, since his primary intention in the *Prior Analytics* was to enumerate the syllogisms essentially useful in demonstration. [Averroes, 1983b, page 197.6–8]

There is much here with which Avicenna would have agreed, even though he belonged to the tradition which believed that the hypothetical had sufficient importance that Aristotle had written another treatise devoted to it.

Secondly, Averroes was able to deal with what Alfarabi had written on the hypotheticals briefly—he thought that Alfarabi was sloppy with his terms, and should have attended more carefully to an important distinction.

> It is clear from what we have said that the kinds of real hypothetical syllogisms are only syllogisms equivocally. The correctness of what Aristotle said emerges, that by them no unknown Question is made evident, and that they are properly part of the *Topics*. The commentators are agreed on this point. But their statements become confused when answering why [Aristotle] left the hypothetical syllogistic out of the *Prior Analytics*. What [Aristotle] said regarding them is that they do not produce a primary Question, and they belong properly to the *Topics*. It appears that this sense relative to the matter of the real hypothetical did not become clearly distinguished for them; we find Alfarabi saying in his *Posterior Analytics*: "As for those demonstrations composed in the hypothetical, the relation of their parts is the relation of those composed in the categorical." But the causes [for production] in the

> hypothetical are the repeated parts of the premises. These [inferences that Alfarabi is talking about] are not real hypotheticals, but merely hypothetical by equivocation. Since he did not distinguish this matter with regard to them, he was therefore not separated from this doubt. [Averroes, 1983b, page 197.9–18]

Lastly, Averroes was not prepared to recognize the Avicennan system of conjunctives and exceptives (see above page 545 f.). Some of his arguments have to do with epistemic matters, and are philosophically the most interesting part of the essay, though beyond the scope of this chapter. This part of the essay concludes:

> Most of the well-known book of this man is full of this sort of thing, both relating to matters logical, and to other matters. Whoever wants to begin in these arts should avoid his books, for they will mislead rather than guide him. [Averroes, 1983b, pages 199.u–200.3]

Averroes then moved on from epistemic claims to the division of the syllogistic into conjunctives and exceptives, first offering a summary of its propositions [Averroes, 1983b, pages 200.4–202.8]. He tried to show that the conjunctives all collapse into categoricals.

> The wonder is that Avicenna posited both these matters together, I mean, he conceded that every hypothetical premise can be reduced to a categorical premise (and similarly that every hypothetical Question can be reduced to a categorical), and then went on to posit that syllogisms composed of hypotheticals are different from syllogisms composed of categoricals. [Averroes, 1983b, page 205.18–21]

This is sufficient for present purposes: Averroes felt able to rectify the Farabian system, but was convinced that Avicenna's system puts forward redundant propositions which can more perspicuously be eliminated.

Averroist logicians I am not sure whether anyone has assembled all the elements of a system with which Averroes would have been content at one or other stage in his life. Important elements in four approaches he followed at various times in his career are presented in [Elamrani-Jamal, 1995], and one systematic overview has been given in [Manekin, 1993]. We know that structurally similar systems, especially that of Kilwardby, came to be important in the middle of the thirteenth century in the Latin West, although the route by which they got there is not entirely clear [Lagerlund, 2000, pages 32–35]. Debate still goes on about how Averroes' texts were transmitted [Burnett, 1999]; whether the Jews were the only path for that transmission matters rather less than the fact that they were one path. Whatever, Averroes' contemporary Maimonides (d. 1204) was not part of this process—Efros hesitates between whether Avicenna or Alfarabi was the greater influence on Maimonides [Maimonides, 1937/38, pages 19–21]. (I doubt whether the form of Maimonides' tract is such that we can ever really answer that question.) But later Jewish scholars such as Gersonides (d. 1344) were certainly reading Averroes, and adopted many, though not all, of his solutions.

> Levi ben Geshom said: Inasmuch as we saw some things in Aristotle's *Book of the Syllogism* as understood by the philosopher Averroes that appear to us to be incorrect—namely, in the conversion of modal sentences and the mode of the conclusion of modal syllogisms, simple and mixed—we have seen fit to investigate the truth of these matters in this book. [Manekin, 1992, pages 53–54]

Even more important than the lines of transmission is the fact that Averroes was transmissible at all. Because he addressed himself so directly to the Aristotelian corpus, he fell squarely within the problematic on which the Latin scholars were fixing their attention. This is why Averroes figured so much more in the West than the Avicennan logicians ever did. It was not a question of availability of texts, but of common interest. Latin and Hebrew writers, however, fall outside the confines of this chapter.

By contrast, Averroes rarely appears in later Arabic treatises on logic. One of those rare appearances is in an epistle on modal propositions which has been edited [El-Ghannouchi, 1971], but never studied beyond that edition, to the best of my knowledge. Ibn-Malīḥ ar-Raqqād, about whom we know nothing beyond the name, wrote an epistle, *On absolute, possible and necessary propositions*, in which he referred to Avicenna and Averroes and their solutions to the various problems mentioned above.

This is a strange little text, and I am not sure if it merely relays the stock Baghdad response to Avicenna's counterexamples, or something different and more developed. Whatever the provenance of its doctrine, the epistle is motivated by the fact that "people have raised doubts against Aristotle regarding the conversion of propositions, especially Avicenna" [El-Ghannouchi, 1971, page 207.u], doubts which can be laid to rest by inductively ascertaining the matter in Aristotle's examples and limiting his claims by these non-formal criteria. In answering Avicenna's objections, however, Averroes has been driven to distinctions which are unAristotelian. On one of Averroes' distinctions regarding the absolute, Ibn-Malīḥ says

> All those who sought to solve this problem imposed on it matters which are not fitting for the doctrine of Aristotle, especially Averroes. He did not conceive the absolute proposition [properly], and in consequence he made three kinds of absolute: the most-part, the least-part, and the in-between, as is the situation with the possible. [El-Ghannouchi, 1971, page 209.20–21]

These distinctions mean that the Averroist absolute "is not the absolute of the Philosopher" [El-Ghannouchi, 1971, page 209.24]—sufficient grounds to reject the Averroist absolute. The cult of Aristotle did not die with Averroes, at least not entirely; nor did its members entirely concur with Averroist doctrines.

5.2 *Rescher's 'Western school'*

This and the following section sketch the interests and pedagogical affiliations of the post-Avicennan logicians in Baghdad and the realms east of Baghdad. As mentioned in the introduction to this chapter, this is where I think Rescher's historical model of Arabic

logic is least helpful and needs to be set aside. I recapitulate the important elements of that account, because doing so provides the point of departure for this and the next sub-section. I offer what seems to me to be the correct version of developments below (see page 579).

Rescher's history of later Eastern logic According to Rescher, from the mid-eleventh century Avicenna's writings determined the course of philosophical doctrines and discussions in the realms east of Baghdad. But in the early twelfth century, a major philosopher, Abū-l-Barakāt al-Baġdādī (d. 1165), began systematically to challenge these doctrines. The spirit in which he did so is characterized as Farabian, which in turn made his logical writings somewhat resemble the work of the Andalusian logicians. Further, his impact was such in Baghdad and further east that one may speak of a school of logicians once again writing in the way of Alfarabi or, at least, challenging Avicenna's logic in ways congenial with Alfarabi's writings. One of the most important scholars to be influenced by Abū-l-Barakāt was Faḫraddīn ar-Rāzī (d. 1210), a prolific Western logician; he taught, among others, Afḍaladdīn al-Kāšī (d. c. 1213). So powerful was the influence of the Western refutation of Avicennan logic that it was not until Naṣīraddīn aṭ-Ṭūsī (d. 1274) that a convincing set of counter-arguments in support of Avicennan logic was produced. In constructing these counter-arguments, Ṭūsī was working within what Rescher calls the Eastern tradition, badly debilitated but holding on to solutions of, among others, 'Umar ibn-Sahlān as-Sāwī (d. 1145). Such was Ṭūsī's achievement that by the early fourteenth century there were two great schools, the Eastern and the Western. The reconciliation of these two schools in the fourteenth century by (among others) Quṭbaddīn ar-Rāzī at-Taḥtānī (d. 1365) was the most important logical event of the period.

There is an abbreviated variant of this history given in another study by Rescher [Rescher, 1967a], which dates the origin of the Western school to Rāzī. The abbreviated version may be rejected for a subset of the reasons that lead to the rejection of the longer account in [Rescher, 1964].

Obviously, if there had been a distinct Western school of logicians including Rāzī and Kāšī, this would be an important consideration in setting about the study of their logical writings. The account of the Western and Eastern schools, however, suffers from some problems, and considering these problems serves to reveal a more complex reality. Briefly, the problems are that (1) Abū-l-Barakāt was not simply reviving Farabian logical doctrine, or even mainly reviving such doctrine; (2) there is no record of a school originating with Abū-l-Barakāt; but anyway (3) Rāzī did not follow his doctrine, at least not in the modal or the hypothetical syllogistic. There is no pedagogical succession from which one may discern a Western school beginning with Abū-l-Barakāt and being carried on by Rāzī. In any event, the Eastern school was not clearly distinct from the Western in terms of doctrine; (4) Rāzī was closer, logically speaking, to Ṭūsī than to either Abū-l-Barakāt or Alfarabi. Further, (5) Rāzī was actually an important, if not the most important, route by which Ṭūsī came to receive and understand earlier logical writings—not merely the writings of Alfarabi, but those of Sāwī as well. Examining the way that the logicians in question relate to one another and to earlier scholars reveals a picture more complex than that of two schools clashing. In the remainder of this sub-section, I examine problems (1)

and (2) with Rescher's account noted above, and I examine the further problems in the following sub-section.

Abū-l-Barakāt al-Baġdādī's logic Abū-l-Barakāt al-Baġdādī (d. 1165), who is referred to by Rescher as Ibn-Malkā, was Jewish by birth, from a small town near Mosul. Abū-l-Barakāt moved to Baghdad and, in old age, he converted to Islam. His great work is *The tried and tested book*, apparently modeled loosely on Avicenna's *Cure*. He interacted with the Avicennan tradition in complex ways, some of which will appear in what follows. According to Tanakabunī, a nineteenth-century Persian writer, his work had a major impact on Faḫraddīn ar-Rāzī, most especially on Rāzī's *Eastern investigations* (*al-Mabāḥiṯ al-mašriqiyya*), a work which does not touch on the logic. Further, on this account, if Naṣīraddīn aṭ-Ṭūsī had not countered the writings of Rāzī, Avicennan philosophy would have been discarded [Pines, 1960–]. Whatever truth there may be in this account for the history of Islamic philosophy generally, it does not hold for the logic.

And so to aspects of Abū-l-Barakāt's logic. Abū-l-Barakāt mentioned, but was largely indifferent to, the *waṣfī* readings; most of what he did has to do with modals in the *ḏātī* reading. Secondly, though he was working in ways that are influenced by and yet different from *both* Alfarabi and Avicenna, Avicenna is incomparably the predominant influence. No one after Avicenna could contribute to the Arabic logical tradition without paying attention to what he had written. But Abū-l-Barakāt did more than merely mention Avicenna to refute him—he accepted a number of arguments and inferences from the Avicennan account, implicitly rejecting the related Farabian arguments.

One of the best opportunities to examine a move typical of those Abū-l-Barakāt made is in his argument to save the e-conversion of the absolute proposition against Avicenna's counterexample (see above page 548 ff.). Abū-l-Barakāt warned people against being like one

> according to whose view, imprecise as it is, the e-proposition does not convert as an e-proposition (as Aristotle had said). He gives an example for that view: laughter may be negated of every person actually at a certain time, so that is an absolute negation. Yet it does not convert, that is, its converse is not true (that no one laughing is a man, for rather, everyone laughing is a man). But he has not considered his words 'at a certain time' and 'actually'. The absolute is absolved (*muṭlaq*) of these and other matters; no given time is mentioned in it, nor any condition. Rather the predicate and the subject are mentioned, and the quantifier in an affirmative, and the particle of negation in a negative, without anything further. If [the proposition] is said like that, then the example offered is not credible, since no one who conceives things accurately on hearing it would accept *no man is laughing* as an absolute statement because [each man] is not laughing at some times, while he would accept that every man is laughing because [each man] is laughing at some times. So the form of the words in affirmation does not convey perpetuity, yet in negation the form does convey perpetuity, such that the negation has to be a negation in accordance with that.

> Reflect on these words, and how they fall in with comprehensibility and conceivability—dispense with all they go on about, and ascertain the correctness of Aristotle's doctrine in his extremely clear words that do without the subtleties used above. [Baġdādī, 1357 A.H., pages 120.20–121.8]

Abū-l-Barakāt accepted the Avicennan truth-conditions for the a-proposition in the absolute, and merely argued against rejecting the 'conventional' truth-conditions for the e-proposition (that is to say, his a- and i-propositions are like proposition 10 in appendix two, and his e- and o-propositions are like proposition 5). Abū-l-Barakāt preserved the immediate inferences of the Aristotelian assertoric, but he did it while accepting Avicennan truth-conditions for the absolute a-proposition.

So much for the first move in the Avicennan modification of the assertoric syllogistic. Abū-l-Barakāt went on to give the accounts not only of conversion, but also of assertoric contradiction, to be found in the first seven books of the *Prior Analytics*. He then moved on to the modals. Unlike Alfarabi, and after him Averroes, however, Abū-l-Barakāt did not use reduplicative propositions to get the conversion of the two-sided possible as a two-sided possible. In fact, he came to the same conversions for his modals that Avicenna proposed, rejecting those defended by Alfarabi and Averroes [Baġdādī, 1357 A.H., pages 121–122]. Abū-l-Barakāt later in his treatise gives the syllogistic moods with both the assertoric second figure, and the fourth figure [Baġdādī, 1357 A.H., page 125 f.; page 148 ff.].

Abū-l-Barakāt accepted the Avicennan division of the syllogistic into conjunctive and exceptive. He also quantified and negated his conditional propositions like Avicennan conditionals. He was largely indifferent, however, to the hypothetical syllogistic, and belonged to the tradition sceptical of positing a lost 'Aristotelian' treatment of the hypothetical syllogistic, a tradition represented more than two centuries before by Alfarabi (see above page 543).

> Regarding syllogisms which are from hypothetical propositions, Aristotle only made mention of the exceptive in his book. What touches on conjunctive [hypothetical] syllogisms, both pure, and mixed with categoricals, is clear from what he says, and the sound mind will recognize them from what has been said. He omitted mentioning them in his book either due to how little benefit they are in the sciences, and he disliked the thought of dwelling on them; or because he relied on the fact that minds which have come to know the categoricals may conclude from them to [the hypotheticals], so that you will recognize them from what you have come to know in the categoricals; or [he omitted mention of them] for both [reasons]. A certain later scholar said that Aristotle had written a special book on them which had not been translated into Arabic; this is baseless conjecture. Had he wanted to mention them, why did he move them from here, their proper place? Anyway, there is not enough concerning them that would merit a separate book with separate principles and conclusions. [Baġdādī, 1357 A.H., page 155.11–18]

Further, Abū-l-Barakāt did not analyse the proofs by reduction using the distinction [Baġdādī, 1357 A.H., page 188.11–12].

We can hardly claim that Abū-l-Barakāt represents a modification of the Avicennan system in the spirit of Alfarabi. He did not adopt any of the Farabian solutions attacked by Avicenna in *The Cure* (noted above, page 547 ff.), and he accepted the modal conversions that Avicenna specified. He also accepted the Avicennan division of the syllogistic into conjunctive and exceptive, though he did not emphasize or use it as much as Avicenna did. It is true that he was not like other Avicennan logicians, described in the next section, who were particularly interested in the extensions of the *waṣfī* readings. Nor was he a precursor to what Averroes did, and although an Aristotelian, his Aristotelianism is much more textually attenuated than that of Averroes.

'Abdallaṭīf al-Baġdādī That said, there is at least one scholar in Baghdad, somewhat later than Abū-l-Barakāt, who did direct his philosophical project towards a recovery of Aristotle by way of Alfarabi. This philosopher was the rather idiosyncratic 'Abdallaṭīf al-Baġdādī (d. 1231), who, having studied Avicennan philosophy, went from Baghdad westwards, travelling widely. On his travels, he met scholars who convinced him that the philosophy he had studied was not as good as Alfarabi's. He came to write more along Farabian than Avicennan lines, and apparently composed a number of commentaries on Alfarabi's works [Gutas, 1993, page 50]. This is very interesting, although it must be said that we do not to this day have in published form a logical treatise by 'Abdallaṭīf. What we do have is a manuscript in Brusa which may well be vital, not merely for understanding 'Abdallaṭīf, but also for reconstructing Alfarabi's lost *Long commentary on the Prior Analytics*. The relevant part is paraphrased by Stern as follows:

> Some particular points in Aristotle's logic have been criticised, but it turned out he was right and his critics did not understand his meaning; this has been explained by al-Fārābī in his great commentary on the *Prior Analytics*. It is altogether a great mistake to think that the modern works are clearer in exposition or style than those of the ancients. [Stern, 1962, page 63]

Until we reassemble the logical writings of 'Abdallaṭīf, we can do little more than note that at least one scholar worked in a Farabian rather than an Avicennan line.

5.3 The Avicennan tradition

Neither Abū-l-Barakāt nor 'Abdallaṭīf represents a larger school which had returned to Farabian doctrine, at least in the syllogistic. Nor was Faḫraddīn ar-Rāzī the representative of a school at war with the school to which the great Naṣīraddīn aṭ-Ṭūsī belonged. In dealing with this second set of objections to Rescher's claim for a Western school (that is, objections 3, 4 and 5 at page 568 above), I propose to show two things: in terms of the later reception of Avicennan logic, Rāzī was rather more one of Ṭūsī's sources than a target for criticism, and, secondly, in terms of substance, Rāzī and Ṭūsī were interested in broadly the same questions and came to roughly the same answers. In short, I approach the material here firstly source-critically, and then in terms of its substantive logical doctrine.

Rāzī, Ṭūsī and the logical tradition In coming to terms with the scholarly relation between Rāzī and Ṭūsī, one has first to negotiate a tendency in studies on the history of Islamic philosophy to overstress the differences between the two men. This arises in large part because of the excoriating attacks made by Ṭūsī in commenting on Rāzī's *Validated philosophy of the ancients and the moderns* (*Muḥaṣṣal afkār al-muta'aḫḫirīn wal-mutaqaddimīn*), a text which does not deal with the logic. But Ṭūsī had at least a grudging admiration for Rāzī's commentary on *Pointers*. This was so in spite of the fact that it is often said in histories of Islamic philosophy that Ṭūsī thought Rāzī's commentary was a "diatribe not a commentary" [Fakhry, 1983[2], page 320, to cite one of many possible examples]. What Ṭūsī wrote is this:

> Among those who have already commented on this book is the eminent and erudite Faḫraddīn, prince of the controversialists, Muḥammad ibn-'Umar ibn-al-Ḥusayn al-Ḫaṭīb ar-Rāzī. He made an effort to explain as clearly as possible everything in it that was hidden, and strove to express in the best way that which was obscure; he followed in hot pursuit of what was meant, and, in searching out what was deposited therein, he reached the furthest path of penetration. He was excessive, however, in responding to Avicenna in the course of his essay, and in refuting his principles overstepped the bounds of justice. By these efforts he only detracted from his own work, and because of them a certain wit has called his commentary a diatribe. [Ṭūsī, 1971, page 112.1–6]

Ṭūsī thought Rāzī's commentary was overly oppositional in expositing Avicenna's system, but when Ṭūsī was asked to write a commentary on *Pointers* himself, he said of its notoriously laconic doctrinal payload: "I gained it from the first commentary, mentioned above, and from other famous books..." [Ṭūsī, 1971, page 112.18–19].

I have dwelt on this because I think it is why Rescher came to decide that Rāzī and Ṭūsī were at loggerheads in the logic. They were not, even though they may have been in metaphysics. More important than any explicit appraisal of Rāzī by Ṭūsī, however, is how they both approached the logical tradition and its problems. As it turns out, Ṭūsī named the scholars he was drawing on, and how he differed from them. The logicians to whom Ṭūsī referred may be divided into three groups, groups which are mentioned to serve different functions in the course of Ṭūsī's exposition. The first group of logicians he mentioned consists of Greeks from classical times and late antiquity: Aristotle, Theophrastus, Eudemus, Alexander, Themistius, Porphyry. The second group, or really pair, of logicians mentioned is made up of Avicenna and Alfarabi (Ṭūsī rightly took 'the eminent later scholar' of *The Cure* to be Alfarabi). There are, lastly, four scholars whose names are invoked, who either died somewhat before Ṭūsī was writing, or were his contemporaries. The oldest of these scholars is 'Umar ibn-Sahlān as-Sāwī (d. 1145) [Brockelmann, 1936–1949, Supplementary volume I, page 830], who wrote *Insights into logic for Naṣīraddīn* [Sāwī, 1898]. The next oldest source quoted by Ṭūsī is Rāzī. The third logician is Afḍaladdīn Kāšī—death-dates for Kāšī vary widely, from 1213 or 1214 to the early fourteenth century, but the earlier date seems preferable [Chittick, 1982–]. Rescher has Kāšī as Rāzī's student, though there is little evidence to support this [Rescher, 1964,

page 68]. Kāšī's second longest work is an Arabic treatise on logic called the *Clear Path* [Kāšī,]. The last of the logicians mentioned is Aṯīraddīn al-Abharī (d. 1264); Abharī studied under Kamāladdīn ibn-Yūnus, who was probably also Ṭūsī's teacher. Abharī wrote a number of logical works, among them the famous *Introduction to logic* [Calverley, 1933]. Sadly, I have not seen any of Abharī's longer works on logic—I think none has been printed.

I list these sources because it allows us to examine which sources Ṭūsī shared with Rāzī, and whether he read them in the same way that Rāzī did. Rāzī did not mention the Greek logicians (at least on my reading), but he did mention both Alfarabi and Sāwī. Before I turn to how Rāzī read these logicians, and influenced Ṭūsī's reading of them, I should say that although Rāzī did not mention the Greeks whom Ṭūsī mentioned, he did deal with all the logical doctrines that Ṭūsī addressed in referring to the Greeks. (I return to this point; see below page 577 f.)

Rāzī referred to Alfarabi by name to bring out his doctrine of the ampliation of the subject-term. I don't think he made such a reference in *Gist* or his longer commentary on *Pointers*, but in his *Summary of philosophy and logic*, he wrote that "Alfarabi claimed that in all Js one should not consider the occurrence of actual Js, but rather what may be describable as J" [Rāzī, a, folio 23a.13]. In the longer commentary on *Pointers* itself he gave the Alexandrian proof for e-conversion without, however, ascribing it to anyone [Rāzī, b, folio 41b.12 et seq.]. Ṭūsī gave accounts of precisely these Farabian doctrines (the ampliation of the subject-term [Ṭūsī, 1971, page 282.1–4]; e-conversion [Ṭūsī, 1971, page 325.8–12]). Both men, that is to say, were reading exactly the same things out of Alfarabi, or perhaps relaying the same things—I wonder if either had actually read Alfarabi.

Again, Ṭūsī read Sāwī the same way Rāzī did. At every point that Ṭūsī consulted Sāwī in his commentary, Rāzī had preceded him. In treating the conversion of the *waṣfī* non-perpetual, Ṭūsī wrote almost verbatim [Ṭūsī, 1971, page 328.5 et seq.] what we find in Rāzī [Rāzī, b, folio 41b.pu et seq.]; so too for the first-figure syllogisms with mixed *waṣfī* and *ḏātī* premises (compare [Ṭūsī, 1971, page 400.12 et seq.] and [Rāzī, b, folio 53b.pu et seq.]), and for the second-figure syllogisms with the same mix (compare [Ṭūsī, 1971, page 416.19 et seq.] and [Rāzī, b, folio 57b.18 et seq.]). In fact, I doubt that Ṭūsī had actually read Sāwī for himself, because he credited Rāzī with coining the term 'conventional' (*'urfiyya*), even though we find it in Sāwī [Sāwī, 1898, page 73.5] (although there of course it may be being used pre-technically).

None of this is surprising; it is exactly what Ṭūsī announced he would do at the beginning of his commentary. To picture Ṭūsī and Sāwī in an Eastern school, doing things logically different from Rāzī—this just misrepresents what was going on. Further, Rāzī and Ṭūsī were both closer to each other than either of them was to Avicenna. Clinging to the idea of an Eastern and a Western school makes it difficult to understand why scholars who appear as either Eastern or Western are doing such similar things (and things so different from what Abū-l-Barakāt was doing), and why 'Easterners' draw so freely on 'Westerners' and vice versa. Rescher's account of the Western school [Rescher, 1964, page 57] needs to be rejected.

Along with the rejection of the 'clash of the schools', we need further to reject the reconciliation of the schools in the *Arbitration between the two commentaries* [Taḥtānī, 1375 AH solar] by Quṭbaddīn ar-Rāzī at-Taḥtānī (d. 1365). (My comments here do not in any way go to what Taḥtānī may or may not have been doing in the physics and metaphysics.) There are about forty references in Ṭūsī's commentary on the logic of *Pointers* to Rāzī, and they clearly did differ on many points. But following Taḥtānī's *Arbitration* as it goes through the logic, one cannot fail to be struck by how often Taḥtānī had nothing to say on the points of difference and, on those few occasions he did say something, how rarely it consituted a synthesis or reconciliation of two seemingly incompatible views. As it happens, attitudes of some logicians in Iran and further east differed from the attitudes of others, in a way I will examine below. There were not, however, two schools differing fundamentally on matters of substantive logical doctrine.

No logician after Avicenna defended the Avicennan syllogistic pure and simple. Changes were put forward for (among other things) the *d̲ātī* propositions, the *waṣfī* propositions, the temporals (*waqtiyyāt*), and the analysis of the proof by reduction. I note some aspects of each in turn, but I stress that these notes fail badly in conveying the depth and range of logical analysis of each aspect, because they are limited to the discussions which issued in the doctrine of the *madrasa* texts. It is this development of logic which was said by Ibn-H̲aldūn to be penetrating, constituting a discipline in its own right (see below page 580); the same development was compared by Ibn-Taymiyya to carrion (see above page 560).

***D̲ātī* propositions** It was noted (see page 552 above) that Avicenna's syllogistic with *d̲ātī* premises includes some puzzling inferences or, more strictly, sets of puzzling inferences. Take, for example, the claim that absolute a- and i-propositions (see proposition 10 in appendix 2) convert as absolute i-propositions, that one-sided possible a- and i-propositions convert as one-sided possible i-propositions, and yet that syllogisms with possible minors and absolute majors produce. In fact, syllogisms with possible minors are central to the development of Avicenna's system. There may well be a way to show the compatibility of all the inferences Avicenna proves, but I cannot see what it is. More importantly, nor could the thirteenth-century logicians. Early on, efforts were made by Faḫraddīn ar-Rāzī to save most of Avicenna's modal syllogisms, seemingly by taking the subject-term to ampliate to the possible. This meant having the absolute a- and i-propositions convert as possibles (see e.g. [Rāzī, 1355^2 A. H., page 24.16–20]).

I am not sure if the inferences in Rāzī's alternative system actually square any better than Avicenna's, but in any event, no one seems to have adopted his approach. By the time Ṭūsī's student Nağmaddīn al-Kātibī (d. 1276) was writing, it is clear that scholars had come to agree that syllogisms with possible minors do not produce, at least not if the propositions are read according to the ordinary way of taking the subject-term. Kātibī put forward the following distinction: In a world where it so happens that there is no geometrical figure apart from triangles, *all figures are triangles* is true. If, however, we are concerned not with how things happen to be, but with how meanings relate, *all figures are triangles* is untrue, even in that world where it so happens that all figures are triangles. The subject-term was to be read in the first way.

Ṭūsī and Kātibī differed from Rāzī over how to take the subject-term, and the Kātibī view won out, at least in the later tradition that has so far been examined in modern studies. In Muḥammad ibn-Fayḍallāh aš-Širwānī (fl. 15th century?), for example, we find Avicenna characterised as taking the subject-term as referring to that which is "actual, that is, at a given time, whether it be at the time of the judgement, or in the past, or in the future" [Širwānī, , folio 96a.6–10], a characterisation which would be less problematic if it were not directly linked to the untrue claim that Avicenna did not allow possible affirmative propositions to convert. Strange to say, Rāzī was more Avicennan than Kātibī on this point. It is a major point, affecting the fabric of the entire *ḏātī* syllogistic; Kātibī's decision means that all first-figure syllogisms with possible minors fail to produce, and all the syllogisms which depend on these mixes for their proofs also fail. And Rāzī was not just defending syllogisms with possible minors in an act of commentatorial charity; we find him committed to them not only in his commentaries on Avicenna [Rāzī, 1355^2 A. H., pages 33–34] but also when he is speaking in his own voice, in *The summary of philosophy and logic* [Rāzī, a, folio 46b.9].

We should note one other point of comparison between the thirteenth-century logicians and Avicenna in their presentation of the syllogistic with *ḏātī* modal premises, which has to do with their respective concern for Aristotelicity. Avicenna mentioned the perpetual (proposition 5 in appendix two) in his development of the syllogistic only when giving the contradictory of the absolute, but at no other place. I can only speculate about the reason for this, but I think it is because Avicenna's presentation is developed in conversation with Aristotle's account, and thus he can find no place for a perpetual, because there is no perpetual in the *Prior Analytics*. Whether or not I am right about this, it is none the less true that the short thirteenth-century accounts have the perpetual as a proposition fully investigated throughout the presentation. Further, some of the converses of *ḏātī* propositions are no longer *ḏātī* propositions (these points should become clear by comparing appendices one and two). I think all of this is symptomatic of increasing indifference to the Aristotelian account (see further page 577 below).

***Waṣfī* propositions** All the logicians I have read were agreed that there were serious problems with Avicenna's account of propositions in the descriptional reading (the *waṣfiyyāt*). Avicenna claimed, among other things, that the contradictory of *all Js are always Bs while Js* is *some Js are always not Bs while Js* (it is not clear to me that this is what Avicenna actually claimed, but anyway, this is what he was taken to have claimed). Among the limited sources I have examined, this concern began with Sāwī [Sāwī, 1898, page 70.10 et seq.], and his approach to the problem is reflected in Rāzī's *Gist* [Rāzī, 1355^2 A. H., page 22.1–4]. It is interesting to follow the concerns about Avicenna's claims that a perpetual in the *waṣfī* reading is contradicted by another perpetual in the *waṣfī* reading as they gradually gather clarity and, ultimately, technical terms to describe the concerns. By the time of Ṭūsī, it was taken as settled that a solution had been reached, and that the descriptional perpetual (a *waṣfī* reading, proposition 6 in appendix two), must be contradicted by a *waṣfī* absolute, called by Ṭūsī *muṭlaqa waṣfiyya* [Ṭūsī, 1971, page 313.8] (proposition 7 in appendix two). Kātibī called it the *ḥīniyya* [Kātibī, 1854, page 16.8], though it is not given in his treatise as one of the propositions in the preliminary list-

ing. *Ḥīniyya* is adopted as the usual term for the proposition in the subsequent literature [Širwānī, , folio 86a.12], and it joins the other propositions in the preliminary listing. We can be sure that Abharī was also working on questions to do with the *waṣfī* propositions, because Ṭūsī mentioned in passing that he took a conventional existential o-proposition to convert (this is not a point Abharī makes in his *Introduction*, and I cannot check it). Avicenna's claims regarding the *waṣfī* readings have been the subject of a recent study which proceeds by adopting an understanding of the proposition proposed by Ṭūsī [Thom,].

In the case of the *ḏātī* readings and the perpetual, the thirteenth-century logicians were indifferent to the fact that Aristotle had not used a perpetual in his account in the *Prior Analytics*. In treating the *waṣfī* readings, they betray no feeling of pressure to find a proposition which conforms to the immediate inferences required of the Aristotelian assertoric. Nor are they primarily interested in syllogisms with purely *waṣfī* premises, which I think was Avicenna's primary concern, but with the ways mixes of *waṣfī* and *ḏātī* readings produce. This takes up a later and less central concern of Avicenna.

Temporals One of the extrinsic conditions on a proposition which occurred in Avicenna's list was the as-of-now. Avicenna used it once in the course of his exposition to save the Aristotelian account of contradiction. Ṭūsī's student Kātibī introduced it into the propositions "into which it is usual to inquire," using it to produce a modalized proposition. Much later, the temporals were exhaustively analysed by Širwānī (see appendix two, propositions 3, 4, 8, 9, 12, 14, 17, 18, 22 and 23). As with the *waṣfī* readings, however, the thirteenth-century logicians were not concerned to use the as-of-now to preserve the Aristotelian account, and changed its truth-conditions so it no longer squared even with the way Avicenna used it.

Proofs by reduction Another point that needs to be considered is how the Avicennan logicians dealt with the proof by reduction. (I will not try to describe their extensive examination of the hypothetical syllogistic.) Sāwī took it to be a combination of a conjunctive and an exceptive [Sāwī, 1898, page 104.pu–u], just as Avicenna had. Rāzī followed Sāwī and Avicenna on this point [Rāzī, 1355[2] A. H., page 43.17–18]. But Kāšī, on the relation between the categorical and the hypothetical syllogistics, differed from Avicenna and offered an alternative analysis which is treated sympathetically and, I think, actually adopted by Ṭūsī and subsequent logicians whom he influenced.

Kāšī's argument seems to me to amount to no more than an argument by assertion, though perhaps more sensitive and acute study of the problem will turn up a different conclusion. What Kāšī concluded is:

> The deduction involving a contradiction is an exceptive syllogism whose minor premise is hypothetical with a compound antecedent, and whose major premise is categorical, being the contradictory of the consequent. So it produces the contradictory of the first proposition of the two parts of the antecedent of the minor. This is its form:
>
> > If Zayd is writing, and everyone who is writing moves his fingers,
> > then Zayd is moving his fingers;

But Zayd is not moving his fingers;
Therefore Zayd is not writing. [Kāšī, , folio 72a.1–5]

Kāšī felt compelled to put forward his alternative view of reduction because of arguments which had become common.

> The reason for this disquisition is to alert people to the truth of the matter concerning how a deduction involving a contradiction is composed. This is because what is to be understood from a certain verifying scholar is other than what we have mentioned. He said rather that this is not the case, but that the deduction involving a contradiction is composed of two syllogisms, one conjunctive and the other exceptive; just as when it is said that the hypothetical proposition is composed of two categorical propositions, from which it is to be understood that the hypothetical proposition is something other than these two. The deduction involving a contradiction is not like that—it is just an exceptive; it is an exceptive syllogism whose minor premise is composed of two categorical propositions sharing a term, from the granting of which there follows as a consequent of the first proposition with its two parts, and the major premise, a categorical proposition which contradicts the antecedent of the minor. This has been determined before, and illustrated. [Kāšī, , folio 71b.u–72b.13]

It is worth noting the form of Kāšī's claim. He did not derive his view of the matter from the Averroist critique, or from its antecedents.

The cult of Avicenna Ṭūsī seems to have adopted Kāšī's analysis of the reduction. It is instructive to note what he took to be Avicenna's reasons for taking the view he did on the hypothetical syllogistic.

> Aristotle placed the deduction involving a contradiction among the hypothetical syllogisms, yet in his writings there were only exceptive hypotheticals; consequently most logicians simply called [the exceptive] a hypothetical syllogism.
>
> However, Avicenna thought that the conjunctive hypotheticals had been treated in a separate text which was not translated into Arabic. [This was] simply an assumption which Avicenna was compelled to hold due to his good opinion of Aristotle. When the later logicians wanted to analyse this syllogism, and reduce it to the above-mentioned syllogisms, that analysis was difficult for them to accept, and they differed completely from Avicenna. [Ṭūsī, 1971, page 453]

This exemplifies the way that Ṭūsī approached the Avicennan system, and how his approach differed from Rāzī's. The difference is even starker when Ṭūsī mentioned Greek philosophers at another important point in his commentary, as will emerge.

Ṭūsī was among the many who accepted the modifications to Avicenna's doctrines regarding the *waṣfī* propositions. Unlike other scholars, however, Ṭūsī did so while at

the same time presenting Avicenna as responding to an ancient peripatetic debate regarding what place the absolute proposition has in scientific discourse. Avicenna's kinds of absolute proposition (corresponding roughly to propositions 6, 10, 20 and 24 in appendix 2), Ṭūsī claimed, find places for the various doctrines of Aristotle, Themistius and Theophrastus, and Alexander [Ṭūsī, 1971, pages 268.pu–269.20]. The problem is that the *d̲ātī* reading of the absolute does not find contradictories or convert as an Aristotelian assertoric, so Avicenna stipulated a reading of the absolute that he thought did. Ṭūsī wrote:

> What spurred him to this was that in the assertoric syllogistic Aristotle and others sometimes used contradictions of absolute propositions assuming them to be absolute; and that was why so many decided that absolutes did contradict absolutes. When Avicenna had shown this to be wrong, he wanted to give a way of construing those [examples from Aristotle]. [Ṭūsī, 1971, page 312.5–7]

The modified, convertible absolute proposition presented by Avicenna is also able, according to Ṭūsī, to accommodate the opposing interpretations of the absolute proposition put forward by Theophrastus and Alexander.

> We have mentioned that the validating scholars of this art had two views in explaining the absolute. The first of them is that it includes the necessary, as Themistius held, and that is the general. The second is that it does not include the necessary, as Alexander held, and that is the special. Avicenna wanted to show that each of the two views could be specified in the way which he puts forward here, so that it is compatible with contradiction in the absolute according to both of the two views.
>
> His explanation is that the conventional can be taken to cover the necessary, and be general; or it can be taken not to cover the necessary, and be special. The conventional general absolute agrees with the first view; and the special... agrees with Alexander. [Ṭūsī, 1971, pages 313.16–314.6]

On the fact that Avicenna took such pride in the fact that his two readings of the propositions allowed him to square a set of examples where other logicians had failed (see above page 551), Ṭūsī wrote:

> He means the majority of logicians were not able to escape the consequences of their doctrine, that is, that absolutes contradict absolutes. This is because they were not able to construe the absolute mentioned in the First Teaching in all places according to their doctrine. Among relevant examples in the First Teaching are the absolutes *All who wake sleep* and *All who sleep wake*, and others like them which cannot be construed as conventional. Similarly in usage, since the First Teaching used the absolute where it is not possible to use the conventional. [Ṭūsī, 1971, pages 314.12–315.2]

There are no references to Greek scholars in the corresponding passages in Rāzī's texts (at least on my reading). Both scholars were convinced that Avicenna's system needed

repair by either extension or restriction, and both proceeded to implement such changes, sometimes Ṭūsī more than Rāzī (especially for example in the *ḏātī* propositions). But Rāzī made his changes with little outward show of respect; at one point in *Gist*, for example, we find him saying:

> Once you have come to understand what I have mentioned here, you will realise that this commentary, in spite of its brevity, is more explanatory and rigorous than what is in Avicenna's book, in spite of its length. [Rāzī, 1355[2] A. H., page 22.14]

Ṭūsī, by contrast, nearly always found a way to justify the position Avicenna had adopted. That is the role the Greek authors play in his commentary—they indicate where Avicenna's doctrine is open for contestation and reinterpretation, while saving Avicenna from any charge of logical error.

Post-Avicennan logic in Baghdad and further east At this point, I offer a short and tentative outline of the history of post-Avicennan logic in Baghdad and further east.

In Baghdad, attention continued to be paid to the ways Aristotle had organised his logic and proved his inferences, though by 1150 at the latest it was clear that Avicennan counterexamples had problematized the Aristotelian account. We can assume that Alfarabi's texts were being read in Baghdad, though the fact that someone like 'Abdallaṭīf al-Baġdādī had to learn his Farabian logic outside of Baghdad shows that the tradition of reading these texts was growing ever weaker. References to Alfarabi among these logicians are so stereotypical that we must wonder if his texts were available at all. By the time of Barhebraeus (d. 1286), even the Syriac Christians had become Avicennan.

Further east, by the first half of the twelfth century, an Avicennan tradition was well advanced in the process of modifying Avicenna's system. The Avicennan tradition may be called 'Avicennan', as has been mentioned, because it accepted the division of the syllogistic into the conjunctive and exceptive, it accepted the conditions proposed by Avicenna as the relevant ones within which to investigate modalities and temporalities, and it accepted the stipulation of truth-conditions for the absolute such that it was contradicted by a perpetual. This tradition proceeded with little or no reference to the Aristotelian corpus, at least in its early days, producing a modified system by the end of the thirteenth century which by and large continued to be accepted down till well into last century. Among some of its later adherents, the tradition started to refer once again to the writings of Aristotle; these references are associated with a cult of Avicenna which used the Greek references as a way to excuse some of Avicenna's less easily defended moves. But scholarly courtesy to the great Avicenna in no way prevented changes being made to his logical system.

5.4 *The Handbooks of the* Madrasa

At roughly the same time that a consensus was emerging that the major questions in Avicenna's formal syllogistic had been settled, the *madrasa* was reaching a period of institutional stability and influence. Ġazālī's opinion that logic was useful for Muslim scholars was an important factor in having the subject included in the syllabus of the *madrasa*. The

doctrine of the *madrasa* handbooks was determined by the Avicennan tradition described in the last section. We can be fairly sure that this was the dominant tradition by the mid-thirteenth century not only from the fact that even the Syriac Christians used Avicennan logic, but also from the account of the great polymath, Ibn-Ḫaldūn, who wrote about some of the important handbooks of the time.

The emergence of the handbooks Ibn-Ḫaldūn received his training in logic from these handbooks, and in his *Prolegomena*, he described both the changes in emphasis over the years within the discipline of logic, and which books were used in teaching. After describing the composition of the Organon and the logical issues each part of it addresses, the *Prolegomena* continues:

> Its sections came to be nine; and all were translated in the Islamic community, and the philosophers dealt with [these books] by commentary and exposition. Alfarabi did [this], and Avicenna, and Averroes among the Andalusian philosophers—Avicenna wrote *The Cure*, in which he took in all seven philosophical disciplines. Then the later scholars came and changed the technical terms of logic; and they appended to the investigation of the five universals its fruit, which is to say the discussion of definitions and descriptions which they moved from the *Posterior Analytics*; and they dropped the *Categories* because a logician is only accidentally and not essentially interested in that book; and they appended to *On Interpretation* the treatment of conversion (even if it had been in the *Topics* in the texts of the ancients, it is none the less in some respects among the things which follow on from the treatment of propositions). Moreover, they treated the syllogistic with respect to its productivity generally, not with respect to its matter. They dropped the investigation of [the syllogistic] with respect to matter, which is to say, these five books: *Posterior Analytics*, *Topics*, *Rhetoric*, *Poetics*, and *Sophistical Fallacies* (though sometimes some of them give a brief outline of them). They have ignored [these five books] as though they had never been, even though they are important and relied upon in the discipline. Moreover, that part of [the discipline] they have set down they have treated in a penetrating way; they look into it in so far as it is a discipline in its own right, not in so far as it is an instrument for the sciences. Treatment of [the subject as newly conceived] has become lengthy and wide-ranging—the first to do that was Faḫraddīn ar-Rāzī and, after him, Ḫūnağī (on whose books Eastern scholars rely even now). On this art, Ḫūnağī has written *The Disclosure of Secrets*, which is long, and an abridgement, *The Short Epitome*, which is good for teaching, and another abridgement, *The Digest*, which in four folios takes up the cruces and principles of the discipline—students use it frequently to this day and benefit from it.
>
> The books and ways of the ancients have been abandoned, as though they had never been... ([Ibn-Ḫaldūn, 1858, pages 112.8–113.12]; cf. [Ibn-Ḫaldūn, 1958, pages 142–143])

The formal syllogistic, Ibn-Taymiyya's camel carrion, survived the ending of interest in the 'material' disciplines. Ibn-Ḫaldūn dated this change from Rāzī's work, though it is apparent already in Sāwī's *Insights* and even in Avicenna's *Pointers and Reminders*.

The other scholar in Ibn-Ḫaldūn's account is Muḥammad ibn-Nāmwar al-Ḫūnağī (d. 1249), who had been a judge in Cairo. I think the point Ibn-Ḫaldūn was trying to make in mentioning his treatises is that logicians writing in the West were doing logic so well that even logicians in the East had to take notice. An examination of his *Digest* reveals just what Ibn-Ḫaldūn meant when he talked about the change in focus and depth of treatment. Of fourteen pages, the first three are given over to utterances, universals, definitions and descriptions; all the rest are given over to the formal study of propositions and the syllogistic. The presentation of the different kinds of propositions begins on 76a, and proceeds by laying out the modalities and temporalities (necessity and perpetuity and their duals), goes on to the readings (*ḏātī* and *waṣfī*), and then the temporal constants (the temporal and the spread). Mention is made of the different doctrines on the absolute, and of the different doctrines on how the subject-term can be taken. What we find, in short, is precisely the same sort of approach to the propositions that we find in Rāzī and Ṭūsī—though it is important to stress that the doctrine presented is not uniquely that of Rāzī, but rather reflective of the main options in his and Ṭūsī's tradition. Again, in the division of the syllogisms, Ḫūnağī adopted the Avicennan distinction between conjunctive and exceptive, though it is impossible to gauge from his reference to reduction [Ḫūnağī, , folio 77a.11–13] how he analysed it. Proportionally, an extraordinary amount of attention is devoted to the conditionals [Ḫūnağī, , folio 78a–80a].

Two standard texts It was two contemporaries of Ḫūnağī, however, who wrote the treatises most widely used as introductions to the subject over the centuries: Abharī (see [Gutas, 1993, page 63 note 161]) and Kātibī (see [Kātibī, 1854]). In Calverly's translation, Abharī's *Introduction* comes to eight pages [Calverley, 1933]. In much the same way as Avicenna's introductory *Philosophy for 'Alā'addawla* [Avicenna, 1971], it is is very elementary and general. Aside from the fact that it divides the syllogistic into the conjunctive and exceptive, it could come from just about any tradition derived from Peripatetic logic. But it presents its logic in the order and with the terms that make it the perfect preliminary to a more difficult text.

For most students through the centuries that text was Kātibī's *Logic for Šamsaddīn* [Kātibī, 1854]. In Sprenger's edition it comes to twenty-nine pages. It has been translated, mostly by Sprenger and Kaye, and by Rescher [Rescher, 1967b]. The whole text is probably ready for a new translation, complete with annotations and the semantics due to Rescher and vander Nat [Rescher and vander Nat, 1974] (given in appendix two below). I offer a rapid sketch of the book.

Kātibī did not compare logic to grammar. But he wrote that to acquire the demonstrative sciences and come in contact with the angelic intelligences, the loftiest pusuit for man, one cannot do without logic [Kātibī, 1854, page 1.5–7]. Kātibī went on by introducing the terms *taṣawwur* and *taṣdīq* in the first section of his treatise [Kātibī, 1854, page 2.5–7], naming the subject-matter of logic as "the objects of cognition, both conceptual and prone to assent" (*al-ma'lūmātu t-taṣawwuriyya wat-taṣdīqiyya*) [Kātibī, 1854,

page 2.18]. The theory of definition deals with conception, while the theory of proof deals with assent [Kātibī, 1854, page 2.u]—this is straightforwardly Avicennan. There is no reference to the reduction of legal arguments to the syllogism, and the context theory is confined to the section on syllogistic matter (*fī mawāddi l-qiyāsāt*). That said, the equation of merely presumed matter (*maẓnūn*) with rhetoric would have signalled clearly where juristic arguments were thought to fit into the theory.

Like Abharī, Kātibī accepted the division of the syllogistic into conjunctive and exceptive, though the text does not devote much attention to conjunctive hypotheticals, nor to the Avicennan analysis of the deduction involving a contradiction. The text then presents a version of the assertoric syllogistic that squares with Aristotle's account, but goes on to say that this will not work for the absolute proposition. Kātibī's subsequent account of the syllogistic with the various modalities and temporalities in both the *d̠ātī* and *waṣfī* readings is a modification and extension of Avicennan ideas. Overall, his logic is Avicennan only in the attenuated sense that Ṭūsī's logic is.

Kātibī and his teacher Ṭūsī also had complicated discussions about a number of abstruse points of logic, discussions which, though recorded, probably only ever attracted the attention of a few scholars (for example, *Logical Discussions*; see [Mohaghegh and Izutsu, 1974, pages 279–286]). By contrast, *Logic for Šamsaddīn* was read by nearly every student. One may reflect that it was Ġazālī who had made its inclusion in the curriculum possible, even though the treatise deals with logical points that are almost entirely absent from Ġazālī's treatises. It is ironic that a scholar with interests confined to the material application of logic had done more than anyone else to find a permanent place for a treatise dealing with entirely formal questions.

6 CONCLUDING REMARKS

These concluding remarks are, strictly speaking, more of an apology for the narrow focus and relentlessly historical emphasis of this chapter, and for the tentative nature of its claims. To trace a set of logical discussions from fragments written in Abbasid times through to the introductory *madrasa* texts of the late thirteenth century entails finding a topic common to all the texts. That common topic, the syllogistic, is apt for study due to the existing scholarship. I further think (to repeat my introductory comments) that tracing such a common topic is worthwhile above all because it begins the process of delineating the framework of logical traditions which in turn determine the system that is the primary object of discussion and dispute for any given logician. The very first thing to do when setting out to study an Arabic logical work is to assign the work to its proper systematic context, that is, the texts it addresses and the methods by which it engages with those texts. I cannot say that Muslim scholars adhered to the traditions apparent in the syllogistic when they contributed to other areas of logic, but I think it is likely that they did, and that the historical account put forward in this chapter may serve provisionally for the study of other debates.

Here then is a summary of the points made in this chapter regarding the syllogistic traditions. The account differs most significantly from Rescher's in speculating that Avicenna belonged to an existing tradition already fundamentally different from Alfarabi's,

and in rejecting his claims regarding a 'Western' school of logic [Rescher, 1964, chapters five and six].

Syllogistic traditions Alfarabi was the first truly independent logician writing in Arabic. We may discern three factors in his writings. Above all, he was a product of late Alexandrian Aristotelianism, and drew, however remotely, on the texts and techniques of that tradition. These he modified in response to the fact that he was a Muslim scholar working on a foreign and pagan intellectual tradition. Lastly, Alfarabi came to be conscious of how badly the Aristotelian corpus was served by the interpretations in the existing Alexandrian and Syriac works. This led Alfarabi to try to revive a true Aristotelianism after a period of rupture, which he did by writing commentaries on Aristotelian texts. Though we no longer have the commentary on the first part of the *Prior Analytics*, we can reconstruct his treatment of the conversion of contingent propositions from references in the works of Avicenna and Averroes. In this treatment, Alfarabi tried to let the Aristotelian text stand by finding an appropriate stratagem (in this case, a distinction prefiguring the Averroist distinction between reading a term *per se* or *per accidens*). Alfarabi in his moments of exegetical exertion was fairly dismissive of the 'commentators', presumably members of the Syriac tradition from which he distanced himself.

Avicenna had, broadly speaking, the same philosophical ancestry as Alfarabi, and claimed Alfarabi as his most eminent forebear after Aristotle. Avicenna's prominence among logicians in Iran and further east roughly parallels that of Alfarabi among the Baghdad logicians. Many of his doctrines which seem idiosyncratic to us are not in the writings of the Baghdad scholars, and are presented as though they are already known to his readers. This procedure may indicate that he was simply modifying an existing tradition, different from that of Baghdad. In any event, Avicenna's syllogistic differed radically from Alfarabi's, and he set out some important points in his system by explicitly stating what was wrong with Alfarabi's corresponding doctrine. The single most important factor determining these differences was the fact that whereas Alfarabi thought that the Aristotelian text would, with sufficient attention, yield a coherent system, Avicenna thought that he already knew the coherent system, and used it to identify obscure parts of the text. Alfarabi bent his system to the text, Avicenna bent the text to his system.

In his logical writings, Avicenna covered the same territory as Alfarabi's Aristotelian commentaries. Avicenna did not go on, however, to deal to the same extent that Alfarabi had with the problems of relating logic to the Islamic disciplines. That strand in Alfarabi's logical writings was taken up by various Andalusian logicians, and by Ġazālī. Ġazālī prepared the ground for the institutional acceptance of logic, a Farabian task, but he did it by basing his formal treatment on the elementary section of Avicenna's *Philosophy for 'Alā'addawla*. Ġazālī's work on the syllogistic, however, was so superficial as to be negligible.

Ġazālī is a special case, because he wrote primarily as a jurist and a theologian. But by his death in the twelfth century, two logical traditions had emerged, one Farabian, the other Avicennan. The finest representative of the Farabian tradition was the Andalusian Averroes, who in his syllogistic developed doctrines found in Alfarabi's writings. In fact, Averroes tells us he took from Alfarabi the distinction between the *per se* and *per acci-*

dens. But Averroes' relation to Alfarabi is complex. As Averroes developed the incipient Aristotelianism of Alfarabi, he became increasingly less satisfied with the Farabian answers to exegetical problems, and sought more global solutions which gave every part of the Aristotelian text due weight. This is an extension of the Farabian attitude to the Aristotelian text, in which every position adopted is intrinsically defeasible in the face of a better stratagem. The other scholar to whom Averroes made constant reference, aside from Aristotle, is Avicenna: Avicenna had problematized the Aristotelian system, and thereby determined those points on which Averroes had to dwell longest.

The other logical tradition, the Avicennan, had by the early twelfth century at the latest come to identify problems and cruces in Avicenna's syllogistic which were to occupy the tradition thereafter. Avicennan logicians ceased to consider anything other than the system Avicenna had used in judging Aristotle's logic and, though referring to Avicenna generally as 'the most eminent of the later scholars', never treated his texts or doctrines as immune to criticism and modification. Some representatives of the tradition, such as Rāzī, were even fairly scathing about Avicenna's expositions, though in the late thirteenth century others began to refer reverentially to Avicenna and find ways to explain away his logical errors. This did not, however, prevent them from modifying his logical system exactly the same way as the earlier Avicennan scholars.

The twelfth century saw the clear delineation of the Farabian and Avicennan traditions, each of which paid attention to the other's founder, but rarely to his epigones. The twelfth century also saw other writers referring to Alfarabi, Avicenna and Aristotle. These writers in some cases tended more to an Avicennan systematic, such as Abū-l-Barakāt al-Baġdādī, in other cases, to a more Farabian, such as 'Abdallaṭīf al-Baġdādī. At least in the case of the former, however, there is no evidence that his syllogistic was developed further by his students, and it really speaks past the interests of the mainstream Avicennan tradition. Neither scholar enjoyed a posterity. Even the Farabian tradition guttered, and after the middle of the thirteenth century, the Avicennan tradition had come to predominate in the Muslim world. The Farabian tradition had been weakened by its continued fixation on the non-Muslim Aristotle; although Avicenna worked in conversation with Aristotle and the later peripatetics, logicians after him worked directly on the system against which he had measured Aristotle.

Avicenna was in one respect too successful in naturalizing the study of Aristotelian logic. Though the Averroist approach was intrinsically less stable and mired in a Greek past, it turned out to be transportable, because it spoke directly to the problematic of Latin writers after the coming of the *logica moderna*. Avicennan logics by contrast were only translated much later (see e.g. [Brockelmann, 1936–1949, Sup. vol. I, page 845]), and aroused no interest. The problem was that it was no longer obvious which parts of the Avicennan system were commensurable with the Aristotelian. It is a problem which still plagues the study of his logic, and research needs to be directed to clearing the ground preparatory to making such assessments.

The work ahead I hope that I have conveyed some sense of how many tasks await attention in the study of medieval Arabic logic. Even in the narrow range of material examined in this chapter, there is much to be done. There has been no sustained effort to reconstruct

Alfarabi's modal syllogistic (though Lameer has announced he is preparing such a study); there has been no plausible interpretation given of Avicenna's modal syllogistic, and there are still many problems in understanding his hypothetical syllogistic; we have no overall picture of what Averroes was doing; and the other post-Avicennan logicians are, with two exceptions among writers on the syllogistic, largely uncharted territory. And this is in the syllogistic, one of the logical disciplines which has been relatively well treated in the scholarly literature. This is not to say that there are not valuable studies in the other logical disciplines; there are (see appendix three for lists of such work), but they suffer from even worse gaps in coverage.

Of the many, many logical issues left out of consideration in this chapter, one is particularly noticeable by its absence, and a few words are in order as to why. I have made no attempt to identify the modal notions which lie behind the syllogistic systems of the various authors. Averroes' modal notions have been compared with the broader range of options explored in the middle ages [Knuuttila, 1982, pages 352–353], and a lengthy study has been made of Avicenna's conception of the modalities [Bäck, 1992]. These are valuable contributions. As a matter of procedure, however, I think the preliminary task should be to lay out as precisely as possible the syntactic outline of any given system, and only then investigate its underlying conceptions of modality.

The most unfortunate consequence of concentrating on the syllogistic, however, is that it leads to minimizing consideration of how Islam influenced the constitution of logic in its realms. It has been noted in the course of this chapter how apologetic tendencies drove early logicians like Alfarabi to argue for logic's utility for and complementarity with the Islamic disciplines of grammar, theology and jurisprudence, and how later theologians and jurists like Ġazālī came to accept these arguments. But there are many more, and more complex, issues to take into account in studying the relation between Islam and logic. The great historical task is working out the precise clashes between the Islamic disciplines and philosophy which left the logical treatises as narrowly focussed as they are, and finding the genres which took over treatment of these previously logical topics. This is really the key point for future research, because as each book of the Organon gave way to a competing Islamic discipline (as for example topics gave way to *ādāb al-baḥṯ*, and rhetoric to *'ilm al-ma'ānī*), aspects of the original Aristotelian discipline either transmuted or decomposed into other disciplines. And if ever we can appreciate those changes, we can speak not only of the contributions of Muslim scholars to logic, but also of the contributions of logic to Islamic culture.

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satisfying for those who work on Western logic. There are a number of other people who, though not having read the chapter, have helped in its construction in one way or another. They did so generally in the course of looking at earlier attempts I have made to study Arabic logic, and making me aware of relevant manuscripts, studies I had overlooked, or various mistaken conceptions. So, thanks to: Ahmad Hasnawi, Dominic Hyde, James Montgomery, Ahmed al-Rahim, and David Reisman. Lastly, as with all who work in the field of Arabic logic, I owe a vote of thanks to Nicholas Rescher, both for his pioneering work, and for his generous words of encouragement.

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APPENDICES

A AVICENNA'S MODALS

Modalized propositions are represented (from left to right) by a modal operator, followed by the predicate, the subject, and a superscripted letter indicating quantity and quality. The modal operators are as follows: X stands for an absolute (*muṭlaqa*) proposition, A for a perpetual (*dā'ima*), M for a possible (*mumkina*) and L for a necessary (*ḍarūriyya*). The default reading is *ḏātī*; *waṣfī* readings are indicated by a superscripted w to the right of the modal operator. Premise-sets are given in order of major, minor and conclusion (if applicable). All references to *Pointers* may be checked in [Inati, 1981], which gives the Arabic page numbers in the margin.

Purely ḏātī *premises*

X_1A **contradictories** See [Avicenna, 1971^2, pages 307–308].

$X_1bj^a \not\equiv Abj^o$
$X_1bj^e \not\equiv Abj^i$
$X_1bj^i \not\equiv Abj^e$
$X_1bj^o \not\equiv Abj^a$

(Square for M_1L is isomorphic; see [Avicenna, 1971^2, pages 318–319].)

X_2A **contradictories** See [Avicenna, 1971^2, pages 309–311]

$X_2bj^a \not\equiv Abj^i \vee Abj^o$
$X_2bj^e \not\equiv Abj^i \vee Abj^o$
$X_2bj^i \not\equiv Abj^a \vee Abj^e$
$X_2bj^o \not\equiv Abj^a \vee Abj^e$

(Square for M_2L is isomorphic; see [Avicenna, 1971[2], pages 319–320].)

Perfect first figure mixes $XXX, XLX, LXL, LLL, MMM, MXM, MLM$. Proofs for some second-figure moods also assume AXA and ALA. See [Avicenna, 1971[2], pages 387–397].

X **conversions** X e-conversion fails. X_1 a- and i-propositions convert as X_1 i-propositions. X_2 a- and i-propositions convert as X_1 i-propositions. See [Avicenna, 1971[2], pages 321–333]

Substituting M_1 for X_1 and M_2 for X_2 gives all M conversions; see [Avicenna, 1971[2], pages 338–340]

L **conversions** L e-proposition converts as L e-proposition. L a- and i-propositions convert as M i-propositions. See [Avicenna, 1971[2], pages 334–337]

Further development In the first figure, there are two imperfect mixes: LML, XMM. See [Avicenna, 1971[2], pages 391–395].

In the second figure, the following are proved: LLL, XLL, LXL, MLL, LML. Premise pairs XX, MM, XM and MX all fail to produce. See [Avicenna, 1971[2], pages 403–407]

In the third figure, the following are proved: XXX, LLL, LXL, XLX, MMM, XMM, MXM, LML, MLM. See [Avicenna, 1971[2], pages 423–426]

Waṣfī *premises*

Avicenna introduces the *waṣfī* as one of his stratagems (along with the temporal) to find a proposition which will have a contradictory and a converse "in its own kind". (The temporal is only considered for contradiction and conversion, and for nothing else.) He takes the *waṣfī* e-proposition (say) to convert as a *waṣfī* e-proposition, and to be contradicted by a *waṣfī* i-proposition. He makes no mention of the temporality being different between the contradictories, and refers to these *waṣfī* propositions as 'absolutes'. Pure *waṣfī* premises will produce all fourteen moods in the three figures.

In *Pointers*, Avicenna also investigates *ḏātī-waṣfī* mixes, proceeding from Barbara L^wLL.

> If the major is absolute, and the time of its assertion is as long as the subject remains described by whatever it is described by, then the conclusion will be necessary, because J is B always, and it has been posited that B, as long as it is B, is A; so J is always A—so here the conclusion is necessary and the major absolute. [Avicenna, 1331[2] A.H., pages 57–58]

B LATER MODAL LOGIC

There is no purely *ḏātī* logic among the later logicians, in that *ḏātī* premises in some cases convert as *waṣfī* propositions. XA **contradictories** are all as in Avicenna, and are still isomorphic with the tables for ML propositions. **Perfect first-figure mixes** differ most significantly in that all syllogisms with possible minors fail. X **conversions** are the same as according to Avicenna, but M propositions all fail to convert. L **conversions** are all different; an L e-proposition converts as A e-proposition, and L a- and i-propositions convert as A^w i-propositions. **Temporals** are treated extensively, but none matches Avicenna's use, nor do they work as assertorics.

The *waṣfī* propositions are differentiated fully as to modality and temporality.

Absolute *waṣfī* contradictories

$A^w bj^a \not\equiv X^w bj^o$
$A^w bj^e \not\equiv X^w bj^i$
$A^w bj^i \not\equiv X^w bj^e$
$A^w bj^o \not\equiv X^w bj^a$

Absolute *waṣfī* conversions $A^w bj^e$ converts to $A^w jb^e$; $A^w bj^a$ and $A^w bj^i$ convert to $X^w jb^i$.

Syllogisms The rule for productivity in the first figure is given as follows; note that it includes the *waṣfī* propositions:

(1) The minor premise must be one of the seventeen actuals.
(2) If the major is not one of $(\Box\mathcal{C})$, $(\forall\mathcal{C})$, $(\Box\mathcal{C}\& \sim \forall\mathcal{E})$, then the mode of the conclusion is that of the major.
(3) If the major is one of these four, then the mode of the conclusion is like that of the minor except that
(a) the *restriction* of the conclusion is same as the restriction of the major [and]
(b) the conclusion is necessitated if and only if both the minor and the major are.
(4) All other moods are non-productive. [Rescher and vander Nat, 1974, page 36]

Rescher's semantics

I give here the names of the propositions, with examples and symbolic rendition, due to Rescher and vander Nat. It may prove a helpful reference for the propositions referred to throughout the text. Note that we cannot be sure that all Avicennan logicians meant the same thing by a given proposition. It is certain that Avicenna did not mean his propositions to be taken this way.

Rescher and vander Nat begin their representation by putting forward R_t as the basic operator for realization-at-time-t (which is described more fully in N. Rescher and A. Urquhart *Temporal logic* (New York and Vienna, 1971) at pages 31–32), and then use it to make the following abbreviations:

$$\begin{array}{lll} \mathcal{T}Q_x = R_{\mathcal{T}}(Q_x) & \Box\mathcal{T}Q_x = \Box R_{\mathcal{T}}(Q_x) & \Diamond\mathcal{T}Q_x = \Diamond R_{\mathcal{T}}(Q_x) \\ \mathcal{S}Q_x = R_{\mathcal{S}}(Q_x) & \Box\mathcal{S}Q_x = \Box R_{\mathcal{S}}(Q_x) & \Diamond\mathcal{S}Q_x = \Diamond R_{\mathcal{S}}(Q_x) \\ \exists Q_x = (\exists t)R_t(Q_x) & \exists\Box Q_x = (\exists t)\Box R_t(Q_x) & \exists\Diamond Q_x = (\exists t)\Diamond R_t(Q_x) \\ \forall Q_x = (\forall t)R_t(Q_x) & \forall\Box Q_x = (\forall t)\Box R_t(Q_x) & \forall\Diamond Q_x = (\forall t)\Diamond R_t(Q_x) \end{array}$$

They go on to say:

> In our symbolizations of modal propositions, *we shall systematically suppress the temporality condition ($\mathcal{E}$)* relation to the existence of the subject.
>
> Concerning the symbolic rendition of modes, we take notice of the following points. First, in adopting the symbolic machinery we have, we assume that all the usual quantificational and modal rules hold. Secondly, in the $\mathcal{E}$-modes the existence condition has been suppressed; fully stated, ($\Box\mathcal{E}$) (All A is B), for example, would be $(\forall x)[(\exists t)R_tA_x \supset (\forall t)\Box R_t(A_x \supset B_x)]$. Thirdly, the modes $\mathcal{T}$ and $\mathcal{S}$ are special time-instantiations, with regard to the existence of the subject, and accordingly, we here use '$\mathcal{T}$' and '$\mathcal{S}$' as time-constants. [Rescher and vander Nat, 1974, page 32]

With these preliminaries in hand, they then go on to offer symbolic renditions of the various a-propositions as presented in a late text [Širwānī,] as follows ([Street, 2000] gives all Arabic terms used to present and define these propositions, and in the order here presented, though note that the translation there of *waṣfī* as 'composite' is wrong, and I would now adopt the Sprenger/Rescher terms for the propositions):

The propositions

1. L: Absolute necessary ($\Box\mathcal{E}$):
   $(\forall x)[\exists A_x \supset \forall\Box B_x]$
   All men are rational of necessity (as long as they exist).

2. L^w: General conditional ($\Box\mathcal{C}$):
   $(\forall x)[\exists A_x \supset \forall\Box(A_x \supset B_x)]$
   All writers move their fingers of necessity as long as they write.

3. Absolute temporal ($\Box\mathcal{T}$):
   $(\forall x)[\exists A_x \supset \Box\mathcal{T}B_x]$
   The moon is eclipsed of necessity at the time when the earth is between it and the sun.

4. Absolute spread ($\Box\mathcal{S}$):
   $(\forall x)[\exists A_x \supset \Box\mathcal{S}B_x]$
   All men breathe of necessity at some times.

5. A: Absolute perpetual ($\forall\mathcal{E}$):
$(\forall x)[\exists A_x \supset \forall B_x]$
All men are rational perpetually (as long as they exist).

6. A^w: General conventional ($\forall\mathcal{C}$):
$(\forall x)[\exists A_x \supset \forall(A_x \supset B_x)]$
All writers move as long as they write.

7. X^w: Absolute continuing ($\exists\mathcal{C}$):
$(\forall x)[\exists A_x \supset \exists(A_x \supset B_x)]$
All writers move while they are writing.

8. Temporal absolute ($\mathcal{T}$):
$(\forall x)[\exists A_x \supset \mathcal{T}B_x]$
All writers move at the time they are writing.

9. Spread absolute ($\mathcal{S}$):
$(\forall x)[\exists A_x \supset \mathcal{S}B_x]$
All men breathe at certain times.

10. X_1: General absolute ($\exists\mathcal{E}$):
$(\forall x)[\exists A_x \supset \exists B_x]$
All men breathe (at some times).

11. Possible continuing ($\Diamond\mathcal{C}$):
$(\forall x)[\exists A_x \supset \exists\Diamond(A_x \supset B_x)]$
All writers move with a possibility while they are writing.

12. Temporal possible ($\Diamond\mathcal{T}$):
$(\forall x)[\exists A_x \supset \Diamond\mathcal{T}B_x]$
The moon is eclipsed with a possibility at the time when the earth is between it and the sun.

13. M_1: General possible ($\Diamond\mathcal{E}$):
$(\forall x)[\exists A_x \supset \exists\Diamond B_x]$
All writers move with a possibility (at some time).

14. Perpetual possible ($\Diamond\mathcal{S}$):
$(\forall x)[\exists A_x \supset \Diamond\mathcal{S}B_x]$
All men breathe with a possibility at all times.

15. Non-perpetual necessary ($\Box\mathcal{E}\& \sim \forall\mathcal{E}$):
$(\forall x)\{\exists A_x \supset [\forall\Box B_x \& \sim \forall B_x]\}$

16. Special conditional ($\Box\mathcal{C}\& \sim \forall\mathcal{E}$):
$(\forall x)\{\exists A_x \supset [\forall\Box(A_x \supset B_x)\& \sim \forall B_x]\}$

17. Temporal ($\Box\mathcal{T}\& \sim \forall\mathcal{E}$):
$(\forall x)\{\exists A_x \supset [\Box\mathcal{T}B_x \& \sim \forall B_x]\}$

18. Spread ($\Box\mathcal{S}\& \sim \forall\mathcal{E}$):
$(\forall x)\{\exists A_x \supset [\Box\mathcal{S}B_x\& \sim \forall B_x]\}$

19. Non-perpetual perpetual ($\forall\mathcal{E}\& \sim \forall\mathcal{E}$):
$(\forall x)\{\exists A_x \supset [\forall B_x\& \sim \forall B_x]\}$

20. Special conventional ($\forall\mathcal{C}\& \sim \forall\mathcal{E}$):
$(\forall x)\{\exists A_x \supset [\forall(A_x \supset B_x)\& \sim \forall B_x]\}$

21. Non-perpetual continuing absolute ($\exists\mathcal{C}\& \sim \forall\mathcal{E}$):
$(\forall x)\{\exists A_x \supset [\exists(A_x\&B_x)\& \sim \forall B_x]\}$

22. Non-perpetual temporal absolute ($\mathcal{T}\& \sim \forall\mathcal{E}$):
$(\forall x)\{\exists A_x \supset [\mathcal{T}B_x\& \sim \forall B_x]\}$

23. Non-perpetual spread absolute ($\mathcal{S}\& \sim \forall\mathcal{E}$):
$(\forall x)\{\exists A_x \supset [\mathcal{S}B_x\& \sim \forall B_x]\}$

24. X_2: Non-perpetual existential ($\exists\mathcal{E}\& \sim \forall\mathcal{E}$):
$(\forall x)\{\exists A_x \supset [\exists B_x\& \sim \forall B_x]\}$

25. Non-necessary existential ($\exists\mathcal{E}\& \sim \Box\mathcal{E}$):
$(\forall x)\{\exists A_x \supset [\exists B_x\& \sim \forall\Box B_x]\}$

26. M_2: Special possible ($\Diamond\mathcal{E}\& \sim \Box\mathcal{E}$):
$(\forall x)\{\exists A_x \supset [\exists\Diamond B_x\& \sim \forall\Box B_x]\}$

C BIBLIOGRAPHICAL NOTES

The best general introduction to the history of Arabic logic is still, sadly (given its age), [Rescher, 1964]. All of the individual logicians listed in its concluding register demand serious further study.

General bibliographical resources The best place to start for a comprehensive list of logical studies is now [Daiber, 1999], updated against *Index Islamicus* and *Bulletin de philosophie médiévale*. The bibliographies of major medieval scholars are listed in [Daiber, 1999], but note on Avicenna especially [Janssens, 1991]. A new bibliography covering the articles, books and editions of more recent years is under preparation.

Terminology There is as yet no sure guide to the technical terms used by logicians writing in Arabic; [Jabre *et al.*, 1996] is extremely helpful, though has some limitations, especially for terms relating to the modal syllogistic. [Endress and Gutas, 1992–] will ultimately provide the most important materials for a complete lexicon. Each sub-discipline within logic has its own set of technical terms. The following works include valuable glossaries: [Black, 1990; Shehaby, 1973b; Zimmermann, 1981]. [Street, 2000] is wrong in translating *waṣfī* as 'composite', but still gives important references that need to be

worked into any putative future lexicon for post-Avicennan usage. [Street,] presents the consecrated phrases by which logicians put forward propositions, proofs and so forth. Individual logicians occasionally have contingent or idiosyncratic usage. Thus especially the early logicians tend to change terminology fairly readily [Lameer, 1994]. The *išrāqī* logicians (who worked in the tradition founded by the twelfth-century logician and metaphysician, Suhrawardī) had their own terms, a number of which are decoded in [Ziai, 1990].

Translation movement, and genres The translation of each work within the Organon is treated in [Goulet, 1989–], though note the following important works which have come out since its publication: [Black, 1991] for *On Interpretation*, [Hugonnard-Roche, 1999] for demonstration, and [Aouad and Rashed, 1999] for the rhetoric.

The genres in which the logicians wrote have been studied in [Gutas, 1993], but this study really stops at the fourteenth century, and many genres which should properly should be thought logical have yet to be examined.

Short treatments (as for example on the heap and the liar paradox) have yet even to be listed as they occur through the literature.

THE TRANSLATION OF ARABIC WORKS ON LOGIC INTO LATIN IN THE MIDDLE AGES AND THE RENAISSANCE

Charles Burnett

In the Middle Ages, and again in the Renaissance, several Arabic texts on logic were translated into Latin. These included not only works by Arabic philosophers, Avicenna, Algazel, Alfarabi and Averroes, but also texts originally written in Greek, i.e. the *Organon* or corpus on logic by Aristotle, on which all Medieval and Renaissance texts were ultimately based.[1] While one can understand how Latin translations of Arabic works on mathematics, medicine, astrology and other practical sciences could useful, it is more difficult to imagine how texts on logic written in, and for, a Semitic language could make much sense in a language which is completely unrelated to it. For Aristotelian logic is, of course, very much language based. Moreover, while Latin scholars were lacking scientific texts in mathematics and medicine, they already had good translations and detailed expositions of at least the first half of the *Organon* (Aristotle's corpus of logical writings, with the Introduction —*Isagoge*—of Porphyry), made by Boethius in the early sixth century. And, when they wished to complete the *Organon*, they were able to do so by translating the texts directly from the Greek. In the mid-twelfth century James of Venice is credited with the translating the *Topics*, the *Prior* and *Posterior Analytics* with 'authentic expositions', and we have from the twelfth century, translations from Greek of the *Topics* and the *Posterior Analytics* (twice). And yet we find Gerard of Cremona translating the latter work from Arabic in the same century, and in the thirteenth century all Averroes' Middle Commentaries on the *Organon* were translated. Why was there any need to do this? This is the question that shall be addressed in this paper.

First, however, one should give some idea of the extent of the translation of logical texts from Arabic into Latin (sometimes through the intermediary of Hebrew). The earliest such translations were made in Toledo in the mid-twelfth century. First, there are those of Gerard of Cremona, the doyen of the translators working

[1] For an overview of the Latin versions of Aristotle made in the Middle Ages, see B. G. Dod, '*Aristoteles Latinus*', in *The Cambridge History of Later Medieval Philosophy*, eds N. Kretzmann, A. Kenny and J. Pinborg, Cambridge University Press, 1982, pp. 43–79.

Handbook of the History of Logic. Volume 1
Dov M. Gabbay and John Woods (Editors)

in Toledo, who lived from 1114 to 1187.[2] He translated the *Posterior Analytics*, together with two texts related to the work: the commentary by Themistius, and a work entitled 'On the syllogism' (the main topic of Aristotle's work) by Alfarabi (d. 950). Then, there are those associated with an archdeacon in Toledo Cathedral, Dominicus Gundissalinus, and a Jewish scholar called Abraham ibn Daud. Ibn Daud came to Toleda as an exile from Islamic Spain in ca. 1160, and sought the patronage of the Archbishop of Toledo that for translating the great philosophical encyclopedia of Avicenna, the *Shifa'*, giving as a specimen translation of the opening section 'On Universals'.[3] His suggestion was presumably accepted, for we soon find him collaborating with Dominicus on other parts of the *Shifa'*, including the whole of the *jumal* on logic. Dominicus collaborated with another scholar — John of Spain — in translating the 'Aims of the Philosophers' of Algazel. This was largely derivative from another philosophical compendium of Avicenna, the *Danishpazeh.* Avicenna's and Algazel's logical works were extremely popular in the Islamic world, and had virtually replaced the original works of Aristotle on which they were ultimately based.

In Cordoba in the late twelfth century, there was, however, an isolated attempt amongst a small group of philosophers to return to Aristotle. Aberrant though this was in the history of Islamic philosophy, its impact on Western philosophy was immense. For the group included Averroes (Ibn Rushd) whose Long Commentaries, Middle Commentaries and Epitomes of Aristotle's works started to become known to Latin scholars within a few years of his death in 1198. Among these works were the set of Middle Commentaries on the *Organon* (*Talkhis al-Mantiq*), which began with a commentary on Porphyry's *Isagoge*, and included (as was normal in the Arabic context) the *Rhetoric* and *Poetics*.[4] The first three texts of these Middle Commentaries — those on the *Isagoge*, the *Categories* and the *De interpretatione* — were almost certainly translated by William of Luna in Naples, in the 1220s.[5] William of Luna is likely also to have translated the Middle Commentaries on the *Prior* and *Posterior Analytics.* William must have had some connection with the new university of Naples which had been founded by the Holy Roman Emperor, Frederick II, in 1224. Frederick himself was very interested in Arabic logic, since he persuaded the Mamluk Sultan to send one of the most distinguished Islamic philosophers to his court in Palermo, Siraj ad-Din al-Urmawi, where 'he wrote a

[2]For the following paragraph I am indebted to H. Hugonnard-Roche, 'Les oeuvres de logique traduites par Gérard de Crémone', in *Gerardo da Cremona*, ed. P. Pizzamiglio, Annali della Biblioteca statale e libreria civica di Cremona, XLI, 1990, Cremona, 1992, pp. 45–56. See also C. Burnett, 'The Coherence of the Arabic-Latin Translation Program in Toledo in the Twelfth Century', *Science in Context*, 2001 (in press).

[3]See D. N. Hasse, *Avicenna's* De anima *in the Latin West*, London: The Warburg Institute, 2000, pp. 4–7.

[4]See D. Black, *Logic and Aristotle's Rhetoric and Poetics in Medieval Arabic Philosophy*, Leiden: Brill, 1990.

[5]See *Commentum Medium super libro Peri Hermeneias Aristotelis translatio Wilhelmo de Luna Attributa*, ed. R. Hissette, Leuven: Peeters, 1996, pp. 1*–4*.

book on logic for him' (We do not know what this may have been).[6] After Frederick's death in 1250, his son Manfred continued his intellectual interests, and, if 'the translator of king Manfred' is the same as Herman the German, one can see a continuation of the project for translating Averroes. For Hermann the German, on his return to Toledo, translated the commentaries on the *Rhetoric* and *Poetics* in 1256.

The interest in Arabic texts on logic continued in Spain in the thirteenth century. For the Dominican, who taught Arabic and Hebrew in the Dominican *studium* in Barcelona, Ramon Martí, quotes (apparently directly from the original Arabic), from Galen's *Book on Proof*, and Averroes's Commentary on the *Topics*,[7] and Ramon Llull, the indefatigable preacher and pamphleteer, wrote his own adaptation (from the Arabic) of Algazel's logic, in Catalan verse.[8]

At the same time as William of Luna was translating Averroes into Latin, Jacab Anatoli translated the Middle Commentaries on the *Isagoge*, *Categories*, *De interpretatione*, and *Prior* and *Posterior Analytics* into Hebrew. He too was working in Naples, and specifically thanks Frederick II for his patronage. Anatoli belonged to a family of Jewish translators, the Tibbonids, who translated other texts of Averroes. This Hebrew tradition of Averroes's works impinged on the Latin tradition from the late fifteenth century onwards, when Hebrew texts of Arabic works began to be translated into Latin. The culmination of this process is represented in the most elaborate and 'definitive' edition of the works of Aristotle in Latin, first printed with great pomp and ceremony by the Giunta brothers in Venice in 1550–52,[9] and reprinted several times thereafter. Accompanying the Latin texts of Aristotle were the commentaries of Averroes, as the title proclaims: 'Aristotelis omnia quae extant opera ... Averrois cordubensis in ea opera omnes qui ad haec usque tempora pervenere, commentarii' ('All the extant works of Aristotle ... and all the commentaries on these works of Averroes of Cordova which have survived to these times').[10] To give an example of the richness of this publication, one may list of the works included in the volumes on logic:

[6] C. Burnett, 'The "Sons of Averroes with the Emperor Frederick" and the Transmission of the Philosophical Works of Ibn Rushd', in *Averroes and the Aristotelian Tradition*, eds. G. Endress and J. A. Aertsen, Leiden: Brill, 1999, 259–99 (p. 267) and D. N. Hasse, 'Mosul and Frederick II Hohenstaufen: Notes on Aṯīraddīn Al-Abharīī and Sirāğaddīn al-Urmawī', in *Occident et Proche-Orient: Contacts scientifiques au temps des Croisades*, eds. I. Draelants, A. Tihon and B. van den Abeele, Turnhout: Brepols, pp. 145–63.

[7] The citation from the 'Book on Proof' occurs within a long passage translated by Marti from ar-Razi's *Doubts on Galen*, edited in C. Burnett, 'Encounters with Rāzī the Philosopher: Constantine the African, Petrus Alfonsi and Ramon Marti', in *Pensamiento medieval hispano: homenaje a Horacio Santiago-Otro*, Madrid: CSIC, 1998, pp. 974-92.

[8] C. Lohr, 'Raimundus Lullus' Compendium Logicae Algazelis', Ph. D., Freiburg im Breisgau, 1967 and id., '*Logica Algazelis*. Introduction and Critical Text', *Traditio*, 21, 1965, pp. 223–90.

[9] *Aristotelis Stagiritae omnia quae extant opera... Averrois Cordubensis in ea opera omnes... commentarii*, ed. G. B. Bagolini, 11 vols, Venice: Giunta, 1550–52.

[10] See C. Burnett, 'The Second Revelation of Arabic Philosophy and Science: 1492–1562' in *Islam and the Italian Renaissance*, eds A. Contadini and C. Burnett, London: The Warburg Institute, 1999, pp. 185–98.

1. Porphyry's *Isagoge*, the *Categories,* and *De interpretatione* with the Middle Commentary of Averroes and Levi Gersonides's 'supercommentary', both translated by Jacob Mantinus.

2. The *Prior Analytics*, with Averroes's *Middle Commentary* as translated by Johannes Franciscus Burana.

3. The *Posterior Analytics*, with Averroes's *Large Commentary* in three translations from Hebrew, those of Abraham de Balmes, Johannes Franciscus Burana and Jacob Mantinus. These translations are set out in three parallel columns, up to the point where Mantinus's 'golden' (*aureus*) translation finishes *morte preventus*, and continues to the end in two columns, covering *in toto* 1,136 pages! Also, Averroes's *Middle Commentary* translated by Johannes Franciscus Burana.

4. Averroes's Epitomes and Questions concerning the whole of logic, translated by Abraham de Balmes.

5. This is followed by an extraordinary series of letters on specific topics in logic, attributed without reserve to Arabic authors, also translated by Burana:

   Averroes, *Epistola de primitate praedicatorum in demonstrationibus*

   Abualkasis Benadaris (i.e. Abu'l-Qāsim ibn Idris), *Quaesita de notificatione generis et speciei.*

   Alhagiag bin Thalmus (i.e. Ibn Tumlūs), *Quaesitum*

   Abuhalkasim Mahmath ben Kasam (i.e. Abu'l-Qāsim Muḥummad ibn Q& sim),*Quaesitum*

   Abuhabad Adhadrahman ben Iohar (Abū 'Abdarraḥmān ibn Jawhār ?), *Epistole*

6. The *Topics* and *Sophistici Elenchi* with Averroes's Middle Commentaries translated by Abraham de Balmes, and an incomplete translation of the Middle Commentary on the *Topics* by Jacob Mantinus.

Thus we can see that there was considerable interest in Arabic logic, especially in the court of Frederick II, and among Aristotelian philosophers in the mid-sixteenth century. The Medieval translations of the Middle Commentaries of Averroes do not survive in many manuscripts, with the exception of that on the *Poetics*, which served instead of Aristotle's original *Poetics* throughout the Middle Ages. Nevertheless, the manuscript evidence only partially reflects the popularity of a text. For Roland Hissette, who has produced the most detailed edition of any of these commentaries so far (that on the *De interpretatione*) has shown that, although only three manuscripts survive, the work was used in 1229 by an early master in the university of Paris, Iohannes Pagus, by two Danes also studying in Paris, Martin and John of Dacia, and at least one anonymous writer; in the Renaissance

its potential readership was large, since it was included in twelve editions of Aristotle's works printed between 1483 and 1560.[11] Moreover, references in Albert the Great and brief surviving fragments show that, aside from Averroes' commentaries, Alfarabi's summaries of at least the *Categories*, and the *De interpretatione*, and his commentaries on the *Prior* and *Posterior Analytics* were known in Latin in the Middle Ages.[12] Moreover, a summary of the *Posterior Analytics* had been included in the Arabic encyclopedia known as the *Brethren of Purity* and was translated into Latin with an attribution to Alkindi.[13] But the fact that these logical texts had an Arabic origin caused problems to Latin scribes and readers.

First of all, it must be pointed out that, for the majority of translators in the Middle Ages, including Gerard, Gundissalinus, and William of Luna, an extremely literal translation of the original was the deliberate aim. The result was 'barbaric Latin', as was frequently pointed out by Renaissance humanists. (Only Llull's poetic paraphrase of Algazel's logic falls outside this extreme literality). Examples of this 'barbarous' Latin are the use of 'invenire' (the root W-J-D; literally 'to find') for 'esse' ('to be') - hence 'inventum' (literally 'the found thing') for 'the existent thing' — and 'intentio' ('ma'nā', meaning both 'meaning' and 'subject') for 'thing'.[14] One may compare the translation of William of Luna with that of Jacob Mantinus, where the relevant words are italicized:[15]

William of Luna, ed. Hissette, p. 3: Et nomen et verbum similantur *intentionibus* simplicibus, que non sunt vere neque false, et sunt ille que *inveniuntur* preter divisionem et compositionem: verbi gratia: sermo noster 'homo' et 'albedo' quoniam, cum non coniungitur ei '*invenitur*' aut 'non *invenitur*', non est adhuc neque verum neque falsum; sed significat quidem rem cui innuitur preter quod disponatur res illa per verum et falsum. Et propter hoc sermo noster 'hyrococerus' et 'acnhagaribach' non disponitur per verum neque falsum, dum non coniungitur cum eo sermo noster '*invenitur*' aut 'non *invenitur*', aut absolute aut in tempore,

[11] *Commentum Medium*, ed. Hissette, pp. 4*-7* and 19*–24*.

[12] M. Grignaschi, 'Les traductions latines des ouvrages de la logique et l'abrégé d'Alfarabi', *Archives d'histoire doctrinale et littéraire du moyen âge*, 39, 1972, pp. 41 107.

[13] Edited by A. Nagy in *Die philosophischen Abhandlungen des Ja'qub ben Ishaq al-Kindi*, in *Beiträge zur Geschichte der Philosophie des Mittelalters*, 2, 1897, pp. 41–64.

[14] A. Maierù, 'Influenze arabe e discussioni sulla natura della logica presso i latini fra XIII e XIV secolo' in *La diffusione delle scienze islamiche nel medio evo europeo*, ed. B. Scarcia Amoretti, Rome: Accademia nazionale dei Lincei, 1987, pp. 243-267.

[15] Charles Butterworth translates the Arabic as follows: 'The noun and the verb resemble uncombined ideas which are neither true nor false, that is, the ones which are taken without being combined or separated. An example of that is our saying "man" and "whiteness". For as long as "exists" or "does not exist" is not joined to it, it is neither true nor false. Instead it signifies a designated thing, without that thing having truth or falsehood attributed to it. Therefore, neither truth nor falsehood can be attributed to our saying "goat-stag" and "griffon" unless "exists" or "does not exist" is joined to it — whether without qualification or according to a particular time — and we then say "a goat-stag is existent", "a goat-stag is not existent" or "a goat-stag exists or does not exist"': C. E. Butterworth, *Averroes' Middle Commentaries on Aristotle's* Categories *and* De interpretatione, Princeton: Princeton University Press, 1983, p. 126.

et dicatur 'hyrcocervus *inventus*', 'hyrcocervus non *inventus*', aut 'hyrcocervus *invenitur*' aut 'non *invenitur*'.

Mantinus, ed. Giunta, I, fol. 68v: Nomen autem et verbum similia sunt *rebus* simplicibus, quae neque verum neque falsum significant, eo quod sunt sine aliqua compositione vel divisione, ut homo vel album, quoniam, si non additur ei '*est*' vel 'non *est*', tunc nec verum neque falsum significat. Sed significat rem individuam, sine tamen aliquo vero vel falso. Et ideo cum dicimus 'hircocervum' aut 'chimeram', neque verum neque falsum significamus, nisi addiderimus eis '*est*' vel 'non *est*' sive simpliciter vel secundum tempus, et dicamus 'hircocervus *est*' vel 'non *est*', vel 'chimera *fuit*' vel 'non *fuit*'.

Examples from Gerard of Cremona's translation of the *Posterior Analytics* show another characteristic of the Greek–Arabic–Latin transmission. Arabic cannot form compound words. So *hypothesis* becomes 'aṣl mawḍū' ('placed root'), which naturally becomes in Gerard's translation 'radix posita', and *enthymema* becomes 'qiyās muḍmar' 'secret/covered syllogism' which yields 'syllogismus occultus' (This use of 'syllogismus' is obviously confusing).[16]

As a result of the success of the translating-enterprise in Toledo, this literary style, including the use of the same Latin translations of the same Arabic terms, was employed for all translations from Arabic. It is evident that Scholastic philosophers of the Middle Ages were accustomed to the style and the peculiar meanings of the words, to an extent that we find difficult to appreciate. Scholars such as Albertus Magnus and Thomas Aquinas have a remarkably accurate understanding of the doctrines of Averroes, Avicenna and the other Arabic philosophers, even though they only knew them through Latin translations, and they would differentiate (for example) between the instances where 'inventum' meant 'found' and where it simply meant 'existing', or 'intentio' meant 'intentio' or simply 'a thing, the subject'. Nevertheless, there are aspects of the Latin translation which would have confused, or would have been unintelligible even to them. One is in the same passage quoted above. For the mythical beast 'hyrcocervus' ('goat-stag') mentioned in Aristotle's text, Averroes, quite sensibly, added the nearest equivalent in Arabic mythology: the "anqā' mughrib' — 'the phoenix/griffon that excites the curiosity'. William of Luna simply transiliterates this unintelligible word into Latin ('anchagaribach' — in fact, suggesting that he read a variant not attested in the Arabic MSS: "anqā' gharība' — 'the strange phoenix'), which soon became corrupted in the Latin manuscripts and editions: 'anchagaribach, anquaganba, anquagauba, auquagariba etc.' Mantinus, however, does for his Latin audience what Averroes had done for his Arabic readers: he finds an equivalent which is familiar to them, in this case, the chimera.

[16]For more details see Hugonnard-Roche, 'Les oeuvres de logique', pp. 50–51.

Another example where a literal translation from Arabic produced incomprehension is a discussion of the use of cases in the noun:[17]

William of Luna, ed. Hissette, p. 7: Et nomen etiam, cum genitivatur aut accusativatur aut mutatur mutatione alia huiusmodi, non dicitur nomen absolute, sed nomen declinatum...differentia est inter declinatum et non declinatum (et illud est in casu 'u' in lingua arabica)...

Mantinus, ed. Giunta, I, fols. 69v-70r: Nomen preterea cum est in genitivo, vel accusativo, vel alio casu, vel mutatur aliqua alia simili mutatione, tunc non dicitur simpliciter nomen, sed nomen casuale...Interest tamen inter obliquum et non obliquum nomen, ut in lingua Arabic patet ...

Here, William of Luna probably produced an accurate translation of the Arabic text ('the case ⟨ending in⟩ 'u'), but this already confused the scribes, who wrote 'in caū' (*vel. sim.*), and Mantinus simply glossed over the phrase.

The problem with the Latin transmission of the Middle Commentaries of Averroes in general can be summarised as follows:

Averroes' intention in the Middle Commentaries is to paraphrase Aristotle's text (without directly quoting it), in a way that both brings out the logical sequence of Aristotle's arguments (hence his use of the 'Porphyrian tree' for the arrangement of the subject matter in these commentaries), and makes the subject matter intelligible to an Arabic audience.[18]

The literal Latin translations of the Middle Commentaries make no concession to their audience. In some Arabic-Latin translations the translator adds a marginal gloss explaining the meaning of certain things specific to the Arabic language and culture, while not changing the text itself. There is little evidence that this was done by William of Luna.

The most obvious example of this mode of transmission can be seen in the case of the Middle Commentary on the *Poetics*, which, as mentioned above, served instead of Aristotle's *Poetics* for the entire Middle Ages. Aristotle had included many examples of Greek poetry to illustrate his text. Averroes systematically replaced these examples with well-chosen illustrations from Arabic poetry. Hermann the German, when translating Averroes's text into Latin, did *not* substitute examples from Latin poetry, but faithfully translated all the excerpts from Arabic poems

[17] Butterworth (p. 128) translates: 'Moreover, when a noun is put into the accusative or genitive case or altered in some similar way, it is not said to be a noun in an absolute sense, but an inflected noun...The difference between the inflected noun and the uninflected noun – which, in the speech of the Arabs, is the noun in the nominative case (literally: the case ending with 'u')...'

[18] See J. Puig Montada, 'Averroes' Commentaries on Aristotle: to Explain and to Interpret', in the proceedings of *Il commento filosofico nell'Occidente latino (saec. XIII–XV), Firenze-Pisa, 19-21 Oct. 2000* (in press).

into Latin, thus unintentionally producing the only Latin anthology of Arabic poetry in the Middle Ages.[19]

Moreover, in respect to recovering the text of Greek works on logic, the Arabic can be seriously misleading. For example, the Arabic texts of the *Posterior Analytics* and the commentary on it by Themistius have been shown by Hugonnard-Roche to be paraphrases of the Greek texts, and indeed it is this paraphrase that was also used by Averroes. One result of this paraphrase was a distortion of Aristotle's own conception of the role of logic/dialectic in respect to the science.[20] According to the teaching of Aristotle, logic examines but does not prove the first principles. The Arabic version of the *Posterior Analytics* that Gerard translated, on the contrary, stated that the *ars dialectica* attempts to demonstrate the common propositions in each science. So, given the ambiguities introduced by Latin translations of Arabic logical texts in the Middle Ages, what were the reasons for translating them in the first place?

The question can be answered most easily, perhaps, in the case of Gerard of Cremona. In the context of Toledo in the twelfth and, indeed, thirteenth century, there was no question of translating anything from Greek. Rather, Arabic culture was so dominant, and so advanced in the area, that the task was simply to replicate, as far as possible, that culture in Latin. Moreover, both Gerard of Cremona and Gundissalinus had a model-curriculum on which to base their replication: i.e. the *Classification (or Enumeration) of the Sciences* of Alfarabi, which both scholars translated. Alfarabi not only provided a template for the subjects to be covered in a course of 'philosophy' in the Aristotelian sense, but also referred to the textbooks to be used in that course. The second chapter of Alfarabi's book (after a chapter on grammar) was on logic ('dialectica' Gerard; 'logica' Gundissalinus), and in it he systematically went through the subject-matter of the *Isagoge*, the *Categories*, etc. finishing with the eight books of the *Topics*, and the *Prior* and *Posterior Analytics*. For Alfarabi logic is a necessary propaedeutic to the other divisions of philosophy dealt with in the work: *Mathematics*, *Physics*, *Metaphysics* and the moral sciences. Gerard translated Arabic texts in all these subjects, and there is good evidence that he taught 'Arabic science in Latin' (as one could say) in Toledo, where he is referred to as 'dictus Magister'–i.e. 'the Teacher *par excellence*'.

Gerard would have known the necessity for logic as a basis for the study of mathematics (in particular, geometry) from another text which he translated, and which was well-known to his students (*socii*), who quote from him, namely Aḥmad ibn Yūsuf ibn Ibrāhīm al-Daya's *Letter on Ratio and Proportion*.[21] The subject of the text is geometry, but ibn Yūsuf starts with a long preface, taking the form

[19]See W. F. Boggess, 'Hermannus Alemannus' Latin Anthology of Arabic Poetry', *Journal of the American Oriental Society*, 88, 1968, pp. 657-70.

[20]Hugonnard-Roche, 'Les oeuvres de logique', pp. 52–4.

[21]Ibn Y⊃suf lived in Cairo in the late ninth and early tenth century, and served the Tulunid Sultans there.

of a conversation between geometricians of different kinds, supposedly infront of the Prince that he serves. The whole point of the conversation is to demonstrate that logic is a necessary propaedeutic to geometry. Having practical knowledge of mathematics, or knowing the theoretical texts off by heart, is not sufficient; one must understand the *principles* of the art, which can only be gained by having recourse to a higher art, namely logic.[22]

It is in this context that one must see the endeavour of Gerard of Cremona himself. He had no need to translate the texts of the 'old logic' which had been known since the translations of Boethius, but he felt compelled to translate from the available Arabic version the *Posterior Analytics*, especially since it was particularly relevant to the arguments used in science, dealing with the different kinds of syllogism, the rules for different kinds of argumentation, and in particular the rules for demonstrative argument, whose importance in the work is indicated by the Arabic title for the *Posterior Analytics* —' kitā al-Burhān' for which Gerard gave the literal translation *De demonstrationibus*. In addition, however, Gerard translated, still from Arabic, the commentary of the late-fourth-century Greek philosopher, Themistius, on the *Posterior Analytics*,[23] as a help for understanding Aristotle's notoriously difficult text. The bibliography of Gerard's works composed just after his death in 1187 also mention 'Alfarabi *De syllogismo*', which has not been identified in Latin, but is presumably Alfarabi's commentary (or part of such a commentary) on the *Posterior Analytics*. The priority of logic in a curriculum of philosophy is further indicated by the fact that the *socii* of Gerard, in compiling a list of his works after his death, put logic first.

For William of Luna, unfortunately, we can only guess why he undertook the translation of Averroes' Middle Commentaries, since he wrote no dedications, and we have no references to his activities. All that I can suggest is that, after it had become known to Jewish and Christian scholars that Averroes had paraphrased (in the Middle Commentaries) and written word-for-word expositions (in the Large Commentaries) on the whole range of Aristotle's works, these scholars felt it important to put *all* the Commentator's works into Hebrew and Latin. This was appropriate, especially since the expositions of Aristotle of the ancient Greek philophers were only known fragmentarily (Byzantine philosophers were, at the same time as Averroes, attempting to fill the gaps), and Averroes was known to use as the starting point of his own commentary, that of Alexander of Aphrodisias, and also brought into discussion the comments of other Greek philosophers, such as Themistius and Philoponus. Thus we can see the translation of Averroes's *Middle Commentaries* of the logical texts as part of a much larger enterprise, and, indeed, the manuscript and printing edition of these works show that they travelled exclusively with other commentaries by Averroes. In addition to this, we can

[22]See C. Burnett, 'Dialectic and Mathematics according to Aḥmad ibn Yūsuf: A Model for Gerard of Cremona's Programme of Translation and Teaching?" in *Langage, sciences, philosophie au xiie siècle*, ed. J. Biard, Paris: Librairie philosophique J. Vrin, 1999, pp. 83 92.

[23]A direct translation of this work from the Greek was not made until 1481.

point to a particularly strong interest in Islamic logic in Frederick II's entourage, encouraged by the patronage and example of the emperor himself.[24]

In the Renaissance, we can see an even greater desire for comprehensiveness, when all the commentaries of Averroes on Aristotle's *Organon*, and much Arabic logic besides, were included in the Giunta edition of 1550-52. Only in recent times has such an intensive interest in texts on Arabic logic been revived.

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[24]See p. *** above.

INDEX